

# Preface

According to a widely held view, chaos is intimately linked to nonlinearity. It is usually taken to be self-evident that a linear system behaves in a predictable manner.

However, as early as 1929, G.D. Birkhoff obtained an example of a linear operator that possesses an important ingredient of chaos: the existence of a dense orbit. Later, G.R. MacLane (1952) found the same phenomenon for the differentiation operator, which, after all, is the fundamental operator in analysis. And S. Rolewicz (1969) showed that not only nonlinear shifts but also linear shifts can have dense orbits. Motivated by these sporadic examples, researchers began in the nineteen-eighties to study the dynamical properties of general linear operators; henceforth, operators with a dense orbit were called hypercyclic. As a first important result a useful condition for hypercyclicity, the so-called Hypercyclicity Criterion, was obtained.

A further decisive step was taken by G. Godefroy and J.H. Shapiro (1991). Not only did they come up with whole new classes of hypercyclic operators, they also proposed to accept Devaney's definition of (nonlinear) chaos as the right definition for linear chaos: a linear operator is chaotic if it has a dense orbit, it has a dense set of periodic points and it has sensitive dependence on initial conditions. They then showed that many linear operators are chaotic, including the three classical operators of Birkhoff, MacLane and Rolewicz.

The fact that chaos for linear systems has only been discovered recently is easily explained: as Rolewicz showed, hypercyclicity, and hence also linear chaos, requires an infinite-dimensional setting.

Over the last quarter of a century, the study of hypercyclic and chaotic operators has turned into a fascinating and very active research area. It has produced an astounding number of deep and beautiful results. As representative examples we mention here only Ansari's theorem that every power of a hypercyclic operator is hypercyclic, the Ansari–Bernal theorem that every infinite-dimensional separable Banach space supports a hypercyclic operator, Grivaux's theorem that every Hilbert space operator is the sum of two

chaotic operators, and the Bourdon–Feldman theorem that any orbit that is somewhere dense is (everywhere) dense.

It seems fair to say that while research in linear dynamics is still expanding in both depth and breadth, the foundations have reached a certain stage of maturity. At the same time the basic ideas as well as the applications of the field have a broad appeal also to nonspecialists.

It is therefore our aim to make the theory of hypercyclic operators and linear chaos accessible to a wider audience. The book is aimed at advanced undergraduate or beginning graduate students, both as a basis for a lecture course and for self-study. We have strived at a self-contained exposition. Each chapter contains a large number of exercises and ends with a section that gives references and directs the reader to further literature.

We have tried to keep the necessary prerequisites for reading this book to a minimum. Since the concept of a hypercyclic operator requires both a topological and a linear structure, the reader is supposed to be familiar with metric spaces (up to the Baire category theorem) and with the basic theory of Hilbert and Banach spaces, as it is often presented in advanced undergraduate courses on analysis. Moreover, since many examples in the theory are given by operators on spaces of holomorphic functions the reader is also expected to have had an introductory course on complex analysis. Additional, more advanced tools that are only needed occasionally will be provided in the two appendices.

The book is divided into two parts. Part I presents an introduction to the dynamics of linear operators. Its chapters form a unity and are best studied in that order. In contrast, Part II covers selected topics from linear dynamics. Its chapters are largely independent so that they can be read in an arbitrary order. An occasional cross reference should pose no problem.

More specifically, Chapter 1 introduces the reader to the fundamental concepts of the theory of (not necessarily linear) dynamical systems. Its highlights are the Birkhoff transitivity theorem, which is of fundamental importance for all that follows, and a close study of the various concepts of maps with complicated behaviour, including chaotic maps. In Chapter 2, the notions and results from the first chapter are revisited in the context of linearity. Among other things it is proved that the operators of Birkhoff, MacLane and Rolewicz are chaotic, and that linear dynamics can be as complicated as nonlinear dynamics. We begin the chapter with an introduction to a straightforward generalization of Banach spaces, the so-called Fréchet spaces; they provide the setting for some important chaotic operators. Chapter 3 presents several criteria for hypercyclicity and chaos, in increasing order of sophistication. It culminates in the Hypercyclicity Criterion, which is discussed in detail. In Chapter 4, some important classes of hypercyclic and chaotic operators are described: weighted shift operators on sequence spaces, differential and composition operators on spaces of holomorphic functions, and adjoint multiplication operators. In addition to the shift operators, which are studied throughout the book, the reader may want to concentrate on one or two ad-

ditional classes, depending on his or her own personal preference. In Chapter 5 we discuss the spectral properties of hypercyclic and chaotic operators. As an application we derive properties that preclude hypercyclicity or chaos. Finally, Chapter 6 presents some of the deepest, most beautiful and most useful results from linear dynamics. It contains, among other things, Ansari's theorem on the powers of hypercyclic operators, the Bourdon–Feldman theorem on somewhere dense orbits and the León–Müller theorem on the hypercyclicity of unimodular multiples of hypercyclic operators.

In the second part, Chapter 7 discusses the continuous analogue of hypercyclic and chaotic operators in the form of semigroups. While the theories run parallel in great parts, hypercyclic and chaotic semigroups have important applications to partial differential equations. In Chapter 8 we obtain, among other things, the Ansari–Bernal theorem on the existence of hypercyclic operators. We also discuss here the richness of the set of hypercyclic operators. The contents of Chapter 9 are motivated by recent work on the application of ergodic theory to linear dynamics. While the technical difficulties involved prevent us from studying these tools here, we will discuss a new concept that has come out of these investigations, the frequently hypercyclic operators. Chapter 10 is devoted to the question of whether there is, for a given operator, an infinite-dimensional closed subspace all of whose nonzero vectors are hypercyclic, while Chapter 11 studies the existence of common hypercyclic vectors for (uncountable) families of operators. The final Chapter 12 treats hypercyclicity and linear chaos in their most natural (and most general) setting, the topological vector spaces. After a brief introduction to such spaces we revisit many of the results previously obtained in the book and show that they hold in great generality.

At this point it seems important to add a disclaimer concerning our strategy for attaching names to theorems. In keeping with the usual practice in mathematics we have attributed results to the author(s) who first proved them. But the form in which the result is presented in the book may well be due to additional contributions from further authors. The reader is advised to consult the relevant sources and comments section for complete references.

We would like to say a few words about the differences with a recent monograph by F. Bayart and É. Matheron [44] on the topic of linear dynamics. While their book is intended to be accessible to readers with a reasonable background in functional analysis at the graduate level, we have tried to make our text more basic and self-contained, so that students should be able to follow it at an earlier stage of their studies without additional material. In a certain sense, the two books are complementary. While we cover the foundations and the main body of linear dynamics in detail, Bayart and Matheron proceed to present some technically demanding topics like the counterexamples to Herrero's problem due to De la Rosa–Read and Bayart–Matheron, the applications of ergodic theory to linear dynamics, or Read-type operators for which every nonzero vector is hypercyclic.

The starting point of this book was a mini-workshop on hypercyclicity and linear chaos at Oberwolfach in August 2006, where we agreed that a book on this topic ought to be written. Since then, stays at the Berlin Mathematical School/Technische Universität Berlin (2007), the Centre International de Rencontres Mathématiques in Marseille (2008, 2009, 2010), and the Universitat Politècnica de València/Universitat de València (2008), made it possible, with their support and their perfect working conditions, that the book was written. We are, in particular, grateful to Günter M. Ziegler (Berlin), Pascal Chossat (CIRM) and Manuel Maestre (València) for the invitations to their institutions.

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