

Preface

Thirty years ago mathematical, as opposed to applied numerical, computation was difficult to perform and so relatively little used. Three threads changed that:

- The emergence of the personal computer, identified with the iconic Macintosh but made ubiquitous by the IBM PC.
- The discovery of fiber-optics and the consequent development of the modern internet culminating with the foundation of the World Wide Web in 1989 made possible by the invention of hypertext earlier in the decade.
- The building of the *Three Ms*: *Maple*, *Mathematica*, and MATLAB. Each of these is a complete mathematical computation workspace with a large and constantly expanding built-in “knowledge base”. The first two are known as “computer algebra” or “symbolic computation” systems, sometimes written *CAS*. They aim to provide exact mathematical answers to mathematical questions such as what is

$$\int_{-\infty}^{\infty} e^{-x^2} dx,$$

what is the real root of $x^3 + x = 1$, or what is the next prime number after 1,000,000,000? The answers, respectively, are

$$\sqrt{\pi}, \quad \frac{\sqrt[3]{108 + 12\sqrt{93}}}{6} - \frac{2}{\sqrt[3]{108 + 12\sqrt{93}}}, \quad \text{and } 1,000,000,007.$$

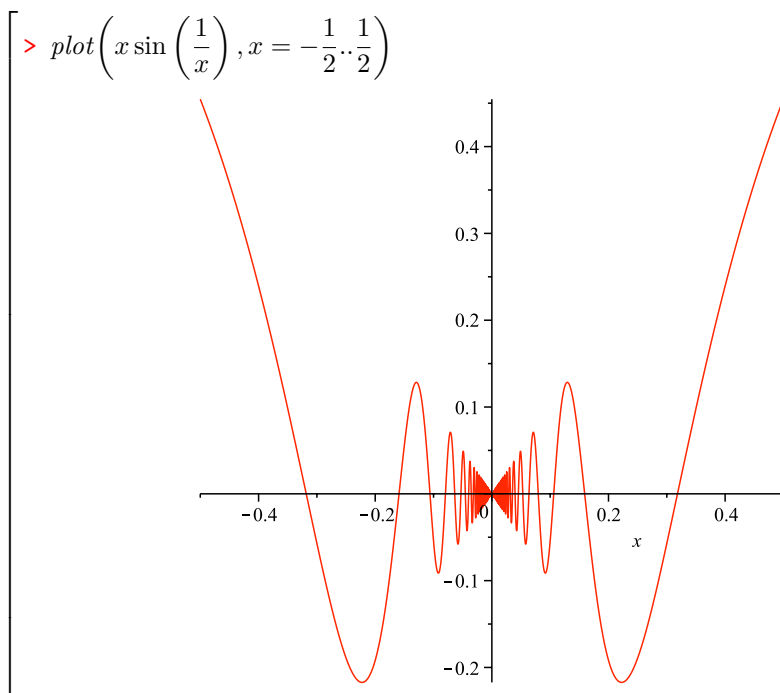
The third M is primarily numerically based. The distinction, however, is not a simple one. Moreover, more and more modern mathematical computation requires a mixture of so-called *hybrid* numeric/symbolic computation and also relies on significant use of geometric, graphic and, visualization tools. It is even possible to mix these technologies, for example, to make use of MATLAB through a *Maple* interface; see also [6]. MATLAB is the preferred tool of many engineers and other scientists who need easy access to efficient numerical computation.

Of course each of these threads relies on earlier related events and projects, and there are many other open source and commercial software packages. For example, *Sage* is an open-source CAS, *GeoGebra* an open-source interactive geometry package, and *Octave* is an open-source counterpart of MATLAB. But this is not the place to discuss the merits and demerits of open source alternatives. For many purposes *Mathematica* and *Maple* are interchangeable as adjuncts to mathematical learning. We propose to use the latter.

After reading this book, you should find it easy to pick up the requisite skills to use *Mathematica* [14] or MATLAB.

Many introductions to computer packages aim to teach the *syntax* (rules and structure) and *semantics* (meaning) of the system as efficiently as possible [7, 8, 9, 13]. They assume one knows why one wishes to learn such things. By contrast, we intend to persuade that *Maple* and other like tools are worth knowing assuming only that one wishes to be a mathematician, a mathematics educator, a computer scientist, an engineer, or scientist, or anyone else who wishes/needs to use mathematics better. We also hope to explain how to become an experimental mathematician while learning to be better at proving things. To accomplish this our material is divided into three main chapters followed by a postscript. These cover the following topics:

- **Elementary number theory.** Using only mathematics that should be familiar from high school, we introduce most of the basic computational ideas behind *Maple*. By the end of this chapter the hope is that the reader can learn new features of *Maple* while also learning more mathematics.
- **Calculus of one and several variables.** In this chapter we revisit ideas met in first-year calculus and introduce the basic ways to plot and explore functions graphically in *Maple*. Many have been taught not to trust pictures in mathematics. This is bad advice. Rather, one has to learn how to draw trustworthy pictures.



- **Introductory linear algebra.** In this chapter we show how much of linear algebra can be animated (i.e. brought to life) within a computer algebra system. We suppose the underlying concepts are familiar, but this is not necessary. One of the powerful attractions of computer-assisted mathematics is that it allows for a lot of “learning while doing” that may be achieved by using the help files in the system and also by consulting Internet mathematics resources such as *MathWorld*, *PlanetMath* or *Wikipedia*.

- **Visualization and interactive geometric computation.** Finally, we explore more carefully how visual computing [10, 11] can help build mathematical intuition and knowledge. This is a theme we will emphasize throughout the book.

Each chapter has three main sections forming that chapter's core content. The fourth section of each chapter has exercises and additional examples. The final section of each chapter is entitled "Further Explorations," and is intended to provide extra material for more mathematically advanced readers.

A more detailed discussion relating to many of these brief remarks may be followed up in [2, 3, 4] or [5], and in the references given therein.

The authors would like to thank Shoham Sabach and James Wan for their help proofreading preliminary versions of this book.

Additional Reading and References

We also supply a list of largely recent books at various levels that the reader may find useful or stimulating. Some are technical and some are more general.

1. George Boros and Victor Moll, *Irresistible Integrals*, Cambridge University Press, New York, 2004.
2. Jonathan M. Borwein and Peter B. Borwein, *Pi and the AGM: A Study in Analytic Number Theory and Computational Complexity*, John Wiley & Sons, New York, 1987 (Paperback, 1998).
3. Christian S. Calude, *Randomness and Complexity, from Leibniz To Chaitin*, World Scientific Press, Singapore, 2007.
4. Gregory Chaitin and Paul Davies, *Thinking About Gödel and Turing: Essays on Complexity, 1970-2007*, World Scientific, Singapore, 2007.
5. Richard Crandall and Carl Pomerance, *Prime Numbers: A Computational Perspective*, Springer, New York, 2001.
6. Philip J. Davis, *Mathematics and Common Sense: A Case of Creative Tension*, A.K. Peters, Natick, MA, 2006.
7. Stephen R. Finch, *Mathematical Constants*, Cambridge University Press, Cambridge, UK, 2003.
8. Marius Giaguinto, *Visual Thinking in Mathematics*, Oxford University, Oxford, 2007.
9. Ronald L. Graham, Donald E. Knuth, and Oren Patashnik, *Concrete Mathematics*, Addison-Wesley, Boston, 1994.
10. Bonnie Gold and Roger Simons (Eds.), *Proof and Other Dilemmas: Mathematics and Philosophy*, Mathematical Association of America, Washington, DC, in press, 2008.
11. Richard K. Guy, *Unsolved Problems in Number Theory*, Springer-Verlag, Heidelberg, 1994.
12. Reuben Hersh, *What Is Mathematics Really?* Oxford University Press, Oxford, 1999.
13. J. Havil, *Gamma: Exploring Euler's Constant*, Princeton University Press, Princeton, NJ, 2003.
14. Steven G. Krantz, *The Proof Is in the Pudding: A Look at the Changing Nature of Mathematical Proof*, Springer, New York, 2010.
15. Marko Petkovsek, Herbert Wilf, and Doron Zeilberger, *A=B*, A.K. Peters, Natick, MA, 1996.

16. Nathalie Sinclair, David Pimm, and William Higginson (Eds.), *Mathematics and the Aesthetic. New Approaches to an Ancient Affinity*, CMS Books in Math, Springer-Verlag, New York, 2007.
17. J. M. Steele, *The Cauchy-Schwarz Master Class*, Mathematical Association of America, Washington, DC, 2004.
18. Karl R. Stromberg, *An Introduction to Classical Real Analysis*, Wadsworth, Belmont, CA, 1981.
19. Richard P. Stanley, *Enumerative Combinatorics*, Volumes 1 and 2, Cambridge University Press, New York, 1999.
20. Terence Tao, *Solving Mathematical Problems*, Oxford University Press, New York, 2006.
21. Nico M. Temme, *Special Functions, an Introduction to the Classical Functions of Mathematical Physics*, John Wiley, New York, 1996.
22. Fernando R. Villegas, *Experimental Number Theory*, Oxford University Press, New York, 2007.

Finally many useful links are maintained by the authors of *Mathematics by Experiment* [3] at www.experimentalmath.info.

Jonathan Borwein
Matthew Skerritt
May 21, 2011



<http://www.springer.com/978-1-4614-0121-6>

An Introduction to Modern Mathematical Computing
With Maple™

Borwein, J.; Skerritt, M.P.

2011, XVI, 216 p., Hardcover

ISBN: 978-1-4614-0121-6