

# Contents

<b>1</b>	<b>Overview</b>	1
1.1	General comments and historical facts	7
<b>2</b>	<b>Elementary Facts about Baire and Baire-Type Spaces</b>	13
2.1	Baire spaces and Polish spaces	13
2.2	A characterization of Baire topological vector spaces	18
2.3	Arias de Reyna–Valdivia–Saxon theorem	20
2.4	Locally convex spaces with some Baire-type conditions	24
2.5	Strongly realcompact spaces $X$ and spaces $C_c(X)$	36
2.6	Pseudocompact spaces, Warner boundedness and spaces $C_c(X)$	46
2.7	Sequential conditions for locally convex Baire-type spaces	56
<b>3</b>	<b>K-analytic and Quasi-Suslin Spaces</b>	63
3.1	Elementary facts	63
3.2	Resolutions and K-analyticity	71
3.3	Quasi-(LB)-spaces	82
3.4	Suslin schemes	91
3.5	Applications of Suslin schemes to separable metrizable spaces	93
3.6	Calbrix–Hurewicz theorem	101
<b>4</b>	<b>Web-Compact Spaces and Angelic Theorems</b>	109
4.1	Angelic lemma and angelicity	109
4.2	Orihuela’s angelic theorem	111
4.3	Web-compact spaces	113
4.4	Subspaces of web-compact spaces	116
4.5	Angelic duals of spaces $C(X)$	118
4.6	About compactness via distances to function spaces $C(K)$	120
<b>5</b>	<b>Strongly Web-Compact Spaces and a Closed Graph Theorem</b>	137
5.1	Strongly web-compact spaces	137
5.2	Products of strongly web-compact spaces	138
5.3	A closed graph theorem for strongly web-compact spaces	140

<b>6</b>	<b>Weakly Analytic Spaces</b>	143
6.1	A few facts about analytic spaces	143
6.2	Christensen's theorem	149
6.3	Subspaces of analytic spaces	155
6.4	Trans-separable topological spaces	157
6.5	Weakly analytic spaces need not be analytic	164
6.6	More about analytic locally convex spaces	167
6.7	Weakly compact density condition	168
6.8	More examples of nonseparable weakly analytic tvs	174
<b>7</b>	<b>K-analytic Baire Spaces</b>	183
7.1	Baire tvs with a bounded resolution	183
7.2	Continuous maps on spaces with resolutions	187
<b>8</b>	<b>A Three-Space Property for Analytic Spaces</b>	193
8.1	An example of Corson	193
8.2	A positive result and a counterexample	196
<b>9</b>	<b>K-analytic and Analytic Spaces <math>C_p(X)</math></b>	201
9.1	A theorem of Talagrand for spaces $C_p(X)$	201
9.2	Theorems of Christensen and Calbrix for $C_p(X)$	204
9.3	Bounded resolutions for $C_p(X)$	215
9.4	Some examples of K-analytic spaces $C_p(X)$ and $C_p(X, E)$	230
9.5	K-analytic spaces $C_p(X)$ over a locally compact group $X$	231
9.6	K-analytic group $X_p^\wedge$ of homomorphisms	234
<b>10</b>	<b>Precompact Sets in <math>(LM)</math>-Spaces and Dual Metric Spaces</b>	239
10.1	The case of $(LM)$ -spaces: elementary approach	239
10.2	The case of dual metric spaces: elementary approach	241
<b>11</b>	<b>Metrizability of Compact Sets in the Class <math>\mathfrak{G}</math></b>	243
11.1	The class $\mathfrak{G}$ : examples	243
11.2	Cascales–Orihuela theorem and applications	245
<b>12</b>	<b>Weakly Realcompact Locally Convex Spaces</b>	251
12.1	Tightness and quasi-Suslin weak duals	251
12.2	A Kaplansky-type theorem about tightness	254
12.3	K-analytic spaces in the class $\mathfrak{G}$	258
12.4	Every WCG Fréchet space is weakly K-analytic	260
12.5	Amir–Lindenstrauss theorem	266
12.6	An example of Pol	271
12.7	More about Banach spaces $C(X)$ over compact scattered $X$	276
<b>13</b>	<b>Corson's Property <math>(C)</math> and Tightness</b>	279
13.1	Property $(C)$ and weakly Lindelöf Banach spaces	279
13.2	The property $(C)$ for Banach spaces $C(X)$	284

<b>14</b>	<b>Fréchet–Urysohn Spaces and Groups</b>	289
14.1	Fréchet–Urysohn topological spaces	289
14.2	A few facts about Fréchet–Urysohn topological groups	291
14.3	Sequentially complete Fréchet–Urysohn spaces are Baire	296
14.4	Three-space property for Fréchet–Urysohn spaces	299
14.5	Topological vector spaces with bounded tightness	302
<b>15</b>	<b>Sequential Properties in the Class <math>\mathfrak{G}</math></b>	305
15.1	Fréchet–Urysohn spaces are metrizable in the class $\mathfrak{G}$	305
15.2	Sequential $(LM)$ -spaces and the dual metric spaces	311
15.3	$(LF)$ -spaces with the property $C_3^-$	320
<b>16</b>	<b>Tightness and Distinguished Fréchet Spaces</b>	327
16.1	A characterization of distinguished spaces	327
16.2	$\mathfrak{G}$ -bases and tightness	334
16.3	$\mathfrak{G}$ -bases, bounding, dominating cardinals, and tightness	338
16.4	More about the Wulbert–Morris space $C_c(\omega_1)$	349
<b>17</b>	<b>Banach Spaces with Many Projections</b>	355
17.1	Preliminaries, model-theoretic tools	355
17.2	Projections from elementary submodels	361
17.3	Lindelöf property of weak topologies	364
17.4	Separable complementation property	365
17.5	Projectional skeletons	369
17.6	Norming subspaces induced by a projectional skeleton	375
17.7	Sigma-products	380
17.8	Markushevich bases, Plichko spaces and Plichko pairs	383
17.9	Preservation of Plichko spaces	388
<b>18</b>	<b>Spaces of Continuous Functions over Compact Lines</b>	395
18.1	General facts	395
18.2	Nakhmanov’s theorem	398
18.3	Separable complementation	399
<b>19</b>	<b>Compact Spaces Generated by Retractions</b>	405
19.1	Retractive inverse systems	405
19.2	Monolithic sets	409
19.3	Classes $\mathcal{R}$ and $\mathcal{RC}$	411
19.4	Stability	412
19.5	Some examples	415
19.6	The first cohomology functor	418
19.7	Compact lines	422
19.8	Valdivia and Corson compact spaces	425
19.9	Preservation theorem	432
19.10	Retractional skeletons	434
19.11	Primarily Lindelöf spaces	438
19.12	Corson compact spaces and WLD spaces	440
19.13	A dichotomy	442

19.14	Alexandrov duplications . . . . .	446
19.15	Valdivia compact groups . . . . .	448
19.16	Compact lines in class $\mathcal{R}$ . . . . .	451
19.17	More on Eberlein compact spaces . . . . .	456
<b>20</b>	<b>Complementably Universal Banach Spaces</b> . . . . .	<b>467</b>
20.1	Amalgamation lemma . . . . .	467
20.2	Embedding-projection pairs . . . . .	469
20.3	A complementably universal Banach space . . . . .	471
<b>References</b>	. . . . .	<b>475</b>
<b>Index</b>	. . . . .	<b>491</b>

Descriptive Topology in Selected Topics of Functional  
Analysis

Kąkol, J.; Kubiś, W.; López-Pellicer, M.

2011, XII, 496 p., Hardcover

ISBN: 978-1-4614-0528-3