

Preface by Arne Næss

This introduction to The General Theory of Relativity and its mathematics is written for all those, young and old, who lack or have forgotten the necessary mathematical knowledge to cope with already published introductions. Some of these introductions seem, at the start to require only moderately much mathematics. Very soon, however, there are frightful ‘jumps’ in the exposition, or suddenly new concepts or notations appear as if nearly self evident. The present text starts at a lower level than any other, and leads the reader slowly and faithfully all the way to the heart of relativity: Einstein’s field equations.

Who are those who seriously desire to get acquainted with General Relativity, but have practically no mathematical knowledge? There are tens of thousands of them, thanks to the great general interest in relativity, quantum physics and cosmology of every profession, including those with education only in the humanities.

Slowly many of these interested persons understand the truth of what one of the last Century’s most brilliant physicists and populariser, Sir Arthur Eddington, told us already in the 1920s: that strictly speaking, mathematical physics cannot be understood through popularisations. Mathematics plays a role that is not merely instrumental, like cobalt chemicals for the paintings of Rembrandt. Mathematical concepts enter in an essential way, and readers of popularisations are mislead. Their intelligence is insulted and bullied when they ask intelligent questions that their popular text cannot answer except by absurdities. The honest readers may end up in a quagmire of paradoxes, and may get the usually false idea that there is something wrong with their intelligence. What they read they consider beyond their intellectual grasp.

It is a widespread expectation that a mathematical understanding of general relativity involves difficult calculations. Actually, coping with the few somewhat lengthy strings of symbols in this text may be felt as a relief from abstract thinking. What takes time is the thorough understanding of the relations between a few basic concepts. They require close, repeated attention and patient work. This surprising feature to some degree justifies that we have not included exercises in the text. To

be honest, some should have been included, but they would have been of a rather strange kind: exercises in articulating conceptual relations.

But what about the formidable calculations one may read about in popularisations? They affect applications, for instance particular solutions of Einstein's field equations. Examples of such calculations are found in appendices B and C. Even if the reader should not expect to be an operator of relativistic physics, he or she should be well acquainted with it having read this book. We venture to suggest that the understanding acquired by the reader may be deeper than what is necessary for completing graduate courses intended to make the student an expert in calculations involving general relativity, but requiring only crude discussions of the all-important conceptual framework.

The present text shows, we hope, that only patience is needed—no special talent for mathematics. Personally I have never shown any such talent, only a persistent wonder at strange mathematical phenomena, like the endless number 3.1415... with the very short name 'pi'. (Caution: the length of the circumference of a circle divided by the diameter may have any value. Only in the special case of a flat space will you get the number 3.1415... More about that later!) Again and again I refused to comply with the long streams of strange mathematical symbols which Øyvind Grøn, my patient Guru of mathematics and physics, rapidly wrote on our gigantic blackboard. "Stop, stop! I don't want that equation! How did you jump from that one to the next?" To the astonishment of both of us it was possible to break down the long deductions into small and easily understandable steps.

A serious weakness of those courses, in my view as a humanist, is the implicit appeal to make the student accept what is going on without wondering. Along the road to Einstein's field equations, feats of artistic conceptual imagination abound. Also postulates and assumptions of seemingly arbitrary kinds are made. Some of them are seen to have a rational aspect when properly understood, but far from all. Einstein did not find all this wonderful. What deeply moved him in his wonder was that the concepts and equations he (and others, like Minkowski) *invented*, could be tested in the real world and in part *confirmed*. Somehow there must be, wondered Einstein, a kind of 'pre-established harmony' between inventive conceptual imagination and aspects of reality itself.

The present text tries to keep wonder alive, a wonder *not* due to misunderstandings.

Some people, as part of their religion, creep on their knees all around the holy mountain Kailas in Tibet. The present text would not have been produced if it had not been clearly felt as a way of honouring Albert Einstein not only as a persistent, fully committed truth seeker, but as a person combining this, and the 'egocentricity' going with it, with perfect generosity. He used his name and his time to work for the persecuted, for emigrants, for the hungry. And, in addition, feeling the absurdity of the political developments, he partook in depressing world affairs, even compromising his deeply felt pacifism. And, last but not least, he retained a sense of humour, and even as a superstar, was unaware of his outer appearance to the extent of neglecting to keep his worn trousers properly shut when lecturing. He would perhaps laugh if he got to know that this text is an expression of personal devotion.

Preface by Øyvind Grøn

One day, early in the Autumn 1985, the seventy three year old philosopher Arne Næss appeared at my graduate course on general relativity. He immediately decided that a new type of introduction to the general theory of relativity is needed; an introduction designed to meet the requirements of non-science educated people wanting to get a thorough understanding of this, most remarkable, theory.

The present text is the result of our efforts to provide a useful book for these people. It is neither a popular nor a semi-popular account of the theory. The book requires a rather large amount of patience from the reader, but nearly no previous knowledge of physics and mathematics. Our intention is to give an introduction that leads right up to Einstein's field equations and their most important consequences, starting at a lower level than what is common. The mathematical deductions are made with small steps so that the mathematically inexperienced reader may follow what happens. The meaning of the concepts that appear are explained and illustrated. And in some instances we mention points of a more philosophical character.

We devote a whole chapter to each of the topics 'vectors', 'differentiation', 'curves' and 'curved coordinate systems'. Tensors are indispensable tools in a formulation of the general theory of relativity intended to give the reader the possibility to apply the theory, at least to some simple, but nonetheless non-trivial, problems. The metric tensor is introduced in chapter 5, which also provides a most important discussion of the kinematic interpretation of the spacetime line element.

Albert Einstein demanded from his theory that no coordinate system is privileged. And in general curved coordinate systems are needed to describe curved spaces. In such coordinate systems the basis vectors vary with the position. This is described by the Christoffel symbols. In chapter 6 we give a thorough discussion, of a geometrical character, showing how the basis vectors of plane polar coordinates change with position, and relate this to the Christoffel symbols. Having worked yourself through this chapter, it is our hope that you have obtained a high degree of familiarity with the Christoffel symbols.

'Covariant differentiation', 'geodesic curves' and 'curvature' are important topics forming a 'package' of prerequisites necessary in order to be able to appreciate

fully Einstein's geometric conception of gravity. These topics are treated in chapters 7, 8 and 9, respectively. The expression for the Riemann curvature tensor is deduced by utilising Green's theorem connecting circulation and curl.

Einstein's law of gravitation is formulated mathematically in his field equations. They tell how matter curves spacetime. The field equations require an appropriate tensor representation of some properties of matter, i.e. of density, stress and motion. The usual representation of these properties is motivated and explained in chapter 10, where the basic conservation laws of classical fluid mechanics are expressed in tensor form. In chapter 11 the expression for Einstein's divergence free curvature tensor is deduced. With this chapter our preparation for a presentation of general relativity has been fulfilled.

Einstein's general theory of relativity is presented in chapter 12. Here we discuss the conceptual contents of the general theory of relativity. We consider the Newtonian limit of the theory, and we give an elementary demonstration of the following theorem: From the general theory of relativity and the assumption that it is impossible to measure velocity relative to vacuum, it follows that vacuum energy acts upon itself with repulsive gravitation.

In chapters 13 and 14 we deduce some consequences of the theory. In particular we discuss the gravitational time dilation, the deflection of light, the relativistic contribution to Mercury's perihelion precession, and we give a detailed explanation of how the theory predicts the possible existence of black holes. Finally the most important relativistic universe models, including the so-called inflationary universe models, are discussed.

Detailed calculations of the form of the Laplacian differential operator in spherical coordinates, needed in chapter 12, and of the components of the Ricci curvature tensor, needed to write down Einstein's field equations for the applications in chapters 13 and 14, are presented in appendices A, B, and C, respectively.

Looking at the stars with Einstein's theory in mind, you may feel it is like a wonder that Einstein managed to reveal such deep secrets of cosmos to us. People do in fact search for black holes now. And there are strong indications that at least a few have been found. What mysterious connection is there that makes nature 'obey' Einstein's great mental construction - the general theory of relativity?

Einstein's Theory

A Rigorous Introduction for the Mathematically
Untrained

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