Preface

The Kepler conjecture asserts that the densest packing of three-dimensional Euclidean space by equal spheres is attained by the "cannonball" packing, or face-centered-cubic (FCC) packing, which fills space with density $\frac{\pi}{\sqrt{18}} \approx 0.74048$. This conjecture, formulated by Kepler in his booklet "*Strena, seu, de Niue Sexangula*," was published in 1611, exactly four hundred years ago. Notably, in 1900 Hilbert included the sphere packing problem in his famous problem list, as part of his 18th problem. More than a century later, in a landmark result, the Kepler conjecture was solved in work of Thomas C. Hales and Samuel P. Ferguson. An abridged version of their proof appeared in the *Annals of Mathematics* in 2005, followed a year later by the publication of a detailed proof.

This book presents the Hales-Ferguson proof of the Kepler conjecture, together with supporting material and commentary. It begins with an introductory overview chapter, followed by a chapter on the local density approach to sphere packing bounds. This is followed by the six papers of Hales and Ferguson giving their detailed proof, as published in 2006 in a special issue of *Discrete & Computational Geometry*. Next comes a 2010 paper by Hales (with five other authors) making a slight revision to the 2006 proof, and listing corrections. It concludes with two of Hales's initial papers on the problem, published in 1997. All chapters except for the first are papers reprinted from *Discrete & Computational Geometry*.

This book is divided into four parts, as follows.

Part I: Introduction and Survey

The editor has written the two chapters in this part. The first chapter is introductory and features a brief summary of the history of work on the problem, together with a description of Hales and Ferguson's 1998 preprints, details of the peer review process of the Hales and Ferguson papers (which took eight years), and subsequent developments. It also includes remarks on the reliability of the proof and the subsequent approach of Hales to obtain a formal proof of the Kepler Conjecture (in a formal logical system), entirely checkable by computer.

The second chapter describes the general program of obtaining sphere packing upper bounds using local density inequalities, an approach that can be applied to sphere packing in any dimension. It was published as a paper in 2003 in *Discrete & Computational Geometry*. After considering the general case, it specializes to the three-dimensional case, and desecribes the main features of the Hales-Ferguson local density inequality as presented in their 1998 preprints; the 2006 published proof established a slightly modified inequality.

Part II: Proof of the Kepler Conjecture

These six chapters comprise the heart of the volume. They reprint the six papers of Hales with Ferguson that together provide the detailed proof of the Kepler Conjecture. As noted earlier, these papers appeared as a special issue (2006) of *Discrete & Computational Geometry*. However, in this book the short index to definitions that appeared in this special issue is omitted; it is replaced by the two indexes at the end of the book.

In 2010 Hales published an addendum and a list of errata to the published proof; we have added asterisks in the margin of the reprinted papers locating these corrections, which are listed in Part III of the volume on pp. 361–374.

Part III: A Revision to the Proof of the Kepler Conjecture

This part presents an important follow-up paper of Hales (with five coauthors) that was published in *Discrete & Computational Geometry* in 2010. It explains Hales's program to obtain a formal proof of the Kepler Conjecture. The initial process of formalization uncovered one logical gap in the original proof, and this follow up paper provides a correction filling that gap. It also supplies a list of errata to the original papers.

Part IV: Initial Papers of the Hales Program

This part presents two early papers of Hales on Kepler's conjecture, which were published in *Discrete & Computational Geometry* in 1997. These papers give his original formulation of an approach to proving the Kepler Conjecture via a local density inequality, and carry out some initial steps of this approach. They explain and establish a basic framework followed in the subsequent proof. As it turned out, to obtain a proof, this approach required some modification. This modification included changes to the local density inequalities as described in Part III. The proofs given in Parts II and III are independent of these two papers.

In reading this volume, it should be helpful to start with the introductory Chapter 1. Next one might study Chapter 2, which describes the general framework for obtaining upper bounds on sphere packing density, in any dimension. At present it is not known in which dimensions optimal such inequalities (i.e., tight inequalities) may exist: they are known to exist in dimensions 1, 2, and now, by the Hales-Ferguson proof, in dimension 3. It seems likely that optimal inequalities might exist in dimensions 8 and 24 as well. One might then read the historical survey of Hales in Chapter 3, which also describes some features of the proof given in the next five chapters. (This chapter could alternatively be read before reading Chapter 2.) Next one could look at some of the details of the formulation of the proof (Chapter 4). It would also be useful to look at the Revision paper (Chapter 9) to see features of the ongoing work towards a formal proof of the Kepler conjecture. Finally the reader may consider the remaining chapters in the volume.

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