Preface to the Third Edition

The main focus of extreme value theory has been undergoing a dramatic change. Instead of concentrating on maxima of observations, large observations are now in the focus, defined as exceedances over high thresholds. Since the pioneering papers by Balkema and de Haan (1974) and Pickands (1975) it is well known that exceedances over high thresholds can reasonably be modeled only by a generalized Pareto distribution. But only in recent years has this fact been widely spread outside the academic world as well.

Just as multivariate extreme value theory was developed roughly thirty years after its univariate basis was established, we presently see the theory of multivariate exceedances and, thus, the theory of multivariate generalized Pareto distributions under extensive investigation.

For that reason, one emphasis of the third edition of the present book is given to multivariate generalized Pareto distributions, their representations, properties such as their peaks-over-threshold stability, simulation, testing and estimation. Concerning this matter, the third edition in particular benefits from the recent PhD-theses of René Michel and Daniel Hofmann, who both made substantial contributions to the theory of multivariate generalized Pareto distributions, mainly concentrated in Section 4.4, Chapter 5 and 6. We are in particular grateful to René Michel, who coauthored these parts of the present edition with high diligence.

Exceedances of stochastic processes and random fields have been further considered in recent years, since the publication of the second edition. These new developments are discussed in additional sections or paragraphs. For instance, we deal with crossings of random processes in a random environment or with random variances, and crossings or level sets of smooth processes. Also maxima of a multi-fractional process, a recently introduced new class of random processes, are investigated.

The following contributions of co-authors are also gratefully acknowledged:

- Isabel Fraga Alves, Claudia Neves and Ulf Cormann: the modeling and testing of super-heavy tails in conjunction with log-Pareto distributions and a class of slowly-varying tails in Section 2.7

- Melanie Frick: testing against residual dependence in Section 6.5.

We are thankful to Holger Drees for pointing out a misarrangement of the text in the first chapter and to Laurens de Haan for correcting the erroneously assigned von Mises condition in the second chapter of the second edition.

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Preface to the Second Edition

Since the publication of the first edition of this seminar book in 1994, the theory and applications of extremes and rare events have received an enormous, increasing interest. This is primarily due to its practical relevance which is well recognized in different fields such as insurance, finance, engineering, environmental sciences and hydrology. The application of extreme value theory in hydrology has a particularly long and fruitful tradition. Meanwhile there are various specialized books available which focus on selected applications.

Different to that, the intention of the present book is to give a mathematically oriented development of the theory of rare events, underlying all applications. In the second edition we strengthen this characteristic of the book. One of the consequences is that the section on the statistical software Xtremes and the pertaining CD are omitted, this software is updated and well documented in [389]. Various new results, which are scattered in the statistical literature, are incorporated in the new edition on about 130 new pages.

The new sections of this edition are written in such a way that the book is again accessible to graduate students and researchers with basic knowledge in probability theory and, partly, in point processes and Gaussian processes. The required statistical prerequisites are minimal.

The book is now divided into three parts, namely,

Part I: The IID Case: Functional Laws of Small Numbers;

Part II: The IID Case: Multivariate Extremes;

Part III: Non-IID Observations.

Part II, which is added to the second edition, discusses recent developments in multivariate extreme value theory based on the Pickands representation of extreme value distributions. A detailed comparison to other representations of such distributions is included. Notable is particularly a new spectral decomposition of multivariate distributions in univariate ones which makes multivariate questions more accessible in theory and practice.

One of the most innovative and fruitful topics during the last decades was the introduction of generalized Pareto distributions in the univariate extreme value theory (by J. Pickands and, concerning theoretical aspects, by A.A. Balkema and L. de Haan). Such a statistical modelling of extremes is now systematically developed in the multivariate framework. It is verified that generalized Pareto distributions play again an exceptional role. This, in conjunction with the aforementioned spectral decomposition, is a relatively novel but rapidly increasing field. Other new sections concern the power normalization of extremes and a LAN theory for thinned empirical processes related to rare events.

The development of rare events of non-iid observations, as outlined in Part III, has seen many new approaches, e.g. in the context of risk analysis, of telecommunication modelling or of finance investigations during the last ten years. Very often these problems can be seen as boundary crossing probabilities. Some of these new aspects of boundary crossing probabilities are dealt with in this edition. Also a subsection on the recent simulation investigations of Pickands constants, which were unknown up to a few values, is added. Another new section deals in detail with the relations between the maxima of a continuous process and the maxima of the process observed at some discrete time points only. This relates the theoretical results to results which are applied and needed in practice.

The present book has benefitted a lot from stimulating discussions and suggestions. We are in particular grateful to Sreenivasan Ravi for contributing the section on power normalization of extremes, to René Michel, who helped with extensive simulations of multivariate extremes, and to Michael Thomas for the administration of our version control system (cvs) providing us with the technical facilities to write this book online. We thank Johan Segers for pointing out an error in one of the earlier definitions of multivariate generalized Pareto distributions in dimensions higher than two, which, on the other hand, actually links them to the field of quasi-copulas.

We would also like to thank the German Mathematical Society (DMV) for the opportunity to organize the symposium *Laws of small numbers: Extremes and rare events* during its annual meeting 2003 at the University of Rostock, Germany. This turned out to be quite a stimulating meeting during the writing of the final drafts of this book. Last, but not least we are grateful to Thomas Hempfling, Editor, Mathematics Birkhauser Publishing, for his continuous support and patience during the preparation of the second edition.

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Preface to the First Edition

In the first part of this book we will develop a theory of *rare events* for which a handy name is *functional laws of small numbers*. Whenever one is concerned with rare events, events with a small probability of occurrence, the *Poisson* distribution shows up in a natural way.

So the basic idea is simple, but its applications are nevertheless far-reaching and require a certain mathematical machinery. The related book by David Aldous entitled “Probability Approximations via the Poisson Clumping Heuristic” demonstrates this need in an impressive way. Yet this book focuses on examples, ranging over many fields of probability theory, and does not try to constitute a complete account of any field.

We will try to take another approach by developing a general theory first and then applying this theory to specific subfields. In prose: If we are interested only in those random elements among independent replicates of a random element Z, which fall into a given subset A of the sample space, a reasonable way to describe this *random* sample (with binomial sample size) is via the concept of *truncated empirical point processes*. If the probability for Z falling into A is small, then the Poisson approximation entails that we can approximate the truncated empirical point process by a Poisson point process, with the sample size now being a Poisson random variable. This is what we will call *first step Poisson process approximation*.

Often, those random elements falling into A follow closely an ideal or limiting distribution; replacing their actual distribution by this ideal one, we generate a *second step Poisson process approximation* to the initial truncated empirical process.

Within certain error bounds, we can therefore handle those observations among the original sample, which fall into the set A, like ideal observations, whose stochastic behavior depends solely upon a few (unknown) parameters. This approach permits the application of standard methods to statistical questions concerning the original and typically non-parametric sample.

If the subset A is located in the center of the distribution of Z, then *regression analysis* turns out to be within the scope of laws of small numbers. If the subset A is however located at the border, then *extreme value theory* is typically covered by our theory.

These specifications will lead to characteristic results in each case, and we will try in the following to convey the beauty of the laws of small numbers and several of their specific applications to the reader. In order to keep a more informal character, the proofs of several results are omitted, but references to detailed ones are given.

As the Hellinger distance provides a more accurate bound for the approximation of product measures in terms of their margins, as does the Kolmogorov-Smirnov or the variational distance, we will focus in the first part of this book on the formulation of laws of small numbers within the Hellinger distance.

The second part of the book concentrates on the theory of extremes and other rare events of non-iid random sequences. The rare events related to stationary sequences and independent sequences are considered as special cases of this general setup. The theory is presented in terms of extremes of random sequences as well as general triangular arrays of rare events.

Basic to the general theory is the restriction of the long range dependence. This enables the approximation of the point process of rare events by a Poisson process. The exact nature of this process depends also on the local behavior of the sequence of rare events. The local dependence among rare events can lead in the non-iid case to clustering, which is described by the compounding distribution of the Poisson process. Since non-stationarity causes the point process to be inhomogeneous, the occurrence of rare events is in general approximated by an extended compound Poisson process.

Part I of this book is organized as follows: In Chapter 1 the general idea of functional laws of small numbers is made rigorous. Chapter 2 provides basic elements from univariate extreme value theory, which enable particularly the investigation of the peaks over threshold method as an example of a functional law of small numbers. In Chapter 3 we demonstrate how our approach can be applied to regression analysis or, generally, to conditional problems. Chapter 4 contains basic results from multivariate extreme value theory including their extension to the continuous time setting. The multivariate peaks over threshold approach is studied in Chapter 5. Chapter 6 provides some elements of exploratory data analysis for univariate extremes.

Part II considers non-iid random sequences and rare events. Chapter 7 introduces the basic ideas to deal with the extremes and rare events in this case. These ideas are made rigorous in Chapter 8 presenting the general theory of extremes which is applied to the special cases of stationary and independent sequences. The extremes of non-stationary Gaussian processes are investigated in Chapter 9. Results for locally stationary Gaussian processes are applied to empirical characteristic functions. The theory of general triangular arrays of rare events is presented in Chapter 10, where we also treat general rare events of random sequences and the characterization of the point process of exceedances. This general approach provides a neat unification of the theory of extremes. Its application to multivariate non-stationary sequences is thus rather straightforward. Finally, Chapter 11 contains the statistical analysis of non-stationary ecological time series.

This book comes with the statistical software system XTREMES, version 1.2, produced by Sylvia Haßmann, Rolf-Dieter Reiss and Michael Thomas. The disk runs on IBM-compatible personal computers under MS-DOS or compatible operating systems. We refer to the appendix (co-authored by Sylvia Haßmann and Michael Thomas) for a user's guide to XTREMES. This software project was partially supported by the Deutsche Forschungsgemeinschaft by a grant.

This edition is based on lectures given at the DMV Seminar on "Laws of small numbers: Extremes and rare events", held at the Katholische Universität Eichstätt from October 20-27, 1991.

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