

Preface

Nowadays mathematical analysis progresses very rapidly and in many different directions. This development is carried by a lot of research groups worldwide that then constantly leads to a great number of new results and achievements. For the individual, active researcher it is difficult to keep track of this development in its full breadth. Thus it has become an imperative to review this rapid development from time to time.

The present volume contributes in this direction. It collects six articles on selected topics in the interface between partial differential equations and spectral theory, written by leading specialists in their respective fields. Apart from the main bodies on attractive fields of current research, with original contributions from the authors, the articles are written in an expository style that makes them accessible to a broader audience. They contain a detailed introduction along with surveys on recent developments, motivations, and necessary tools. Moreover, the authors share their views on future developments, hypotheses, and unsolved problems.

These six articles reflect to some extent the thematic diversity of current research in the area of mathematical analysis:

The first article, by Chen and Dreher, discusses macroscopic models of quantum semiconductor theory, in particular, quantum drift diffusion models and quantum hydrodynamic models. The authors present both the modeling and an rigorous analytic treatment of these models. They formulate the models as nonlinear mixed-order parameter-elliptic systems which then makes semigroup techniques applicable.

The following article, by BelHadjAli, Ben Amor, and Brasche, treats various asymptotic problems related to large coupling convergence of non-negative quadratic forms. This treatment is accompanied by a collection of well-chosen, also higher-dimensional, model problems which are carefully analyzed, where the authors place much emphasis on the interplay between abstract results and concrete applications.

Ben-Artzi presents in his contribution a smooth spectral calculus for a self-adjoint operator in an abstract Hilbert space setting. The author derives a limiting absorption principle under the assumption that the density of states is Hölder continuous and provides as an application a general eigenfunction expansion theorem as well as global space-time estimates for associated inhomogeneous wave equations.

The article by Bauer, Furutani, and Iwasaki studies subelliptic operators in the framework of sub-Riemannian geometry. It gives explicit representations for sub-Riemannian geodesics, heat kernels, and sub-Riemannian structures. Specifically, the authors determine geodesics of the Grushin plane and Grushin sphere and provide the heat kernel for the sub-Laplacian on the six-dimensional free nilpotent Lie group, among others. They also analyze the spectra on certain compact nilmanifolds.

Mendoza discusses in his contribution the singularities of the zeta function for elliptic cone differential operators. The author first recalls the framework of cone-differential calculus and discusses the existence of rays of minimal growth, before he deals with the short-term asymptotics of the heat trace. The constructions rely on a symbolic handling of the resolvent.

The final article, by McKeag and Safarov, is concerned with a coordinate-free approach to pseudodifferential operators. The introduction of the class of pseudodifferential operators is facilitated by choosing a linear connection on the base manifold. The authors discuss elements of a calculus under such an approach and describe an application to approximate spectral projections of the Laplace operator.

The volume addresses people generally interested in an overview of current developments in partial differential equations and spectral theory. It is mainly intended for specialists in partial differential equations, spectral theory, stochastic analysis, and mathematical physics, but it is also suitable for doctoral students who wish to gather first-hand information from leading scientists on these topics.

The idea for this volume originated from an “International Conference on Partial Differential Equations and Spectral Theory” held in Goslar, Germany, August 31 to September, 2008, which was jointly co-organized by the three editors. We would like to express our thanks to the authors for their contributions, to the participants in the conference who made it a very successful event, and to the Birkhäuser publisher for the constant support.

The editors

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