Preface

The integral equation method is an elegant mathematical way of transforming elliptic partial differential equations (PDEs) into boundary integral equations (BIEs). The focus of this book is the systematic development of efficient numerical methods for the solution of these boundary integral equations and therefore of the underlying differential equations.

The integral equation method has a long history that is closely linked to mathematicians such as I. Fredholm, D. Hilbert, E. Nyström, J. Hadamard, J. Plemelj, J. Radon and many others. Here is a list of some of the original works on the subject: [46, 96, 101, 126, 164, 165, 173, 175–177, 181, 182, 188, 214, 229].

With the introduction of variational methods for partial differential equations at the beginning of the twentieth century, integral equations lost some of their importance for the area of analysis. This was due to the difficulty of formulating precise results on existence and uniqueness by means of classical integral equations.

Since the middle of the twentieth century the need for numerical methods for partial differential methods began to grow. This was reflected also in the rapidly increasing interest in integral equation methods. Some advantages of this approach for certain classes of problems compared to domain methods (difference methods and finite elements) are given in the following:

1. The treatment of equations on spatial domains with a complex geometry is simpler with respect to mesh generation – this is the subdivision of the domain into small geometric elements – for boundary integral equations than for domain methods, since only a surface mesh of the domain has to be generated as opposed to an entire volume mesh.
2. The numerical treatment of problems on unbounded domains is especially simple with integral equation methods, while the treatment by means of domain methods requires the generation of a mesh on an unbounded domain, which is rather problematic.
3. For some parameter dependent problems, for example, from the area of electromagnetism at high frequencies, numerical methods for integral equations remain more stable for extreme parameters than for domain discretizations.

4. The large linear systems of equations that appear in almost every discretization method have a better condition number than the systems of equations for domain discretizations. Basic iterative methods thus converge more rapidly.
5. The drawbacks of the integral equation method, such as the numerical integrations that are necessary to generate and solve the systems of equations, are being resolved with numerical methods that have been under constant development since about 1980.

The Nyström or quadrature formula methods and the collocation method are classical numerical solution methods for integral equations. One of the first textbooks on this topic was written by K.E. Atkinson [7] with an extended new edition [8]. These methods are suited to the solution of boundary integral equations of *the second kind*. These are integral equations with operators of the form $I + K$, where I denotes the identity and K an integral operator. They can be implemented on computers relatively easily, although they do have two significant drawbacks: (a) the Nyström and collocation methods cannot be applied to all boundary integral equations that appear in connection with elliptic boundary value problems and (b) the convergence and stability of the methods can only be shown for very restrictive conditions imposed on the underlying differential equation and the smoothness of the physical domain.

Since about 1980–1990 the Galerkin methods for the discretization of boundary integral equations have been gaining importance for practical problems. From a theoretical point of view this method is superior to the alternatives such as the Nyström and collocation methods: stability, consistency and convergence of the Galerkin method can be shown for a very general class of boundary integral equation. The approach is based on a variational formulation of boundary integral equations as opposed to the pointwise, classical approach. This approach is explained in detail in, for example, [72, 74, 80, 167, 171, 238] or in the monographs [137, 162, 170].

The breakthrough for the Galerkin methods for practical, three-dimensional problems was achieved through the development of numerical methods for the approximation of integrals in order to determine the system matrix and through the development of fast algorithms to represent the non-local (boundary integral) operators.

The focus of this book is the systematic development of numerical methods to determine the Galerkin solution of boundary integral equations. All necessary tools from the area of analysis are presented, most of which are proven and derived; some, however, are only cited so that this book does not become too expansive. This book can be used as the basis for a lecture course of four hours a week on the numerics of boundary integral equations, consisting of an intensive short course on functional analysis and with a focus on the numerical methods. Some of the subsections bridge the gap between the textbook and current areas of research or should be seen as complements to the material. They are marked by a star ($\star$). The applications from the area of *electromagnetism* (Maxwell and wave equations, Helmholtz equation for high frequencies), for which integral equation methods are currently being developed intensively, serve as examples. The methods that are dealt with in this book form the basis with which to treat such problems. We will, however, not elaborate on the concrete applications.

So as not to go into too much detail we refrained from representing methods to couple finite elements with boundary elements and domain decomposition methods (see [49, 54, 71, 73, 135, 149]).

First and foremost the aim of this book is to represent and mathematically analyze efficient methods. Its purpose is not the treatment of concrete applications from the area of engineering. For this the books [13, 23, 32] may serve as an introduction.

Other textbooks and monographs from the area of numerical analysis for integral equations include [23, 60, 117, 216].

This book is the translation of the German version [204] and extended by chapters on p-parametric surface approximation and a posteriori error estimates – thanks are due to E. Louw for the translation of the German version. In addition we have corrected some misprints and incorporated additional material at various places.

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