Glossary

Standard symbols

$\overset{d}{\approx}$	approximately distributed as
$\Leftrightarrow$, iff	the logical equivalence standing for 'if and only if'
$\wedge$, $\neg$	the logical conjunction and the logical negation 'not'
$\wedge_{i=1}^{n}$	the logical conjunction of n statements indexed 1, 2, ..., n
$\exists, \forall$	existential and universal quantifiers
$\varnothing$	empty set
$\in, \subseteq, \subset, \supset, \supseteq$	set membership, set inclusion, proper (or strict) set inclusion, and the reverse inclusions
$\cup$, $\cap$, $\setminus$, $\triangle$	union, intersection, difference, and symmetric difference of sets
$+$, $\sum$	may stand for the ordinary addition or for the union of disjoint sets
$f(B)$	if B is a set and f a function, the image of B by f
$\lvert X\rvert$	number of elements (or cardinal number) of a set X
2^X	power set of the set X (i.e., the set of all subsets of X)
$X_1 \times X_2 \times \cdots \times X_n$	Cartesian product of the sets X_1,X_2, ..., X_n
$\mathbb{N}$	the set of all natural numbers (excluding 0)
$\mathbb{N}_0$	the set of nonnegative integers
$\mathbb{Q}$	the set of all rational numbers
$\mathbb{R}$	the set of all real numbers
$\mathbb{R}_+$	the set $[0, \infty[$ of all nonnegative real numbers
$\mathbb{Z}$	the set of all integers
$\mathbb{P}$	a probability measure
$]x, y[$	open interval of real numbers $\{z \in \mathbb{R} \mid x < z < y\}$
$[x, y]$	closed interval of real numbers $\{z \in \mathbb{R} \mid x \leq z \leq y\}$
$]x, y]$, $[x, y[$	real, half open intervals
$\breve{R}$	Hasse diagram of a partial order R
$t(R)$	transitive closure of a relation R
$\square$	marks the end of a proof
$\Diamond$	marks the end the proof of a lemma inserted in the proof of a theorem
l.h.s., r.h.s.	abbreviations for 'left-hand side' and 'right-hand side' (of a formula)
r.v.	abbreviation for 'random variable'
w.r.t.	abbreviation for 'with respect to'

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Non Standard Technical Terms[1]

In the notation '⟨k → j⟩' appearing in an entry, 'k' refers to the numerical marker of the relevant chapter, section or subsection and 'j' to the corresponding page number. Words or phrases set in blue appear elsewhere as entries in this glossary. The $\triangleleft$ superscript signifies that the entry refers to media theory.

accessible. A family of sets $\mathcal{K}$ satisfying Axiom [MA] of an antimatroid. Such a family is also said to be downgradable. $\langle 2.2.2 \to 27\rangle$

acyclic attribution. An attribution σ is acyclic when the relation $\mathcal{R}_\sigma$ defined by the equivalence $q\mathcal{R}_\sigma q' \iff \exists C \in \sigma(q') : q \in C$ (for $q, q' \in Q$) is acyclic. $\langle 5.6.10 \to 98\rangle$

adjacent$^\triangleleft$. Two distinct states S and V in a medium are adjacent if there exists a token τ such that $S\tau = V$. $\langle 10.1.2 \to 166\rangle$

alphabet. In Chapter 9, a set Q of symbols pertaining to a language. The elements of Q are called positive literals. The negative literals are the elements of Q marked with an overbar. Both positive and negative literals are used to write the words of the language. $\langle 9.2.1 \to 155\rangle$

antecedent set. The set A appearing as the first component of a query (A, p). $\langle$Section 15.1 $\to 298\rangle$

antimatroid. Another name for a learning space used in the combinatoric literature and defined there by different axioms. The dual structures are also called 'antimatroid.' Thus the union-closed antimatroids are the well-graded knowledge spaces $\langle 2.2.2 \to 27\rangle$ and the intersection-closed antimatroids are the duals. The latter are also called 'convex geometries.'

apex$^\triangleleft$. A state S is the apex of an oriented medium $(\mathcal{S}, \mathcal{T})$ if $\widehat{S} = \widehat{\mathcal{T}^+}$, that is, if the content of S is made of all the positive tokens in $\mathcal{T}$. $\langle 10.5.11 \to 182\rangle$

ascendent family. Consider a knowledge structure $(Q, \mathcal{K})$, a subset Q' of Q and a state W in the projection $\mathcal{K}_{|Q'}$ of $\mathcal{K}$ on Q'. The ascendent family $\mathcal{K}(Q', W)$ of W is the subfamily of all the states of $\mathcal{K}$ whose trace on Q' is W. Formally, we have $\mathcal{K}(Q', W) = \{K \in \mathcal{K} \,|\, K \cap Q' = W\}$. $\langle 13.7.1 \to 259\rangle$

assessment. The process of uncovering the competence of an individual in a domain of information $\langle$Section 1.3 $\to 10\rangle$.

assessment language for a collection $\mathcal{K} \subseteq 2^Q$. $\langle 9.2.3 \to 155\rangle$ A language L over the alphabet Q that is empty if $|\mathcal{K}| = 0$, has only the word 1 if $|\mathcal{K}| = 1$, and otherwise satisfies $L = qL_1 \cup \bar{q}L_2$, for some q in Q, where

[A1] L_1 is an assessment language for the trace $(\mathcal{K}_q)_{|Q\setminus\{q\}}$;
[A2] L_2 is an assessment language for the collection $\mathcal{K}_{\bar{q}}$ with domain $Q\setminus\{q\}$.

[1] Defined terms in the theory expounded in this book which do not belong to the standard mathematical lingo.

assessment module. The part of a computer educational software, such as the `ALEKS` system, that is devoted to the assessment of students' competence in a subject. ⟨Section 1.3 → 10⟩

assigned. For any item q in Q, the subset $\tau(q)$ of S is referred to as the set of skills assigned to q by the skill map. ⟨6.2.1 → 106⟩

association (relation). A mapping from the collection $2^Q \setminus \{\varnothing\}$ of all nonempty subsets of a domain Q to Q ⟨8.6.1 → 147⟩. If $\mathcal{K}$ is a knowledge space, then its derived entailment is an example of association ⟨7.1.6 → 123⟩.

atom. A set B in a family of sets $\mathcal{F}$ is an atom at some q in $\cup\mathcal{F}$ if B is a minimal set in $\mathcal{F}$ for the property of containing q ⟨3.4.5 → 48⟩. ('Minimal' is to be understood with respect to set inclusion.) In a knowledge space equipped with a base, the atoms are exactly the elements of the base ⟨3.4.8 → 48⟩.

attribution. A function σ mapping a domain Q into 2^{2^Q} (thus linking any element of Q to some family of subsets of Q) satisfying the condition that $\sigma(q) \neq \varnothing$ for any $q \in Q$. ⟨5.1.2 → 83⟩

attribution order. ⟨5.5.1 → 92⟩ A relation $\precsim$ on the collection $\mathfrak{F}$ of all attributions on a nonempty set Q defined by the equivalence

$$\sigma' \precsim \sigma \iff \forall q \in Q, \forall C \in \sigma(q), \exists C' \in \sigma'(q) : C' \subseteq C \qquad (\sigma, \sigma' \in \mathfrak{F}).$$

ball. For a state K in a knowledge structure $(\mathcal{K}, Q)$, the set of all states whose distance from K is at most h is called the ball of radius h centered at K. It is denoted by $\mathcal{N}(K,h)$. We have thus $\mathcal{N}(K,h) = \{L \in \mathcal{K} \mid d(K,L) \leq h\}$. This set is sometimes referred to as the h-neighborhood of K. ⟨4.1.6 → 63⟩

base of a $\cup$-closed family $\mathcal{F}$. A minimal subfamily $\mathcal{B}$ of $\mathcal{F}$ spanning $\mathcal{F}$, where 'minimal' is meant with respect to set inclusion: if $\mathcal{H}$ also spans $\mathcal{F}$ for some $\mathcal{H} \subseteq \mathcal{B}$, then $\mathcal{H} = \mathcal{B}$. ⟨3.4.1 → 47⟩

base†. A concept similar to the base that is instrumental for partial knowledge spaces ⟨Page 262⟩.

basic local independence model. A basic probabilistic model satisfying local independence ⟨11.1.2 → 189⟩. The added qualifier "with no guessing" means that, in such a model, all the lucky guess probabilities are assumed to be equal to zero ⟨11.3.6 → 198⟩.

basic probabilistic model. A quadruple $(Q, \mathcal{K}, p, r)$, in which $(Q, \mathcal{K}, p)$ is a probabilistic knowledge structure and r its response function. ⟨11.1.2 → 189⟩

binary classification language (over a finite alphabet Q). A language L which either consists of the empty word alone or satisfies the two following conditions ⟨9.2.4 → 156⟩:

[B1] a letter may not appear more than once in a word;
[B2] if π is a proper prefix of L, then there exist exactly two prefixes of the form $\pi\alpha$ and $\pi\beta$, where α and β are literals; moreover $\bar{\alpha} = \beta$.

block. The `QUERY` routine proceeds by 'blocks': first Block 1, then Block 2, etc. The responses $A\mathcal{P}q$ with $|A| = k$ appear in Block k. $\langle 15.1.2 \to 299\rangle$

bounded path. In a knowledge structure, a family of states connecting two states and satisfying certain conditions stated in $\langle 4.3.3 \to 70\rangle$.

canonical$^{\triangleleft}$**.** A message $\boldsymbol{m}$ in an oriented medium is called canonical if it is concise and satisfies one of the following three conditions $\langle 10.5.5 \to 180\rangle$:

(i) it is positive, that is, contains only positive tokens;
(ii) it is negative, that is, contains only negative tokens;
(iii) it is mixed, that is, of the form $\boldsymbol{m} = \boldsymbol{n}\boldsymbol{n}'$ where $\boldsymbol{n}$ is a positive message and $\boldsymbol{n}'$ a negative one.

careless error probability. The probability that a student in state K makes an error in responding to an instance of an item in K $\langle$pages 188, 362-364$\rangle$.

cast as. Used in the sense of "assigned the role of", as in: "The relation $\mathcal{R}$ is cast as the attribution σ." $\langle 5.1.4 \to 84\rangle$

child, Q'-child, plus child. The children of the partial knowledge structure $(Q, \mathcal{K})$ are specified by a proper subset Q' of Q. Define the equivalence relation $\sim_{Q'}$ on $\mathcal{K}$ by the formula $L \sim_{Q'} K \iff L \cap Q' = K \cap Q'$. Denote by $[K]$ the equivalence class containing the state K. With $K \in \mathcal{K}$, the set

$$\mathcal{K}_{[K]} = \{M \subseteq Q \,|\, \exists\, L \in [K], M = L \setminus (\cap[K])\}$$

is a child, or a Q'-child, of $\mathcal{K}$ $\langle 2.4.2 \to 32\rangle$. For any non trivial child $\mathcal{K}_{[K]}$ of $\mathcal{K}$, we call $\mathcal{K}^{+}_{[K]} = \mathcal{K}_{[K]} \cup \{\varnothing\}$ a plus child of $\mathcal{K}$ $\langle 2.4.11 \to 37\rangle$.

classification. A nomenclature $\{\mathcal{K}_{|Q_i} \,|\, 1 \leq i \leq k\}$ of a knowledge structure $(Q, \mathcal{K})$ is a classification if $\{Q_1, \ldots, Q_k\}$ is a partition of the domain Q. $\langle 11.8.1 \to 209\rangle$

clause for an item q. Any $C \in \sigma(q)$ where σ is an attribution. A clause for q is also called a foundation of q. $\langle 5.1.2 \to 83\rangle$

closed$^{\triangleleft}$**.** An oriented medium is closed if for any state S and any two distinct tokens τ and μ, both effective for S, we have $S\tau\mu = S\mu\tau$. $\langle 10.5.1 \to 179\rangle$

closed under intersection. See intersection-closed family.

closed under union, closed under finite union. See union-closed family.

$\cup$–**closure.** Property of being union-closed $\langle 2.2.2 \to 27\rangle$. (See also partial $\cup$–closure.)

closure space. A family of subsets of a set which is closed under intersection. $\langle 3.3.1 \to 46\rangle$

closure of a set. In the context of a closure space $(Q, \mathcal{L})$, the closure of a set $A \subseteq Q$ is the unique set A' in $\mathcal{L}$ including A that is minimal for inclusion in $\mathcal{L}$. For any $A, B \subseteq Q$, we have: (i) $A \subseteq A'$; (ii) $A' \subseteq B'$ when $A \subseteq B$; (iii) $A'' = A'$. $\langle 3.3.4 \to 46\rangle$

collection. A family of sets (or of other specified objects). $\langle 3.3.1 \to 46\rangle$

compatible knowledge structures. A knowledge structure $(Y, \mathcal{F})$ is compatible with a knowledge structure $(Z, \mathcal{G})$ if, for any $F \in \mathcal{F}$, the intersection $F \cap Z$ is the trace on Y of some state of $\mathcal{G}$. $\langle 7.3.5 \to 126\rangle$

competency for an item q in a skill multimap $(Q, S; \mu)$. Any set belonging to $\mu(q)$. $\langle 6.5.1 \to 112\rangle$

concise$^{\triangleleft}$ message. A message in a medium is concise if it is stepwise effective, consistent, and has no token occuring more than once. $\langle 10.1.3 \to 167\rangle$

conjunctive model. See delineated.

1-connected. A finite knowledge structure $(Q, \mathcal{K})$ is 1-connected if there is a stepwise path between any two of its (distinct) states. $\langle 4.1.3 \to 62\rangle$

consistent$^{\triangleleft}$ message. In a medium, a message is consistent if it does not contain both a token and its reverse. $\langle 10.1.3 \to 167\rangle$

content$^{\triangleleft}$ of a message. The set containing all the distinct tokens in that message. $\langle 10.3.1 \to 169\rangle$

convex updating rule. A special kind of non permutable updating rule. $\langle 13.4.2 \to 250 \text{ to } 250\rangle$

critical state. In a knowledge structure, a state K is critical for a state L if the inner fringe of L is some singleton $\{q\}$ and $K = L \setminus \{q\}$. $\langle 16.1.1 \to 336\rangle$

delineated knowledge state. Let (Q, S, τ) be a skill map with T a subset of S. A knowledge state K is delineated by T (via the disjunctive model) if $K = \{q \in Q \,|\, \tau(q) \cap T \neq \varnothing\}$ $\langle 6.2.1 \to 106\rangle$. Such a state K is delineated via the conjunctive model if $K = \{q \in Q \,|\, \tau(q) \subseteq T\}$ $\langle 6.4.1 \to 110\rangle$.

derived quasi ordinal space. $\langle 3.8.6 \to 58\rangle$ In the context of Theorem 3.8.5, the quasi ordinal space $\mathcal{K}$ defined from a relation $\mathcal{Q}$ on a domain Q by the equivalence $K \in \mathcal{K} \iff (\forall (p, q) \in \mathcal{Q} : \; q \in K \Rightarrow p \in K)$. The term "derived" is also used in a similar sense in the context of surmise systems and surmise functions $\langle 5.2.1 \to 85\rangle$, and in Chapter 8 $\langle 8.4.5 \to 144 \text{ and } 8.6.1 \to 147\rangle$.

describe (a word describes a state). In Chapter 9, a word is a string belonging to a language. Such a word describes a knowledge state in a knowledge structure if it specifies the state exactly $\langle 9.2.7 \to 157\rangle$. For instance, the word $\bar{a}ed$ specifies the state $\{b, c, d, e\}$ in the knowledge structure

$$\mathcal{G} = \big\{\varnothing, \{a\}, \{b, d\}, \{a, b, c\}, \{b, c, e\}, \{a, b, d\}, \\ \{a, b, c, d\}, \{a, b, c, e\}, \{b, c, d, e\}, \{a, b, c, d, e\}\big\}.$$

descriptive language. $\langle 9.2.7 \to 157\rangle$ A language L is a descriptive language for a partial knowledge structure $\mathcal{K}$ when

[D1] any word of L describes a unique state in K;

[D2] any state in $\mathcal{K}$ is described by at least one word of L.

discrepancy distribution. For two spaces $\mathcal{K}$ and $\mathcal{K}'$, the distribution $f_{\mathcal{K},\mathcal{K}'}$ of the (minimum) distances from the states in $\mathcal{K}$ to the states in $\mathcal{K}'$ is called the discrepancy distribution from $\mathcal{K}$ to $\mathcal{K}'$. $\langle 15.4.5 \to 318$ to $320\rangle$

discrepancy index. The discrepancy index from a knowledge structure $\mathcal{K}$ to a knowledge structure $\mathcal{K}'$ is defined by the mean

$$di(\mathcal{K},\mathcal{K}') = \tfrac{1}{|\mathcal{K}|}\textstyle\sum_{k=0}^{h(Q)} k f_{\mathcal{K},\mathcal{K}'}(k), \qquad (h(Q) = \lfloor \tfrac{1}{2}|Q|\rfloor),$$

of the discrepancy distribution $f_{\mathcal{K},\mathcal{K}'}$ from $\mathcal{K}$ to $\mathcal{K}'$, where Q is the common domain of $\mathcal{K}$ and $\mathcal{K}'$. $\langle 15.4.5 \to 318$ to $320\rangle$

discriminative. A knowledge structure is discriminative if every notion is a singleton $\langle 2.1.5 \to 24\rangle$. A surmise system (Q,σ) is discriminative if whenever $\sigma(q) = \sigma(q')$ for some $q, q' \in Q$, then $q = q'$. In such a case, the surmise function σ is also called discriminative $\langle 5.1.2 \to 83\rangle$.

discriminative reduction. See reduction.

disjunctive model. See delineated.

domain of a (partial) knowledge structure. The set of all its items. If $\mathcal{K}$ is a (partial) knowledge structure, the domain of $\mathcal{K}$ is $\cup\mathcal{K}$. $\langle 2.1.2 \to 23\rangle$

downgradable. A synonym of accessible. A nonempty family of sets $\mathcal{F}$ is downgradable if for every nonempty $S \in \mathcal{F}$, there exists $T \in \mathcal{F}$ such that $S \setminus \{q\} = T$ for some $q \in S$. $\langle 2.2.2 \to 27\rangle$

dual. The dual of a knowledge structure $\mathcal{K}$ is the knowledge structure $\bar{\mathcal{K}}$ containing all the complements of the states of $\mathcal{K}$. Here, 'complement' is to be understood with respect to the domain $\cup\mathcal{K}$ (in the set-theoretical meaning). $\langle 2.2.2 \to 27\rangle$

effective$^{\triangleleft}$. (See also 'stepwise effective.') In a medium, a message $\boldsymbol{m}$ is effective for a state S if $S\boldsymbol{m} \neq S$. $\langle 10.1.3 \to 167\rangle$

entail relation for a nonempty set Q. A relation $\mathcal{Q}$ on $2^Q \setminus \{\varnothing\}$ satisfying Conditions (i), (ii) and (iii) in Theorem 7.1.5. $\langle 7.2.2 \to 124\rangle$

entailment for a nonempty set Q. Any relation $\mathcal{P}$ from $2^Q \setminus \{\varnothing\}$ to Q that satisfies Conditions (i) and (ii) in Theorem 7.1.3. $\langle 7.1.4 \to 122\rangle$

essential distance between two states. This concept applies to non necessarily discriminative knowledge structures. The essential distance between two states K and L is defined by $e(K,L) = |K^* \triangle L^*|$, where K^* (resp. L^*) is the set of all notions q^* with q in K (resp. q in L). $\langle 2.3.1 \to 30\rangle$

fair stochastic assessment process. In the Markov chain procedure of Chapter 14, the case in which the probabilities of lucky guesses are all equal to zero; thus $\eta_q = 0$ for any item q. $\langle 14.2.2 \to 279\rangle$

finitary. A knowledge structure is finitary if the intersection of any chain of states is a state $\langle 3.6.1 \to 52\rangle$. A closure space is $\cap$-finitary if the union of any chain of states is a state $\langle$Problem 13 on page 60$\rangle$.

finitely learnable $\langle 4.4.2 \to 72\rangle$. A discriminative structure is finitely learnable if there is a positive integer l such that, for any state K and any item $q \notin K$, there exists a positive integer h and a chain of states $K = K_0 \subset K_1 \subset \cdots \subset K_h$ satisfying the two conditions:

(i) $q \in K_h$;
(ii) $d(K_i, K_{i+1}) \leq l, \quad$ for $0 \leq i \leq h-1$.

foundation of an item q**.** See clause.

fringe. The fringe of a state K in a knowledge structure is the union of the inner and outer fringes of K, that is, the set $K^{\mathcal{F}} = K^{\mathcal{I}} \cup K^{\mathcal{O}}$. $\langle 4.1.6 \to 63\rangle$

gradation, ∞-gradation. In a finite knowledge structure, a gradation is a tight path from the empty set to the domain $\langle 4.1.3 \to 62\rangle$. A similar concept applies in the infinite case where it is called an ∞-gradation $\langle 4.3.1 \to 69\rangle$.

granular. A knowledge structure is granular if for every state K, any item q in K has an atom at q included in K. $\langle 3.6.1 \to 52\rangle$

guessing probability. See lucky guess probability.

half-split questioning rule. An item q selected by this rule on trial n of an assessment minimizes the quantity $|2L_n(\mathcal{K}_q) - 1|$, where L_n is the current probability distribution on the set $\mathcal{K}$ of states. If two or more items satisfy this condition, the algorithm chooses randomly between them. $\langle 13.4.7 \to 252\rangle$

ε-half-split. A special case of the questioning function in the Questioning Rule Axiom [QM]. $\langle 14.3.2 \to 280\rangle$

hanging, almost hanging. $\langle 16.1.1 \to 336\rangle$ In a knowledge structure, a nonempty state K having an empty inner fringe is called a hanging state. A state K is almost hanging if it has at least two items and its inner fringe is a singleton.

hanging-safe. A query (A, q) is hanging-safe (for a learning space) if there is no clause C for some item r with $A \cap C = \{r\}$ and $q \in C$. $\langle 16.1.9 \to 338\rangle$

Hasse system. $\langle 5.5.8 \to 94\rangle$ Generalization of a Hasse diagram $\langle 1.6.8 \to 15\rangle$. This concept applies to granular knowledge spaces.

height of an item. The height of an item q is the number $h(q) = k - 1$, where k is the size of a minimal state containing the item q. A height of zero for an item q means that $\{q\}$ is a state. $\langle 15.4.4 \to 317\rangle$

incidence matrix. $\langle 14.7.1 \to 293\rangle$ For a collection $\mathcal{K}$ of subsets of the finite domain Q of items, the matrix $\mathbf{M} = (M_{q,K})$ whose rows are indexed by items q in Q, columns are indexed by states K in $\mathcal{K}$, and

$$M_{q,K} = \begin{cases} 1 & \text{if } q \in K, \\ 0 & \text{otherwise.} \end{cases}$$

inclusive mesh. A mesh $\mathcal{K}$ of two knowledge structures $\mathcal{F}$ and $\mathcal{G}$ is called (union) inclusive if $F \cup G \in \mathcal{K}$ for any $F \in \mathcal{F}$ and $G \in \mathcal{G}$. $\langle 7.4.5 \to 128 \rangle$

inconsistent$^{\triangleleft}$. A message in a medium is inconsistent if it contains both some token and its reverse. $\langle 10.1.3 \to 167 \rangle$

ineffective$^{\triangleleft}$. A message is ineffective for a state if it is not effective for that state. $\langle 10.1.3 \to 167 \rangle$

independent states, projections. Let $(Q, \mathcal{K}, p)$ be a probabilistic knowledge structure, and let $\mathbb{P}$ be the induced probability measure on the power set of $\mathcal{K}$. With $Q', Q'' \subset Q$, consider the probabilistic projections $(Q', \mathcal{K}', p')$ and $(Q'', \mathcal{K}'', p'')$. Two states $J \in \mathcal{K}'$, $L \in \mathcal{K}''$ are independent if the events $J^{\diamond} = \{K \in \mathcal{K} \,|\, K \cap Q' = J\}$ and $L^{\diamond} = \{K \in \mathcal{K} \,|\, K \cap Q'' = L\}$ are independent in the probability space $(\mathcal{K}, 2^{\mathcal{K}}, \mathbb{P})$. The projections $(Q', \mathcal{K}', p')$ and $(Q'', \mathcal{K}'', p'')$ are independent if any state $K \in \mathcal{K}$ has independent traces on Q' and Q''. $\langle 11.9.1 \to 210 \rangle$

informative (equally). Two items belonging to the same notion are called equally informative. $\langle 2.1.5 \to 24 \rangle$

informative questioning rule. An item selected on trial n according to this rule and presented to the subject minimizes the expected entropy of the likelihood distribution on the set of states on trial $n + 1$. $\langle 13.4.8 \to 253 \rangle$

inner fringe. In a knowledge structure $(Q, \mathcal{K})$, the inner fringe of a state K is the set $K^{\mathcal{I}} = \{q \in K \,|\, K \setminus \{q\} \in \mathcal{K}\}$. $\langle 4.1.6 \to 63 \rangle$

inner questioning rule. An abstract constraint on the questioning rule requiring that an item q be chosen so that the likelihood of a correct response to q is as far as possible from 0 or 1. $\langle 13.6.4 \to 257 \rangle$

instance. $\langle 1.1.1 \to 2 \rangle$ A particular case of an item that can be used in an assessment, for example by replacing the abstract parameters of an equation by numerical values. This is a pedagogical concept, with no formal definition.

intersection-closed family, $\cap$-closed family. A family $\mathcal{F}$ of subsets of a set X which is closed under intersection: for any subfamily $\mathcal{G}$ of $\mathcal{F}$, we have $\cap\mathcal{G} \in \mathcal{F}$. As we can have $\mathcal{G} = \varnothing \subseteq \mathcal{F}$, we get $\cap\varnothing = X \in \mathcal{F}$. $\langle 2.2.2 \to 27 \rangle$

item, item indicator. An item is an element in the domain of a knowledge structure or partial knowledge structure $\langle 2.1.2 \to 23 \rangle$. The term item is also used in the context of skill maps $\langle 6.2.1 \to 106 \rangle$. An item indicator random variable for an item q is a 0-1 random variable taking value 1 if the response to item q is correct, and 0 otherwise $\langle 11.9.4 \to 212 \rangle$.

jointly consistent$^{\triangleleft}$. Two messages $\boldsymbol{n}$ and $\boldsymbol{m}$ are jointly consistent if $\boldsymbol{nm}$ is consistent. $\langle 10.1.3 \to 167 \rangle$

knowledge space. A pair $(Q, \mathcal{K})$ where Q is a nonempty set and $\mathcal{K}$ is a family of subsets of Q closed under union and containing the empty set and the set $Q = \cup\mathcal{K}$, which is the domain of the knowledge space. Thus, a knowledge space is a $\cup$-closed knowledge structure. The pair $(Q, \mathcal{K})$ can also be referred to as a space. The family $\mathcal{K}$ itself is often called a space. $\langle 2.2.2 \to 27\rangle$

knowledge state. Any set in a knowledge structure or partial knowledge structure $\langle 2.1.2 \to 23\rangle$. Can be abbreviated as state.

knowledge structure. A pair $(Q, \mathcal{K})$ where Q is a nonempty set and $\mathcal{K}$ is a family of subsets of Q containing both the empty set $\varnothing$ and the set Q, which is called the domain of the knowledge structure. The sets in $\mathcal{K}$ are referred to as the states of the knowledge structure. The family $\mathcal{K}$ itself is also called a knowledge structure. $\langle 2.1.2 \to 23\rangle$

L1-chain in a learning space. Let $K \subset L$ be two knowledge states with $|K \setminus L| = p$ and $L_0 = L \subset L_1 \subset \ldots \subset L_p = K$ a chain of states; so, $|L_i \setminus L_{i-1}| = 1$ for $1 \leq i \leq p$. Such a chain is called a L1-chain. $\langle 2.2.1 \to 26\rangle$

language. A distinguished set of words comprising the positive and negative literals. $\langle 9.2.1 \to 155\rangle$

latent. The term 'latent state' is non-technical and has several meanings. It may refer informally to the hypothetical knowledge state determining the subject's responses to the questions in an assessment $\langle 13.3.1 \to 246\rangle$. The term 'latent structure' is used in the psychometric literature with a germane meaning. Finally, it may also qualify the hypothetical knowledge space or learning space governing the responses in the application of the `QUERY` procedure. $\langle$Section 15.1 $\to 298\rangle$

learning function, learning rate. The function ℓe of Axiom [L] in a system of stochastic learning paths, formalizing the idea that the probability of a state at a given time only depends upon the last state recorded, the time elapsed, the learning rate and the gradation $\langle 12.2.3 \to 218$–$12.2.4 \to 220\rangle$. In a special case, the learning rate is a gamma distributed r.v. $\langle 12.4.1 \to 224\rangle$

learning path. A maximal chain in a knowledge structure. $\langle 4.1.1 \to 61\rangle$

learning space. A knowledge structure satisfying Axioms [L1] and [L2] $\langle 2.2.1 \to 26\rangle$; equivalently, a $\cup$-closed knowledge structure which is either well-graded, or finite and downgradable. $\langle 2.2.4 \to 28\rangle$

learnstep number (of a finitely learnable knowledge structure). The smallest number l in the definition of finite learnability. $\langle 4.4.2 \to 72\rangle$

length$^{\triangleleft}$. The length of a message $\boldsymbol{m} = \tau_1 \ldots \tau_n$ is the number of its (non necessarily distinct) tokens. We write then $\ell(\boldsymbol{m}) = n$. $\langle 10.1.3 \to 167\rangle$.

literals, positive, negative. The symbols used to write the words of a language. $\langle 9.2.1 \to 155\rangle$

local independence. $\langle 11.1.2 \to 189\rangle$ The response function r of a basic probabilistic model $(Q, \mathcal{K}, p, r)$ satisfies local independence if for all $K \in \mathcal{K}$ and $R \subseteq Q$, we have

$$r(R, K) = \left(\prod_{q\in K\setminus R} \beta_q\right)\left(\prod_{q\in K\cap R}(1-\beta_q)\right)\left(\prod_{q\in R\setminus K}\eta_q\right)\left(\prod_{q\in\overline{R\cup K}}(1-\eta_q)\right)$$

in which, for each item $q \in Q$, the symbols β_q and η_q denote two parameters measuring a careless error probability and a lucky guess probability, respectively, for that item.

lucky guess probability. Probability of a correct response to an instance of an item which does not belong to the student's knowledge state (cf. for example the Local Independence Axiom [N]). $\langle 12.4.1 \to 224\rangle$

m-states. $\langle 14.4.1 \to 283\rangle$ The states of a Markov chain. The term is used in Chapter 14 to avoid a possible confusion with the states of a knowledge structure.

marking function. In the Markov chain procedure of Chapter 14, the function μ of the Marking Rule Axiom [M]. $\langle 14.2.1 \to 278\rangle$

marked states. In the Markov chain procedure of Chapter 14 those states retained as feasible on a given trial. $\langle 14.1 \to 273$ to $278\rangle$

medium$^{\lhd}$. A pair $(\mathcal{S}, \mathcal{T})$ in which $\mathcal{S}$ is a set of states, $\mathcal{T}$ is a set of transformations on $\mathcal{S}$ and the two axioms [Ma] and [Mb] are satisfied $\langle 10.1.4 \to 167\rangle$. Any discriminative, well-graded family of sets gives rise in a natural manner to a medium; in turn, any medium is obtainable in this way in the sense that the sets of the family are in a one-to-one correspondence with the states of the medium. $\langle 10.4.11 \to 178$–$10.5.12 \to 182\rangle$

mesh, meshable, maximal mesh. A knowledge structure $(X, \mathcal{K})$ is a mesh of two knowledge structures $(Y, \mathcal{F})$ and $(Z, \mathcal{G})$ if: (i) $X = Y \cup Z$; and (ii) $\mathcal{F}$ and $\mathcal{G}$ are the projections of $\mathcal{K}$ on Y and Z, respectively $\langle 7.3.1 \to 126\rangle$. Two knowledge structures having a mesh are said to be meshable $\langle 7.3.1 \to 126\rangle$. For two compatible knowledge structures $(Y, \mathcal{F})$ and $(Z, \mathcal{G})$, the knowledge structure $(Y \cup Z, \mathcal{F} \star \mathcal{G})$ defined by the equation

$$\mathcal{F} \star \mathcal{G} = \{K \in 2^{Y\cup Z} \mid K \cap Y \in \mathcal{F},\ K \cap Z \in \mathcal{G}\}$$

is the maximal mesh of $\mathcal{F}$ and $\mathcal{G}$ $\langle 7.4.1 \to 127\rangle$.

message$^{\lhd}$. A string $\boldsymbol{m} = \tau_1 \ldots \tau_n$ of tokens in a medium. $\langle 10.1.3 \to 167\rangle$

minimal well-graded extension. $\langle 16.3.3 \to 356\rangle$ A minimal well-graded extension of a non well-graded family $\mathcal{F}$ is a well-graded $\cup$-closed family $\mathcal{H}$ such that:

(i) $\mathcal{F} \subset \mathcal{H}$;
(ii) there is no $\cup$-closed, well-graded family $\mathcal{H}'$ satisfying $\mathcal{F} \subset \mathcal{H}' \subset \mathcal{H}$.

mixed$^{\triangleleft}$. A message $\boldsymbol{m}$ in an oriented medium is called mixed if it is concise and of the form $\boldsymbol{m} = \boldsymbol{n}\boldsymbol{n}'$, where $\boldsymbol{n}$ is a positive message and $\boldsymbol{n}'$ a negative one (see 'canonical message'). $\langle 10.5.5 \to 180\rangle$

multiplicative updating rule. A special case of the permutable updating rule. $\langle 13.4.4 \to 251\rangle$

negative token$^{\triangleleft}$. In an orientation $\{\mathcal{T}^+, \mathcal{T}^-\}$ of a medium, any token in $\mathcal{T}^-$. $\langle 10.4.2 \to 174$ to $175\rangle$

neighborhood, neighbor. The subfamily $\mathcal{N}(K, h)$ of all the states at distance at most h from a state K in a knowledge structure $\mathcal{K}$ is referred to as the h-neighborhood of K, or sometimes as the ball of radius h centered at the state K; thus, $\mathcal{N}(K, h) = \{K' \in \mathcal{K} \,|\, d(K, K') \leq h\}$ $\langle 4.1.6 \to 63\rangle$.

The term 'neighborhood' is also used with a different, but related meaning pertaining to a family rather than a state. The ε-neighborhood of a subfamily $\mathcal{F}$ of a knowledge structure $\mathcal{K}$ is the subfamily of $\mathcal{F}$ defined by

$$\mathcal{N}(\mathcal{F}, \varepsilon) = \{K' \in \mathcal{K} \,|\, d(K, K') \leq \varepsilon, \text{ for some } K \text{ in } \mathcal{F}\}.$$

The (q, ε)-neighborhood and the $(\bar{q}, \varepsilon)$-neighborhood of $\mathcal{F}$ are respectively $\mathcal{N}_q(\mathcal{F}, \varepsilon) = \mathcal{N}(\mathcal{F}, \varepsilon) \cap \mathcal{K}_q$ and $\mathcal{N}_{\bar{q}}(\mathcal{F}, \varepsilon) = \mathcal{N}(\mathcal{F}, \varepsilon) \cap \mathcal{K}_{\bar{q}}$. The ε-neighbors are the states in a ε-neighborhood; the terms '(q, ε)-neighbors' and '$(\bar{q}, \varepsilon)$-neighbors' have similar meanings. $\langle 14.3.1 \to 279\rangle$.

nomenclature. Let $Q_1, \ldots, Q_k$ be a collection of nonempty subsets of the domain Q of a knowledge structure $\mathcal{K}$. The collection $\mathcal{K}_{|Q_1}, \ldots, \mathcal{K}_{|Q_k}$ of projections is a nomenclature if the collection $(Q_i)_{1 \leq i \leq n}$ covers Q, that is, if $\cup_{i=1}^{n} Q_i = Q$. $\langle 11.8.1 \to 209\rangle$

notion. If q is an item in a knowledge structure $(Q, \mathcal{K})$, then the set q^* of all the items contained in exactly the same states as q is a notion. We have thus $q^* = \{s \in Q \,|\, \forall K \in \mathcal{K},\ s \in K \Leftrightarrow q \in K\}$. $\langle 2.1.5 \to 24\rangle$

operative. A query (A, q) is said to be operative for a learning space $\mathcal{L}$ if $\mathcal{L} \setminus \mathcal{D}_{\mathcal{L}}(A, q) \subset \mathcal{L}$. $\langle 16.1.9 \to 338\rangle$

ordinal space. A discriminative knowledge space closed under intersection is a (partially) ordinal space (with 'partially' referring to the corresponding partial order). $\langle 3.8.1 \to 56$–$3.8.3 \to 57\rangle$

orientation$^{\triangleleft}$. An orientation of a medium $(\mathcal{S}, \mathcal{T})$ is a partition $\{\mathcal{T}^+, \mathcal{T}^-\}$ of its set of tokens $\mathcal{T}$ such that, for any token τ, we have $\tau \in \mathcal{T}^+ \Leftrightarrow \tilde{\tau} \in \mathcal{T}^-$, with $\tilde{\tau}$ the reverse of τ. By convention, the tokens in $\mathcal{T}^+$ (resp. $\mathcal{T}^-$) are called 'positive' (resp. negative). $\langle 10.4.2 \to 174\rangle$

oriented medium$^{\triangleleft}$. A medium equipped with an orientation is called an oriented medium. $\langle 10.4.2 \to 174\rangle$

outer fringe. In a knowledge structure $(Q, \mathcal{K})$, the outer fringe of a state K is the set $K^{\mathcal{O}} = \{q \in Q \setminus K \,|\, K + \{q\} \in \mathcal{K}\}$. $\langle 4.1.6 \to 63\rangle$

parent family. Let $\mathcal{K}_{|Q'}$ be projection of a knowledge structure $(Q, \mathcal{K})$ on a proper subset Q' of Q. For any $J \in \mathcal{K}_{|Q'}$, the parent family $J^\diamond$ of J is defined by $J^\diamond = \{K \in \mathcal{K} \mid K \cap Q' = J\}$; so, $\cup_{J \in \mathcal{K}_{|Q'}} J^\diamond = \mathcal{K}$. $\langle 11.7.2 \to 207 \rangle$

partial knowledge structure. $\langle 2.2.6 \to 29 \rangle$ A family of sets $\mathcal{K}$ containing the set $\cup\mathcal{K}$. Partial knowledge spaces and partial learning spaces are defined similarly.

partially ordinal space. A discriminative quasi ordinal space $\langle 3.8.1 \to 56 \rangle$. In other words, a space which is derived from a partial order $\langle 3.8.3 \to 57 \rangle$.

partially union-closed. A family $\mathcal{F}$ is partially union-closed (or partially $\cup$-closed) if for any nonempty subfamily $\mathcal{G}$ of $\mathcal{F}$, we have $\cup\mathcal{G} \in \mathcal{F}$. (Contrary to the $\cup$-closure condition, partial $\cup$-closure does not imply that the empty set belongs to the family.) $\langle 2.2.6 \to 29 \rangle$

path in a knowledge structure. See stepwise path or tight path.

Pending-Table. Buffer collecting all the responses $A\mathcal{P}q$ having failed the `HS-test` in the algorithm for building a learning space. $\langle 16.2.11 \to 351 \rangle$

permutable updating rule. An updating rule satisfying the permutability condition $F\big(F(l, \xi), \xi'\big) = F\big(F(l, \xi'), \xi\big)$. This condition implies that the order of the questions asked during the assessment is irrelevant. $\langle 13.4.3 \to 251 \rangle$

positive token$^\triangleleft$. In a medium equipped with an orientation $\{\mathcal{T}^+, \mathcal{T}^-\}$ of a medium, any token in the set $\mathcal{T}^+$. $\langle 10.4.2 \to 174 \text{ to } 175 \rangle$

precede. Let $\precsim$ be the surmise relation of a knowledge structure. When $r \precsim q$, we say that r precedes q, or equivalently that r is surmisable from q. If, moreover, $q \precsim r$ does not hold, we say that r strictly precedes q. $\langle 3.7.1 \to 54 \rangle$

prefix, prefix$^\triangleleft$. The initial segment of a word $\langle 9.2.1 \to 155 \rangle$, or of a message. $\langle 10.5.4 \to 180 \rangle$

probabilistic knowledge structure. A finite partial knowledge structure equipped with a probability distribution on its set of states. $\langle 11.1.2 \to 189 \rangle$

probabilistic projection. $\langle 11.7.3 \to 208 \rangle$ Suppose that $(Q, \mathcal{K}, p)$ is a probabilistic knowledge structure, and let $\mathcal{K}'$ be a projection of $(\mathcal{K}, Q)$ on a proper subset Q' of Q. For any $J \in \mathcal{K}_{|Q'}$, write $J^\diamond = \{K \in \mathcal{K} \mid K \cap Q' = J\}$. The triple $(Q', \mathcal{K}', p')$ is the probabilistic projection induced by Q', if for all $J \in \mathcal{K}'$, we have $p'(J) = \sum_{K \in J^\diamond} p(K)$.

produce, produce$^\triangleleft$. Any attribution σ on a set Q produces a knowledge space $(Q, \mathcal{K})$ via the equivalence $K \in \mathcal{K} \iff \forall q \in K, \exists C \in \sigma(q) : C \subseteq K$ $\langle 5.2.3 \to 86 \rangle$. In media theory, we say that a message $\boldsymbol{m}$ from a state S produces some state $V \neq S$ if $S\boldsymbol{m} = V$ $\langle 10.1.3 \to 167 \rangle$.

progressive. A system of stochastic learning paths is progressive if it does not allow for any forgetting. $\langle 12.2.4 \to 220 \rangle$

projection. Let $(Q, \mathcal{K})$ be a partial knowledge structure. For any nonempty proper subset Q' of Q, the family $\mathcal{K}_{|Q'} = \{W \subset Q' \mid \exists K \in \mathcal{K}, W = K \cap Q'\}$ is the projection of $\mathcal{K}$ on Q'. We thus have $\mathcal{K}_{|Q'} \subseteq 2^{Q'}$. If $W = K \cap Q'$ for some $K \in \mathcal{K}$ and so $W \in \mathcal{K}_{|Q'}$, then W is called the trace of K on Q'. ⟨2.4.2 → 32⟩

prolong. The skill map (Q', S', τ') prolongs the skill map (Q, S, τ) if $Q = Q'$, $S \subseteq S'$, and $\tau(q) = \tau'(q) \cap S$ for all $q \in Q$. ⟨6.3.6 → 109⟩.

PS-QUERY. ⟨15.5.1 → 324 to 331⟩ An extension of the `QUERY` routine in which a response to a query (A, p) is not implemented immediately. Instead, it is put in a buffer, awaiting for a confirmation or a contradiction from a later response.

quadratic discrepancy index. For two knowledge structures $\mathcal{K}$ and $\mathcal{K}'$, this index is the quadratic mean $\overline{di}(\mathcal{K}, \mathcal{K}') = \sqrt{di^2(\mathcal{K}, \mathcal{K}') + di^2(\mathcal{K}', \mathcal{K}')}$ between the two discrepancy indices for $\mathcal{K}$ and $\mathcal{K}'$. ⟨15.6.2 → 330 to 330⟩

quasi learning space. A knowledge structure satisfying quasi learning smoothness and quasi learning consistency. This is a variant of the concept of learning spaces for nondiscriminative structure. ⟨2.3.2 → 31⟩.

quasi ordinal space. A knowledge space which is closed under intersection ⟨3.8.1 → 56–3.8.3 → 57⟩. Equivalently, a space derived from a quasi order ⟨3.8.3 → 57⟩.

quasi well-graded family, or qwg-family. A family $\mathcal{F}$ such that, for any two distinct states $K, L \in \mathcal{F}$, there exists a finite sequence of states $K = K_0$, $K_1, \ldots, K_p = L$ satisfying $e(K_{i-1}, K_i) = 1$ for $1 \leq i \leq p$ and moreover $p = e(K, L)$ (where e denotes the essential distance). ⟨2.3.3 → 31⟩

query. ⟨Section 3.2 → 44⟩ ⟨Section 7.1 → 120–Section 7.6 → 123⟩ A question symbolized as (A, p), posed to experts in scholalrly topic, with the following interpretation: "*Will any student failing all the items in the set A also fail item q?*" If Q is the domain, we thus have $A \subseteq Q$ and $p \in Q$. The response data may also be gathered from assessment statistics. ⟨3.2.3 → 45⟩

QUERY (typeset as `QUERY`). A routine for constructing a knowledge space, based on responses to queries of the type (A, q) "*Does failing all the items in the set A entails failing also item q?*" ⟨Chapter 15 → 297 to 323⟩. These responses can either be obtained from questioning an expert, or can be derived from assessment statistics ⟨15.4.7 → 323⟩.

questioning function. The function τ of the Questioning Rule Axiom [QM] of the Markov chain procedure. ⟨14.2.1 → 278–14.2.2 → 279⟩

questioning rule (general). A function $\Psi : (q, L_n) \mapsto \Phi(q, L_n)$ providing a framework for rules governing the choice of the item to be asked on trial n of an assessment, based on the probability distribution L_n on the set of states on that trial ⟨13.3.3 → 248⟩. Special cases of the function Ψ provide actual questioning rules ⟨Section 13.4 → 249⟩.

qwg-family. Abbreviation for quasi well-graded family.

reduction (discriminative). The discriminative reduction of a knowledge structure $(Q, \mathcal{K})$ is the knowledge structure $(Q^*, \mathcal{K}^*)$ constructed by replacing all the items q by the corresponding notions q^*. We have $\mathcal{K}^* = \{K^* \mid K \in \mathcal{K}\}$ where $K^* = \{q^* \mid q \in K\}$. $\langle 2.1.5 \to 24$ to $25\rangle$

regular updating rule. An abstract constraint on the updating rule generalizing the convex and the multiplicative updating rules. $\langle 13.6.2 \to 256\rangle$

resoluble attribution, resolution order. An attribution σ on a domain Q is called resoluble if there exists a linear order $\mathcal{T}$ on Q such that for any item q in Q: (a) there is some $C \in \sigma(q)$ satisfying $C \subseteq \mathcal{T}^{-1}(q)$; (b) $\mathcal{T}^{-1}(q)$ is finite. The order $\mathcal{T}$ is then a resolution order. $\langle 5.6.2 \to 97\rangle$

resoluble knowledge space. A knowledge space is resoluble when it is produced by at least one resoluble attribution. $\langle 5.6.5 \to 97\rangle$

response function. For a probabilistic knowledge structure $(Q, \mathcal{K}, p)$, a function $r : (R, K) \mapsto r(R, K)$, with $R \subseteq Q$ and $K \in \mathcal{K}$, specifying the probability of the response pattern R for a subject in state K $\langle 11.1.2 \to 189\rangle$. This term, denoted by the same symbol r, is also used with a germane meaning in a system of stochastic learning paths $\langle 12.2.4 \to 220\rangle$.

response rule. The name of Axiom [R] of a stochastic assessment procedure, which states formally that the probability of a correct response to the question asked on trial n is equal to 1 if the question belong to the latent state of the subject, and to 0 otherwise. (In that context, the careless errors and lucky guesses probabilities are a assumed to be 0.) $\langle 13.3.3 \to 248\rangle$

response pattern. The subset R of the domain Q containing all the questions correctly solved by the subject in the course of the assessment. There are thus $2^{|Q|}$ possible response patterns. $\langle 11.1.2 \to 189\rangle$

return message$^{\triangleleft}$. A message which is both stepwise effective and ineffective for some state is called a return message or, more briefly, a return (for that state). $\langle 10.1.3 \to 167\rangle$

reverse$^{\triangleleft}$. $\langle 10.1.2 \to 166$–$10.1.3 \to 167\rangle$ The reverse of a token τ is a token $\tilde{\tau}$ that annuls the effect of τ. More precisely, for any two adjacent states S and V we have $S\tau = V \Leftrightarrow V\tilde{\tau} = S$. The reverse of a message is defined similarly.

root$^{\triangleleft}$, rooted$^{\triangleleft}$ medium. The root of an oriented medium is a state R such that any concise message from R producing any other state is positive. An oriented medium having a root is said to be rooted. $\langle 10.4.6 \to 176\rangle$

R-store. One of the two buffers of Algorithm 16.2.11. In the second stage of the algorithm, the responses are copied from the `Pending-Table` into the `R-Store` prior to evaluation by the `HS-test`. $\langle 16.2.11 \to 351$ to $16.3 \to 353\rangle$

segment$^{\triangleleft}$. Suppose that $\boldsymbol{n} = \boldsymbol{mpm'}$ is a message in a medium, with $\boldsymbol{m}$ and $\boldsymbol{m'}$ two possibly (but not necessarily) ineffective messages, and $\boldsymbol{p}$ an effective one. Then $\boldsymbol{p}$ is called a segment of $\boldsymbol{n}$. $\langle 10.5.4 \to 180 \rangle$

simple (when applied to a closure space). A closure space $(Q, \mathcal{L})$ is simple when $\varnothing$ is in $\mathcal{L}$. $\langle 3.3.1 \to 46 \rangle$

simple learning model. A quadruple $(Q, \mathcal{K}, p, r)$ in which $(Q, \mathcal{K})$ a discriminative knowledge structure, r is a response function and $p : \mathcal{K} \to [0, 1]$ is defined by by the equation

$$p(K) = \prod_{q \in K} g_q \ \prod_{q' \in K^{\complement}} (1 - g_{q'})$$

in which the g_q's and $g_{q'}$'s are parameters. Moreover, the function p is a probability distribution on $\mathcal{K}$. $\langle 11.4.1 \to 199 \rangle$

skill, skill map. $\langle 6.2.1 \to 106 \rangle$ A triple (Q, S, τ), where Q is a nonempty set of items, S is a nonempty set of skills, and τ is a mapping from Q to $2^S \setminus \{\varnothing\}$. When the sets Q and S are specified by the context, the function τ itself is called the skill map. Any element of the set S is a skill.

skill multimap. A triple $(Q, S; \mu)$, where Q is a nonempty set of items, S is a nonempty set of skills, and μ is a mapping that associates to any item q a nonempty family $\mu(q)$ of nonempty subsets of S. $\langle 6.5.1 \to 112 \rangle$

selective with parameter δ (marking function). In the Markov chain procedure of Chapter 14, a special case of the marking function $\mathfrak{m}$ of the Marking Rule Axiom [M]. $\langle 14.3.4 \to 282 \rangle$

space. Abbreviation for knowledge space. $\langle 2.2.2 \to 27 \rangle$

span of a family of sets $\mathcal{G}$. The family $\mathbb{S}(\mathcal{G})$ containing any set that is the union of any subfamily of sets in $\mathcal{G}$. We say then that $\mathcal{G}$ spans $\mathbb{S}(\mathcal{G})$. We always have $\varnothing \in \mathbb{S}(\mathcal{G})$ because, by convention the union of the empty family equals the empty set. $\langle 3.4.1 \to 47 \rangle$

span† of a family of sets $\mathcal{G}$. The family $\mathbb{S}^{\dagger}(\mathcal{G})$ containing any set that is the union of a nonempty subfamily of sets in $\mathcal{G}$. We thus have $\varnothing \in \mathbb{S}^{\dagger}(\mathcal{G})$ if and only if $\varnothing \in \mathcal{G}$. $\langle 4.5.1 \to 74 \rangle$

state, state$^{\triangleleft}$. Shorthand for knowledge state, that is, a set in a knowledge structure or partial knowledge structure $\langle 2.1.2 \to 23$–$2.2.6 \to 29 \rangle$. In media theory, an element in the set $\mathcal{S}$ of a token system $(\mathcal{S}, \mathcal{T})$ $\langle 10.1.2 \to 166 \rangle$.

stepwise effective$^{\triangleleft}$. (See also 'effective'.) A message $\boldsymbol{m} = \tau_1 \dots \tau_n$ is stepwise effective for a state S if, for $0 \leq i \leq n-1$ and with $S_i = S\tau_0 \dots \tau_i$, we have $S_i \neq S_{i+1}$. (We recall that τ_0 is the identity function, which is not a token.) A message $\boldsymbol{m}$ can be stepwise effective for a state without being effective for that state: we may have $S\boldsymbol{m} = S$. $\langle 10.1.3 \to 167 \rangle$

stepwise path. A stepwise path between two sets F and G in a family of sets $\mathcal{F}$ is a sequence $F = F_0, F_1, \ldots, F_p = G$ of sets in $\mathcal{F}$ such that $d(F_{i-1}, F_i) = 1$ for $1 \leq i \leq p$, with d denoting the symmetric difference distance between sets. $\langle 4.1.3 \to 62 \rangle$

stochastic assessment process. This phrase is used in Chapters 13 and 14, with different meanings distinguished by the qualifiers "continuous" and "discrete", respectively. In Chapter 13 a stochastic process satisfying Axioms [U], [Q] and [R] $\langle 13.3.3 \to 248$–$13.3.4 \to 249 \rangle$. In the Markov chain procedure of Chapter 14, a stochastic process $(\mathbf{R}_n, \mathbf{Q}_n, \mathbf{K}_n, \mathbf{M}_n)$ satisfying Axioms [K], [QM], [RM], and [M] $\langle 14.2.1 \to 278$–$14.2.2 \to 279 \rangle$.

stochastic learning paths (system of). A stochastic process satisfying the Axioms [B], [R], [I] and [L], modeling the successive mastery of items over time. $\langle 12.2.3 \to 218$–$12.2.4 \to 220 \rangle$

straight process. The case of a discrete stochastic assessment process in which the probabilities of careless errors and lucky guesses are both equal to zero; thus $\beta_q = \eta_q = 0$ for any item q. $\langle 14.2.2 \to 279 \rangle$

substructure. See projection.

suffix, suffix$^{\triangleleft}$. The terminal segment of a word $\langle 9.2.1 \to 155 \rangle$, or of a message $\langle 10.5.4 \to 180 \rangle$.

superfluous. A query is superfluous if its response can be inferred from the responses to other preceding queries. $\langle 15.1.3 \to 302 \rangle$

support. In the Markov chain procedure of Chapter 14, the set $supp(\pi)$ of all the knowledge states K having a positive probability, that is, $\pi(K) > 0$. This subset of states is called the support of π. $\langle 14.2.2 \to 279 \rangle$

surmise function, system. Let σ be an attribution on a nonempty set Q of items. The attribution σ is a surmise function if the following three conditions are satisfied for all $q, q' \in Q$, and $C, C' \subseteq Q$:

(i) if $C \in \sigma(q)$, then $q \in C$;
(ii) if $q' \in C \in \sigma(q)$, then $C' \subseteq C$ for some $C' \in \sigma(q')$;
(iii) if $C, C' \in \sigma(q)$ and $C' \subseteq C$, then $C = C'$.

The pair (Q, σ) is then called a surmise system. $\langle 5.1.2 \to 83 \rangle$

surmise relation. The surmise relation of a knowledge structure $(Q, \mathcal{K})$ is the relation $\precsim$ on Q defined by the equivalence $r \precsim q \Leftrightarrow r \in \cap\mathcal{K}_q$. In such a case, we sometimes say that r is surmisable from q or that r precedes q. $\langle 3.7.1 \to 54 \rangle$

system of stochastic learning paths See stochastic learning paths.

tense. $\langle 5.5.4 \rightarrow 93\rangle$ An attribution σ on a nonempty set is tense when for any item q and any clause C for q, there is a state K (in the knowledge space derived from σ) which contains q and includes C but no other clause for q.

tight path. A tight path between two sets F and G in a family of sets $\mathcal{F}$ is a sequence $F = F_0, F_1, \ldots, F_p = G$ of sets in $\mathcal{F}$ such that $d(F_{i-1}, F_i) = 1$ for $1 \leq i \leq p$ with moreover $p = d(F, G)$. (Here, d denotes the symmetric difference distance between sets). $\langle 2.2.2 \rightarrow 27$ and $4.1.3 \rightarrow 62\rangle$

token$^{\triangleleft}$. In a medium $(\mathcal{S}, \mathcal{T})$, an element of $\mathcal{T}$, thus a transformation on the set of states of the medium. $\langle 10.1.2 \rightarrow 166\rangle$

token system$^{\triangleleft}$. A pair $(\mathcal{S}, \mathcal{T})$, with $\mathcal{S}$ a set of states and $\mathcal{T}$ a collection of functions $\mathcal{T} : \mathcal{S} \rightarrow \mathcal{S}$, satisfying the three conditions: (i) $|\mathcal{S}| \geq 2$; (ii) $\mathcal{T} \neq \varnothing$; and (iii) the identity τ_0 is not in $\mathcal{T}$. The elements of $\mathcal{T}$ are called 'tokens.' $\langle 10.1.2 \rightarrow 166\rangle$

trace. Let $(Q, \mathcal{K})$ be a partial knowledge structure and Q' a nonempty proper subset Q' of Q. For any state K in $\mathcal{K}$, the intersection $K \cap Q'$ is called the trace of K on Q'. $\langle 2.4.2 \rightarrow 32\rangle$.

trivial child. A child of a partial knowledge structure $(Q, \mathcal{K})$ is trivial if it is equal to $\{\varnothing\}$. $\langle 2.4.2 \rightarrow 32\rangle$

true state. In the Markov chain procedure of Chapter 14, any state K having a positive probability. The set of those states is the support of the knowledge structure. $\langle 14.2.2 \rightarrow 279\rangle$

uniform extension. Let $\mathcal{K}' = \mathcal{K}_{|Q'}$ be the projection induced by a proper subset $Q' \subseteq Q$, and suppose that $(Q', \mathcal{K}', p')$ is a probabilistic knowledge structure. Then $(Q, \mathcal{K}, p)$ is a uniform extension of $(Q', \mathcal{K}', p')$ to $(Q, \mathcal{K})$ if for all $K \in \mathcal{K}$, we have $p(K) = p'(K \cap Q')/|(K \cap Q')^{\diamond}|$ $\langle 11.7.5 \rightarrow 208\rangle$. We recall that, for any $J \in \mathcal{K}_{|Q'}$, the family $J^{\diamond} = \{K \in \mathcal{K} \,|\, K \cap Q' = J\}$ is the parent family of J $\langle 11.7.2 \rightarrow 207\rangle$.

union-closed family, closed under finite union. A family $\mathcal{F}$ of sets which is closed under union, that is: for any subfamily $\mathcal{G}$ of $\mathcal{F}$, we have $\cup\mathcal{G} \in \mathcal{F}$. Notice that when $\mathcal{G} = \varnothing$ (the empty subfamily), we get $\cup\mathcal{G} = \varnothing \in \mathcal{F}$. A family $\mathcal{K}$ is closed under finite union when, for any K and L in $\mathcal{K}$, the set $K \cup L$ is also in $\mathcal{K}$. Note that, in such a case, the empty set does not necessarily belong to the family $\mathcal{K}$. $\langle 2.2.2 \rightarrow 27\rangle$.

unit support. $\langle 14.2.2 \rightarrow 279\rangle$ In the context of a (discrete) stochastic assessment process, the support of a knowledge structure when it contains only one set.

unitary. A special case of a stochastic assessment process with a ε-half-split questioning function and a marking function which is selective with parameter δ. The knowledge structure is assumed to be well-graded and the parameters ε and δ satisfy particular conditions. $\langle 14.5.3 \rightarrow 285\rangle$

updating rule. A function $u : (K, r_n, q_n, L_n) \mapsto u_K(r_n, q_n, L_n)$ modifying the probability of state K on trial n on the basis of the subject's response r_n to the question q_n on that trial and on the current distribution L_n on the set of states on that trial $\langle 13.3.3 \to 248 \rangle$. The function u computes the probability L_{n+1} of the states on trial $n+1$. Several special cases of the function u are considered $\langle 13.4.2 \to 250$–$13.4.4 \to 251 \rangle$.

vacuous$^{\triangleleft}$. A message $\boldsymbol{m} = \tau_1 \cdots \tau_n$ in a medium is vacuous if its set of indices $\{1, \dots, n\}$ can be partitioned into unordered pairs $\{i, j\}$, such τ_i and τ_j are mutual reverses. $\langle 10.1.3 \to 167 \rangle$

well-gradedness, ∞-wellgradedness. $\langle 2.2.2 \to 27$ and $4.3.3 \to 70 \rangle$ A family of sets $\mathcal{F}$ is well-graded if, for any two sets K and L in $\mathcal{F}$, there is a tight path in $\mathcal{F}$ from K to L. For knowledge structures, the wellgradedness condition implies the finiteness of the family. A generalized version of this concept, called ∞-wellgradedness, applies to the infinite case $\langle 4.3.3 \to 70 \rangle$.

word. A string belonging to a language $\langle 9.2.1 \to 155 \rangle$. See also describes.

yielding. A subset $Q' \subset Q$ of a partial knowledge space $(Q, \mathcal{K})$ is yielding if for any state L of $\mathcal{K}$ that is minimal for inclusion in some equivalence class $[K]$ of the partition induced by Q', we have $|L \setminus \cap [K]| \leq 1$. $\langle 2.4.11 \to 37 \rangle$

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