

Preface

An extraordinary amount of research has been conducted on the general topic of Voting Paradoxes. It has been studied for over two centuries by philosophers, mathematicians, economists, political scientists and other interested people from many different backgrounds. It has fascinated numerous people to think about the very strange and counterintuitive outcomes that might possibly be observed when a group of decision makers, or voters, takes on the task of selecting a winning candidate from a set of available candidates. Books have been written to describe many of these paradoxical outcomes and to categorize them according to the types of unusual behaviors that they display.

The most famous of these paradoxical outcomes is Condorcet's Paradox, or the Condorcet Effect, which is named after the renowned eighteenth century French mathematician-philosopher who formally described the phenomenon. Condorcet wrote at length about the possibility that cyclical majorities on pairs of candidates might occur, and he made some attempts to assess the likelihood that such an outcome might happen. Condorcet was also adamant in his assertion that if some candidate, that we call a Pairwise Majority Rule Winner (PMRW), would be capable of defeating each of the other candidates on the basis of paired comparisons by majority rule, then that candidate should be selected as representing the best choice according to the voters' preferences. As a result, this principle has become known as the Condorcet Criterion.

Much effort has been expended since Condorcet's early work to obtain probability representations for the likelihood that voting paradoxes will be observed in election settings. The basic motivation has been to determine if these possible paradoxical events might actually pose real threats to elections. The level of sophistication of the techniques that have been used to assess the probability that voting paradoxes will be observed has advanced at a very significant rate in recent years. These advances have allowed for the introduction of new dimensions into the formal probability representations that can be obtained. These new dimensions specifically allow for the consideration of the degree to which a group of decision makers, or voters, displays various measures of group mutual coherence. This led to the

ultimate conclusion that while Condorcet's Paradox is a fascinating concept to think about, it should actually be a rare event in actual election settings with a small number of candidates, whenever a group of voters displays any significant level of group mutual coherence for any of a number of possible measures of such coherence.

Given that as a starting point, we began this study with two objectives in mind. First, it was of interest to investigate other voting paradoxes to determine if they too would suffer the same fate of being shown to be interesting phenomena to study, while having very little chance of ever being observed in reality. The second objective resulted from the fact that since Condorcet's Paradox should be a relatively rare event, there is a high probability that a PMRW will exist, to make the Condorcet Criterion very relevant. We therefore wanted to investigate the propensity of common voting rules to elect the PMRW, with an emphasis on an analysis of the impact that various levels of group mutual coherence might have on that outcome.

Our goal throughout was to integrate the theoretical results that we were obtaining from formal probability representations with empirical results from other studies. Some voting paradoxes are definitely more paradoxical than others, and it obviously can not be shown that all voting paradoxes should be very rare events. However, the more extreme paradoxes are generally found to pose very little threat to actual elections, in agreement with empirical findings. The study of the propensities of common voting rules to meet the Condorcet Criterion produces mixed results. Most voting rules can perform very well, depending upon the model that describes the mechanism with which group mutual coherence is attained. However, it is found that while Borda Rule is not always the most effective voting rule for selecting the PMRW in all scenarios, it is resistant to the potential problem of performing very poorly. Moreover, scenarios do exist for all other common voting rules in which the possible outcome of very poor performance is a significant issue. Borda Rule is also found to have a number of very interesting additional properties, to make it a very good choice as a voting rule. This all leads us to suggest the *Borda Compromise* position, to avoid the possibility of poor performance with other voting rules, when nothing is known a priori about the general structure of preferences for a group of voters.

A significant effort was made in our literature search to include references to all work that is directly related to the specific topic of interest. Apologies are extended in advance if we accidentally overlooked some relevant related studies. On a personal note, Gehrlein wishes to extend sincere gratitude to the many people who have been supportive and encouraging through the long course of this project. This particularly includes his wife Barbara Eller, who has been the most supportive and encouraging of all. Lepelley is very grateful to Maurice Salles for introducing him to the wonderful world of Voting Theory, to Bill Gehrlein for his trust and to his wife Françoise for her constant support and patience throughout these last 35 years.

Newark, DE, USA
La Réunion Island, FRANCE
September 2010

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Voting Paradoxes and Group Coherence

The Condorcet Efficiency of Voting Rules

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2011, XII, 385 p., Hardcover

ISBN: 978-3-642-03106-9