Due to the given advantages of BODE representation, it will be consistently utilized in the course of further chapters.

3.1.7 Network Representation of Mass Point Systems

With the preliminary considerations it must be noted that for the present only problems can be modeled which perform only directions of motion along one axis. Other directions of motion must be excluded by suitable bearings. In order to achieve a network model for a real technical configuration, it necessitates the determination of components being essential for the considered function. This is achieved by means of abstraction and simplification compared to the real technical configuration, e.g. with omitting nonessential elements. It makes sense to concentrate rigidly connected masses and to represent existing parallel connected spring elements as one combined component.
This is to be demonstrated by using the example of a simple foundation with a source of force. The foundation mass m rests on several spring and friction components which are connected to the firm ground with their other component sides. The motion of foundation in solely perpendicular direction is obtained by guides. The center of mass provides a reference point. Since all spring elements are connected to the reference point with the one side and to the origin with the other side, thus parallel, all spring elements can be concentrated in accordance with Fig. 3.13. Now they provide a combined spring with compliance n. The same considerations are valid for the friction elements. The new friction element is represented by the frictional impedance r. It is common practice to designate the component friction with the frictional admittance $h = 1/r$ as parameter. On the foundation stands a source of force which is expected to be supported at the origin. In the left part of Fig. 3.19, the mechanical representation of this technical problem is illustrated. Here, the source of force is symbolized by a force arrow pointing to the mass.

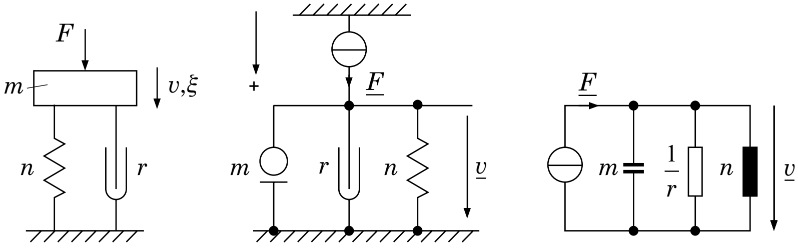


Fig. 3.19. Mechanical representation, mechanical scheme and network representation

A representation level between the mechanical representation and the desired network model provides the *second step* toward a network model. This inter-

mediate step results in the „mechanical scheme". The mechanical scheme is characterized by maintenance of order of the mechanical representation, by replacement of mass elements by their components, by realization of all connections by means of massless and rigid rods and by application of network coordinates. Thus, velocity arrows are plotted across the components, force arrows are plotted into the rods. For that global designation, a positive directional arrow is necessary. If the direction of a force arrow within a connecting rod corresponds to the directional arrow, then, as already discussed before, a pressure state within the rod will be existent (see Sect. 3.1.2). In the middle of Fig. 3.19, the mechanical scheme is illustrated. On the basis of calculation results finally obtained from the network model, motion sequences of real mechanics can be analyzed by correct spatial allocation of the reference points.
In the *third step*, reference points with same direction of motion (e.g. all points of reference at the rigid frame) can be combined. In addition, the sources are usually represented on the left side and the reference points which are connected to the origin, are drawn as bottom line. These rules originate from the representation practice of electrical circuits. The mechanical network representation achieved in this way is shown in Fig. 3.19. It proved to be necessary to proceed in these three steps in order to be able to assign surely the calculation results to the real configuration.
In case of excitation with a force $\underline{F}$, the dynamic behavior of the foundation can be described by calculating the frequency response of velocity $\underline{v}$ or oscillating deflection $\underline{\xi}$. By convention, a sinusoidal excitational force is assumed for this. But it is also possible to calculate system responses, e.g. the response to impulsive or stepwise force excitations. In order to calculate the frequency response, the impedance $\underline{z}$ of the parallel connection of mass, spring and friction is determined at first:

$$\underline{z} = \mathrm{j}\omega m + \frac{1}{\mathrm{j}\omega n} + r$$

$$\underline{z} = \frac{1}{\mathrm{j}\omega n}\left(1 - \omega^2 mn + \mathrm{j}\omega nr\right)$$

In combination with the characteristic values *assigned frequency* ω_0 (resonant frequency) and *quality factor* Q, the frequency response of the oscillating deflection and the oscillating velocity can be formulated in normalized form:

$$\omega_0 = \frac{1}{\sqrt{mn}}, \qquad \omega_0 nr = \frac{1}{Q}$$

$$\underline{z} = \frac{1}{\mathrm{j}\omega n}\left(1 - \left(\frac{\omega}{\omega_0}\right)^2 + \mathrm{j}\left(\frac{\omega}{\omega_0}\right)\omega_0 nr\right)$$

$$\frac{\underline{v}}{\underline{F}} = \underline{h} = \frac{1}{\underline{z}} = \omega_0 n \frac{\mathrm{j}\dfrac{\omega}{\omega_0}}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2 + \mathrm{j}\left(\dfrac{\omega}{\omega_0}\right)\dfrac{1}{Q}} \begin{cases} \mathrm{j}\omega n & \omega \ll \omega_0 \\ Q\omega_0 n & \omega = \omega_0 \\ \dfrac{1}{\mathrm{j}\omega m} & \omega \gg \omega_0 \end{cases} \tag{3.13}$$

The frequency functions are represented in Fig. 3.20. In order that a logarithmic representation of amplitude-frequency responses is achieved, it is necessary to provide dimensionless quantities and to form the absolute values. Typed frequency responses are achieved by a suitable choice of reference quantities. The reduction of the alternating force which is led in the ground often represents the aim of the application of a foundation. Thus, perturbations of environment can be avoided extensively. Now it is easy to calculate the force passing through the springs and friction elements. The force $\underline{F}_\mathrm{B}$ leading in the ground results in:

$$\frac{\underline{F}_\mathrm{B}}{\underline{F}} = \frac{1 + \mathrm{j}\left(\dfrac{\omega}{\omega_0}\right)\dfrac{1}{Q}}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2 + \mathrm{j}\left(\dfrac{\omega}{\omega_0}\right)\dfrac{1}{Q}} \tag{3.14}$$

For large mechanical quality factors Q and for frequencies considerably above the assigned frequency ω_0, the frequency response corresponds approximately to the frequency response of the oscillating deflection (see Fig. 3.20). At resonant frequency, an approximate force $\underline{F}_\mathrm{B} = \underline{F}Q$ affects the ground. Within the considered frequency range, it is considerably larger than without foundation. With foundation and with higher frequencies, the force is considerably smaller than without foundation.

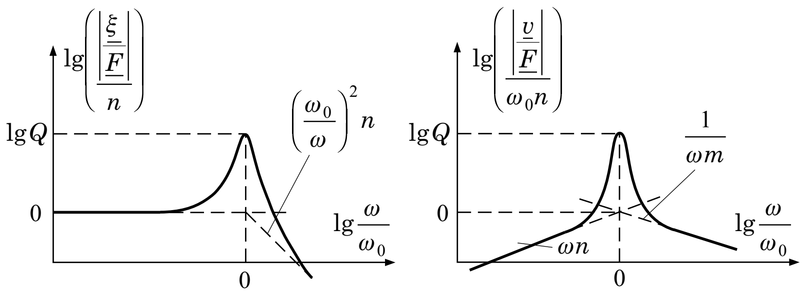


Fig. 3.20. Frequency response of deflection and velocity

3.1.8 Sample Applications

After having considered the quite simple translational system „foundation" in the previous section concerning particularly the aspect of methodical approach, now examples will follow, for which the result is not predictable by implication so easily.

Determination of the Dissipation Factor of a Spring

The relation between mechanical stress T and mechanical strain S within a rod in longitudinal direction and with respect to an allowed transversal contraction is denoted by YOUNG's *modulus* or *modulus of elasticity* E. Here, the relation $E = T/S$ is valid. The relations for a spring without loss presented in this book describe a proportionality between force and deflection. Thus, no phase shift exists between the complex amplitudes of force and deflection.
However, by measurements for nearly all *real* springs a phase shift can be observed. It originates from internal losses caused by deformation of the spring material. It is appropriate to describe these losses by a complex modulus of elasticity. It is expected to be valid

$$\underline{E} = E\,(1 + \mathrm{j}\eta)\,. \tag{3.15}$$

The real part of the complex modulus of elasticity corresponds to the modulus of elasticity mentioned above, whereas the factor η denotes the losses in the material. Generally, the dissipation factor η is frequency-dependent. Mostly, the frequency dependence of the real part of the modulus of elasticity is less than that of the dissipation factor.
The technical circuit interpretation of the complex modulus of elasticity succeeds e.g. by connecting a friction element in parallel to the actual spring element. If the dissipation factor η would be strictly proportional to frequency, the frictional impedance would be constant (frequency-independent). That does not usually apply in such a way. Within a narrow frequency range, e.g. in the neighborhood of the working frequency of a system, a constant frictional impedance can be approximately assumed.
Figure 3.21 shows the circuit representation of a lossy spring by means of a parallel connection. The components of the parallel connection result from following consideration:

$$\underline{z} = \frac{\underline{F}}{\underline{v}} = \frac{AT}{\mathrm{j}\omega l S} = \frac{AE\,(1+\mathrm{j}\eta)}{\mathrm{j}\omega l} = \frac{1}{\mathrm{j}\omega n} + \frac{1}{\left(\dfrac{\omega n}{\eta}\right)} = \frac{1}{\mathrm{j}\omega n} + r$$

Due to the generally quite low losses, for a constant frequency this parallel connection can be also converted to a series connection consisting of a spring and a friction element and showing approximately the same effect.

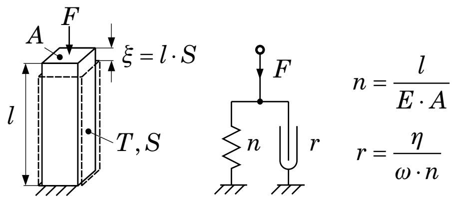


Fig. 3.21. Representation of a lossy spring rod

In practice, losses can often be considered only by specifying estimated resonator performances. Thus, it makes sense to place the component with loss in such a way in the circuit that these estimates can be easily converted to component parameters and that they remain constant within the operating frequency range. In a second step, corrections can be deduced from measured frequency responses or tests of free oscillations by means of a sample set-up providing the model verification.
Considering this described background, a measuring unit for the determination of modulus of elasticity and dissipation factor of a sample of material at a target frequency is of special interest. The sample of a material to be analyzed is set on an oscillating surface. Furhermore, a piece of mass covering all over is put on the sample. Preferably the oscillating surface is represented by the mounting surface of a shaker which can be used for generation of a frequency-variable sinusoidal oscillation. In addition to the sample, also an acceleration sensor is attached to the shaker's mounting surface. The sensor measures the acceleration amplitude of excitation a_0. By means of a second acceleration sensor attached to the mass, the amplitude a_2 is measured. The experimental set-up and the mechanical circuit are illustrated in Fig. 3.22.

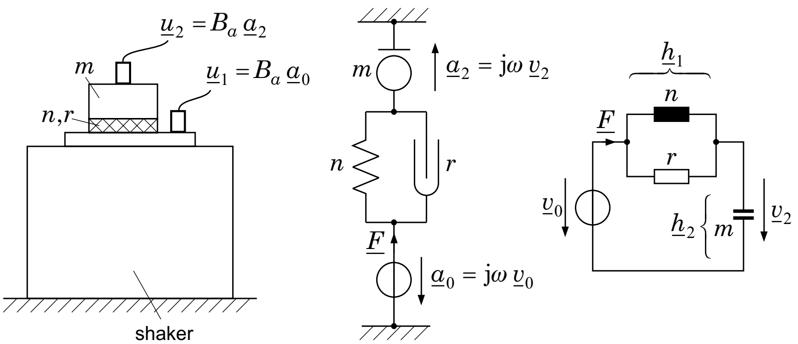


Fig. 3.22. Experimental set-up for determination of modulus of elasticity and dissipation factor

Instead of accelerations, the velocities are charted in the mechanical circuit. The ratio of the absolut amplitude values a_2/a_0 is plotted against the frequency.

The transfer function has the character of a resonant low-pass. By introducing the resonant frequency

$$\omega_0 = \frac{1}{\sqrt{mn}}$$

and the mechanical quality factor

$$Q = \frac{1}{\omega_0 n r},$$

the quantities to be determined result in

$$\eta(\omega_0) = \frac{1}{Q} \qquad \text{and} \qquad E(\omega_0) = \frac{l m \omega_0^2}{A}.$$

In addition to the mechanical dimensions sample height l, sample surface A and mass m (including mass of acceleration sensor), the resonant frequency and the mechanical quality factor have to be determined. If the samples of material do not represent high-loss materials, the resonant frequency can be determined by means of maximum position of the acceleration amplitudes' ratio. The quality factor Q is determined by means of frequency difference $\Delta\omega$ between those two points, at which the transfer function decreases by the factor $1/\sqrt{2}$ with respect to the resonant maximum value (see Fig. 3.23). Thus, the dissipation factor can be formulated in the following way:

$$\eta(\omega_0) = \frac{\Delta\omega}{\omega_0}$$

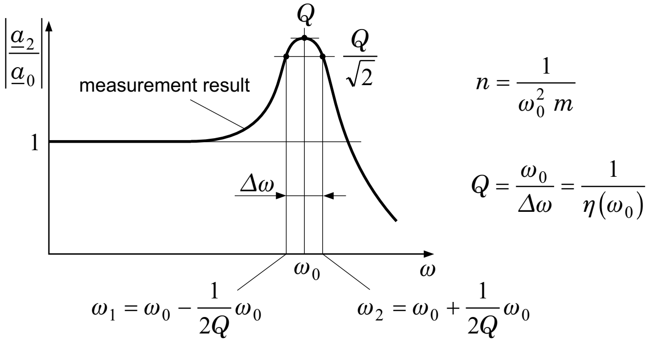


Fig. 3.23. Measurement of resonant frequency ω_0 and quality factor Q

In consideration of high quality factors, this method of determination of quality factor and resonant frequency is very accurate. Even if the quality factor amounts to $Q = 3$, the error of the characteristic values will be less than 3% and is accurate enough for most practical problems.

Vibration Isolation of a Machine

A machine with rotating components which are not completely balanced represents a source of force for that position, at which it is mounted. In order to avoid disturbances of environment, the forces induced by the machine into the mounting place should be as small as possible. For this purpose, usually an elastic element between the machine and the mounting place is attached. Now it is to be analyzed which reduction of force to be induced into the mouting place can be obtained in this way. The model of the machine comprises a rotor with mass m_1 rotating with an imbalance amplitude ϱ and angular velocity ω in a supporting stand with mass m_2. On the one hand, the supporting stand is directly installed on a surface assumed to be stationary (Fig. 3.24 a)), on the other hand, an elastic intermediate layer with internal absorption is installed between housing and surface (Fig. 3.24 b)). Now it is searched for both the force $\underline{F}_1$ which is applied to the mounting place without isolation and the improvement $\underline{F}'_1/\underline{F}_1$ resulting from mounting of the elastic intermediate layer. By means of the assembly, only vertical motions are permitted.
Due to the existing eccentricity ϱ, the shaft performs a motion with a vertical velocity component $\underline{v} = \mathrm{j}\omega\varrho$. The internal admittance $\underline{h}_i$ denotes the ratio of vertical force and velocity with respect to a non-rotating rotor. It is defined by the rotor mass m_1.
The force for short-circuit operation is formally obtained from $\underline{F}_0 = \underline{v}_0/\underline{h}_i = \mathrm{j}\omega m_1 \underline{v}_0$. It is the force which must affect the shaft of the rotating rotor in order to force a rotor velocity of $\underline{v} = 0$. The rotation of rotor with vertically blocked shaft can be replaced by a sinusoidal motion of a mass point with mass m_1 and

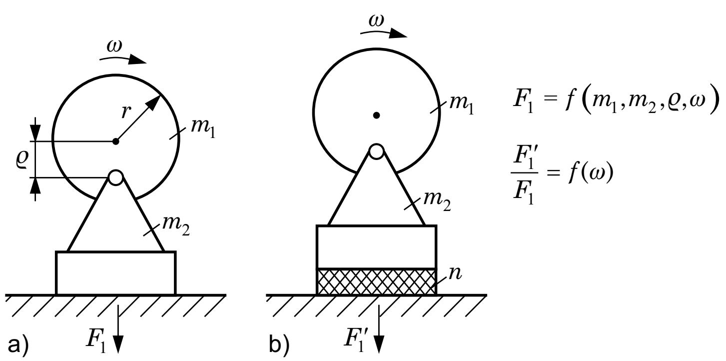


Fig. 3.24. Vibration isolation of a machine with imbalance

amplitude ϱ. This motion generates exactly the force $\underline{F}_0 = m_1\underline{a} = m_1 \mathrm{j}\omega \underline{v}_0$. Thus, the parameters of the active one-port „rotor" are determined. Now there is the task to specify the mechanical scheme for the two cases represented in Fig. 3.24 and to deduce the mechanical circuits. That is accomplished in Fig. 3.25 and Fig. 3.26.

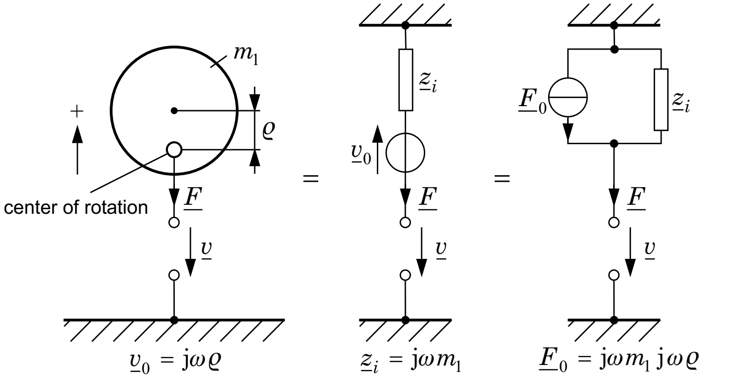


Fig. 3.25. Rotor with imbalance as active mechanical one-port

For case a) of the missing elastic support, the searched force $\underline{F}_1$ is identical to the force of source $\underline{F}_0$. Due to its velocity ($\underline{v} = 0$), the mass m_1 can not absorb forces. For case b), this is not so simple. Here, the force $\underline{F}_0$ must be divided up between both impedances $\underline{z}_1$ and $\underline{z}_2$.

It can be written:

$$\underline{F}_1' = \underline{F}_0 \frac{\frac{1}{\mathrm{j}\omega n} + r}{\frac{1}{\mathrm{j}\omega n} + r + \mathrm{j}\omega\underbrace{(m_1 + m_2)}_{m}} = \underline{F}_0 \frac{\frac{1}{\mathrm{j}\omega n}(1 + \mathrm{j}\eta)}{\frac{1}{\mathrm{j}\omega n}(1 + \mathrm{j}\eta) + \mathrm{j}\omega m}$$

$$\Rightarrow \left|\frac{\underline{F}_1}{\underline{F}_0}\right| = \left|\frac{\underline{F}_1'}{\underline{F}_1}\right| = \frac{\sqrt{1+\eta^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \eta^2}}$$

with

$$\omega_0 = \frac{1}{\sqrt{mn}} \quad \text{and} \quad \eta = r\omega n = \eta(\omega_0)\frac{\omega}{\omega_0}.$$

The elastic support is assumed to be characterized by a frequency-independent friction r. The progression of ratio $\left|\underline{F}_1'/\underline{F}_1\right|$ as a function of frequency is represented in Fig. 3.27.

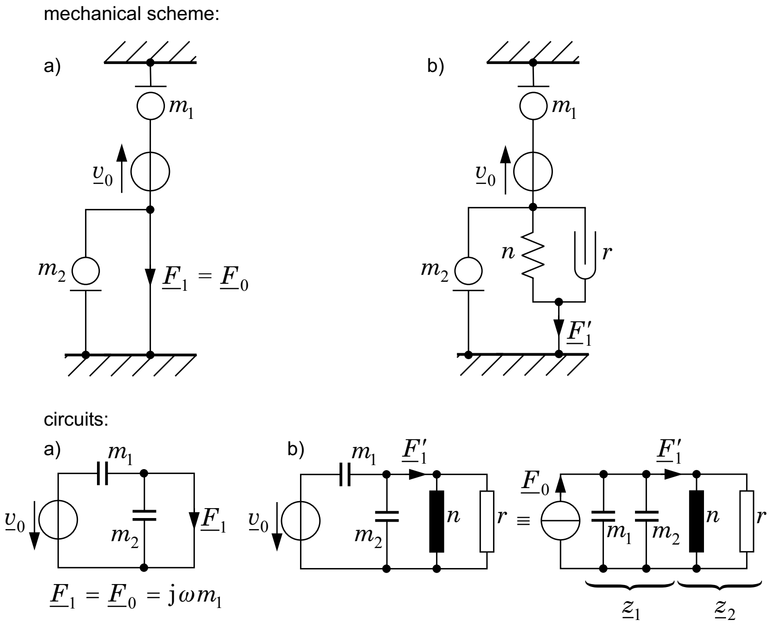


Fig. 3.26. Mechanical scheme and deduced circuits of the configurations illustrated in Fig. 3.24

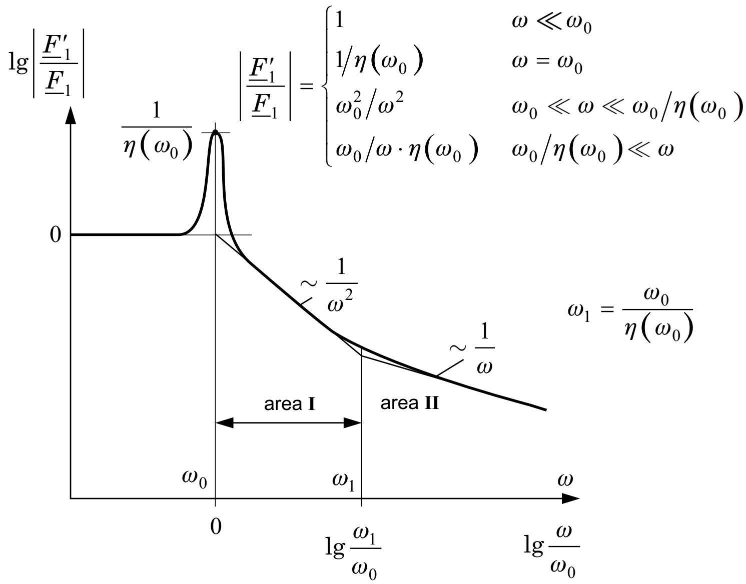


Fig. 3.27. Vibration damping of a vibration-isolated machine

For low frequencies both η^2 and $(\omega/\omega_0)^2$ are much less than 1 and $\underline{F}_1' = \underline{F}_1 = \underline{F}_0$. The elastic support has no influence. For $\omega = \omega_0$ the denominator has its smallest value. $\eta(\omega_0)$ is usually of the order of magnitude of 10^{-1}. The square root in the numerator can be considered to be still 1. The force $\underline{F}_1'$ generated in case of the elastic support is by a factor $1/\eta(\omega_0)$ higher than with permanent coupling. No attenuation but an amplification of the disturbing process is generated.
For $\omega > \omega_0$ the influence of the summand $(1-(\omega/\omega_0)^2)^2 \approx (\omega/\omega_0)^4$ preponderates in the square root of the denominator at first. The ratio $\left|\underline{F}_1'/\underline{F}_1\right|$ decreases with $1/\omega^2$. However, from a certain frequency of ω_1 also $\eta(\omega)$ in the numerator can not be neglected any longer. This frequency is defined by $\eta = 1$. Above ω_1 it is $\sqrt{1+\eta^2} \approx \eta$ and the ratio $\left|\underline{F}_1'/\underline{F}_1\right|$ decreases only with $1/\omega$.

A numerical example is to demonstrate the practically occurring orders of magnitude. The imbalance of a machine may be characterized by a rotor mass of $m_1 = 100\,\mathrm{kg}$ and an imbalance amplitude of $\varrho = 0.1\,\mathrm{mm}$. The operating speed frequency of the machine may be $n = 3000\,\mathrm{min}^{-1} = 50\,\mathrm{Hz}$. The support may be characterized by a resonant frequency of $f_0 = 10\,\mathrm{Hz}$ in combination with the total mass $m_1 + m_2$ of the machine. Its dissipation factor may be $\eta(\omega) = 0.1$ at 10 Hz. So, the amplitude of the force $\underline{F}_0 = \underline{F}_1$ results in

$$\hat{F}_0 = \hat{F}_1 = \omega^2 \varrho m_1 = \begin{cases} 1000\,\mathrm{N} & f = 50\,\mathrm{Hz}, \\ 40\,\mathrm{N} & f = 10\,\mathrm{Hz}. \end{cases}$$

Due to $f_1 = 10 f_0$, for $f = 50\,\mathrm{Hz} = 5 f_0$ you are still in area I illustrated in Fig. 3.27. Therefore, the force $\underline{F}_1'$ results in

$$\text{for} \quad f = 50\,\mathrm{Hz}: \qquad \hat{F}_1' = \hat{F}_1\,(50\,\mathrm{Hz}) \left(\frac{\omega_0}{\omega}\right)^2 = 40\,\mathrm{N}$$

$$\text{for} \quad f = 10\,\mathrm{Hz}: \qquad \hat{F}_1' = \hat{F}_1\,(10\,\mathrm{Hz}) \frac{1}{\eta(\omega_0)} = 400\,\mathrm{N}$$

Thus, compared to the fixed assembly, at operating speed frequency a force reduction around the factor 25 is the result.

Passive Vibration Absorber

The considerations to the translational system „foundation" have shown that the force affecting the ground will only be reduced compared to the source force, if the operating frequency of the force generator is considerably above the resonant frequency. However, depending on the quality factor Q, in the neighborhood of the resonant frequency the ground force is larger. If the ground force ought to be reduced in resonance, a larger attenuation will be necessary. However, the ground force increases with higher operating frequency.

Then the flux of force flows through the damping element into the ground. By applying a second oscillating system, further possibilities are generated which can be utilized purposefully. This additional system is known under the designation „vibration absorber". Such a supplementary unit is appropriate for several fields of application. For the cases outlined in the following sections, a real mechanical system consisting of a simple resonant foundation in combination with an additional oscillating system is represented in Fig. 3.28. The frictional losses in the supplementary system are generated both in the spring and in the region of the guide rods of mass m_2. The guide rods are rigidly connected with the mass m_1. The viscous friction r_2^* may be freely adjustable in this example. Figure 3.28 shows the mechanical scheme of the system and the mechanical circuit, too.
At first, the case of an excitation of foundation with **constant frequency** ω **being effected in direct neighbourhood of the foundation's resonant frequency** ω_0 is to be considered. Without additional oscillating system, a

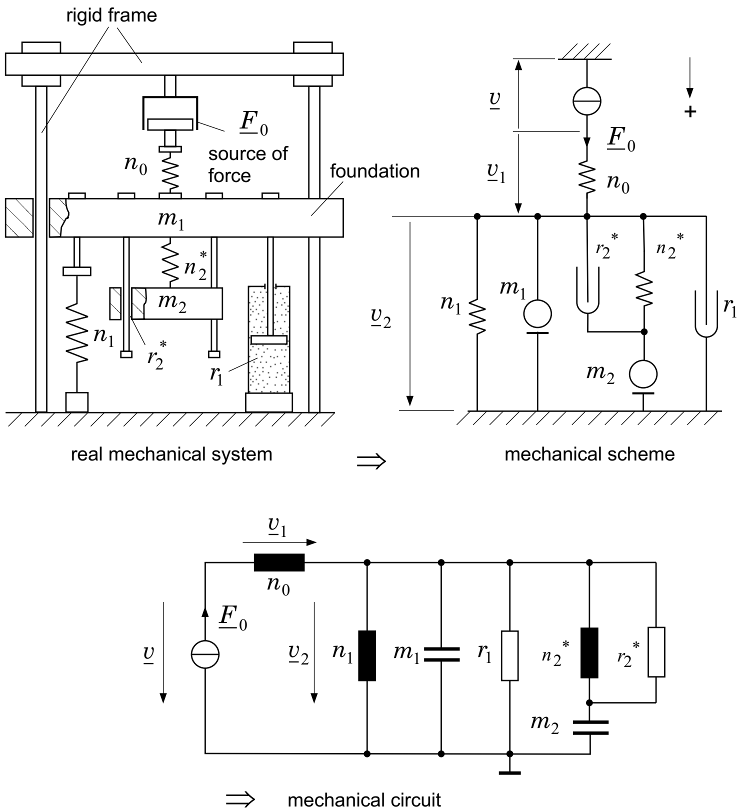


Fig. 3.28. Foundation with vibration absorber

high force amplitude affecting the ground compared to the excitational force amplitude and a high deflection amplitude on foundation mass would be generated in case of this operational mode. A considerable lowering of the resonant frequency ω_0 of the simple resonant foundation could be a solution. However, due to the often limited permitted deflection of foundation mass as a result of the gravity acceleration g

$$\xi_{\text{stat}} = \frac{g}{\omega_0^2}$$

a lowering of the resonant frequency will not be considered. Instead a second mechanical resonator is attached and dimensioned to the same resonant frequency ω_0. In the sample calculation, the mass m_2 of this system may amount to 20% of foundation mass only. In practice, this extension of foundation construction is applicable more easily than the considerable lowering of the resonant frequency specified above. In order to achieve a considerable decrease of the ground force in the neighborhood of resonant frequency, the quality factor of the additional oscillator must be sufficiently high (e.g. higher than 20). For an easy analytical calulation it is convenient to perform approximations and circuit simplifications. So e.g. the parallel connection of spring n_2^* and friction r_2^* can be transformed into a series connection (Fig. 3.29).

For the components it can be written

$$r_2 = \frac{r_2^*}{1+\left(Q_2^* \frac{\omega_0}{\omega}\right)^2} \qquad n_2 = \frac{n_2^*}{1+\left(\frac{\omega_0}{Q_2^* \omega}\right)^2} \qquad \text{with} \qquad Q_2^* = \frac{r_2^*}{\omega_0 n_2^*}.$$

For a high quality factor Q_2^* and for only minor deviation of frequency from the resonant frequency it can be written approximately:

$$\frac{\omega}{\omega_0} \approx 1, \qquad n_2 \approx n_2^* \qquad \text{and} \qquad r_2 \approx \frac{r_2^*}{(Q_2^*)^2}$$

Now the ratio of ground force $\underline{F}$ to excitational force $\underline{F}_0$ can be calculated very easily. The spring n_0 is of no importance for the mentioned problem. At first, the components are combined into complex impedances (Fig. 3.29).

The parallel connection of n_1, m_1 and r_1 provides the complex impedance $\underline{z}_1$. The components n_1 and r_1 are combined into $\underline{z}$. The series connection of n_2, r_2 and m_2 is described by the complex impedance $\underline{z}_2$. Now the force ratios can be described by the impedances $\underline{z}$, $\underline{z}_1$ and $\underline{z}_2$ according to:

$$\frac{\underline{F}_1}{\underline{F}_0} = \frac{\underline{z}_1}{\underline{z}_1 + \underline{z}_2} \qquad \text{and} \qquad \frac{\underline{F}}{\underline{F}_1} = \frac{\underline{z}}{\underline{z}_1}$$

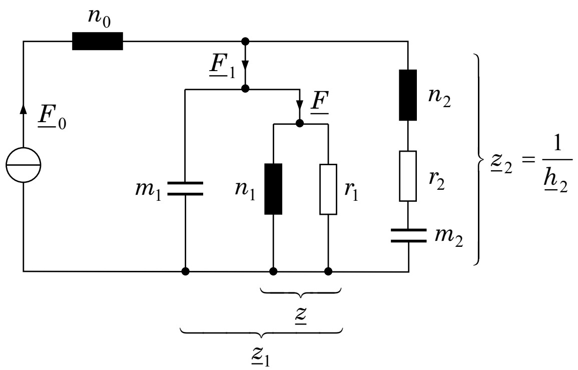


Fig. 3.29. Introduction of ground force $\underline{F}$ and impedances $\underline{z}$, $\underline{z}_1$ and $\underline{z}_2$

This results in:

$$\frac{\underline{F}}{\underline{F}_0} = \frac{\underline{z}}{\underline{z}_1 + \underline{z}_2} = \frac{\underline{z} \cdot \underline{h}_2}{1 + \underline{z}_1 \underline{h}_2} \qquad \text{with} \qquad \underline{h}_2 = \frac{1}{\underline{z}_2}$$

Insertion of the components yields

$$\frac{\underline{F}}{\underline{F}_0} = \frac{\left(\frac{1}{\mathrm{j}\omega n_1} + r_1\right)\left(\mathrm{j}\omega n_2 + \frac{1}{\mathrm{j}\omega m_2} + \frac{1}{r_2}\right)}{1 + \left(\mathrm{j}\omega m_1 + \frac{1}{\mathrm{j}\omega n_1} + r_1\right)\left(\mathrm{j}\omega n_2 + \frac{1}{\mathrm{j}\omega m_2} + \frac{1}{r_2}\right)}$$

and

$$\frac{\underline{F}}{\underline{F}_0} = \frac{(1 + \mathrm{j}\omega n_1 r_1)\left(1 - \omega^2 m_2 n_2 + \mathrm{j}\omega \frac{m_2}{r_2}\right)}{-\omega^2 m_2 n_1 + (1 - \omega^2 m_1 n_1 + \mathrm{j}\omega n_1 r_1)\left(1 - \omega^2 m_2 n_2 + \mathrm{j}\omega \frac{m_2}{r_2}\right)}, \tag{3.16}$$

respectively. With the appropriate *normalized quantities* (2 assigned frequencies and 2 quality factors)

$$\omega_1^2 = \frac{1}{m_1 n_1}, \qquad \omega_2^2 = \frac{1}{m_2 n_2}, \qquad \frac{1}{Q_1} = \omega_1 n_1 r_1, \qquad \frac{1}{Q_2} = \frac{\omega_2 m_2}{r_2}$$

it follows in normalized notation:

$$\frac{\underline{F}}{\underline{F}_0} = \frac{\left(1 + \mathrm{j}\frac{\omega}{\omega_1}\frac{1}{Q_1}\right)\left(1 - \frac{\omega^2}{\omega_2^2} + \mathrm{j}\frac{\omega}{\omega_2}\frac{1}{Q_2}\right)}{-\frac{\omega^2}{\omega_1 \omega_2}\sqrt{\frac{m_2}{m_1}\frac{n_1}{n_2}} + \left(1 - \frac{\omega^2}{\omega_1^2} + \mathrm{j}\frac{\omega}{\omega_1}\frac{1}{Q_1}\right)\left(1 - \frac{\omega^2}{\omega_2^2} + \mathrm{j}\frac{\omega}{\omega_2}\frac{1}{Q_2}\right)} \tag{3.17}$$

According to the assumptions concerning frequency and quality factor, the expression can be simplified:

$$\left| \frac{\underline{F}}{\underline{F}_0} \right| \approx \frac{m_1}{m_2} \frac{1}{Q_2}$$

The smaller the additional mass is chosen, the higher the quality factor must be adjusted, so that an effective decrease of ground force is achieved.

In order to consider a broader frequency range, the mechanical circuit is calculated without using of approximations. Figure 3.30 shows the frequency response of ground force of the circuit illustrated in Fig. 3.28.

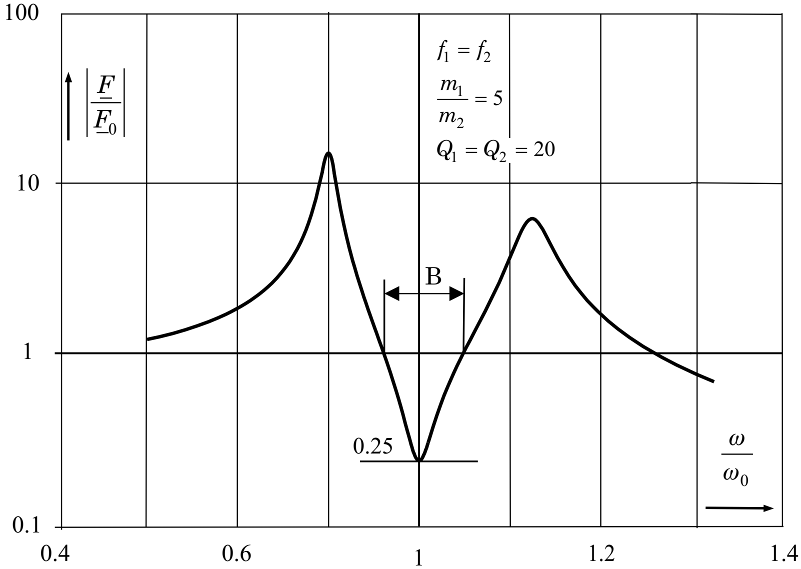


Fig. 3.30. Frequency response of ground force

If the deviation of the operating frequency from the common resonant frequency amounts to more than about 8% (with assumed mass ratio $m_1/m_2 = 5$ and quality factors $Q_1 = Q_2 = 20$), the advantage of the damping effect will be lost. For positive or negative deviations of 20% even new resonance areas are generated. With decreasing ratio m_1/m_2, the frequency area B in Fig. 3.30 increases, in which the operating frequency is allowed to deviate from the common resonant frequency. However, the complexity of the vibration absorber is much higher.
The new resonance areas which can be identified during powering the frequency up to operating frequency, will represent only a problem, if the change

of frequency (speed frequency) of a powering-up or powering-down drive system happens fast enough. The residence time in the critical resonance area must be sufficiently short in order to avoid high oscillation amplitudes.
The system must be dimensioned completely different, if it is to be utilized for the support of a fast decay after **impulsive or sudden loads**. The reduction of motion of a balance plate, on which always approximately same masses are weighed (e.g. in production control of food), represents a typical sample application. Compared to the direct damping of the simple spring-mass system, the application of the vibration absorber enables constructional advantages. For the additional resonator a quality factor of $Q_2^* = 2$ is advantageous. The decay time is longer for lower or higher quality factors. Only mass ratios of 10 or more can be mostly chosen for this application. If the mass ratio amounts to $m_1/m_2 = 10$ and the quality factor of the main oscillator amounts to $Q_1 = 20$, an equivalent quality factor of 5.3 of the total system will be achieved with respect to the additional resonator's quality factor of $Q_2^* = 2$ and thus the decay time will decrease considerably.
The same principle is also used successfully in systems comprising an electromechanical transducer in order to allow for damping the mechanical system electrically. The series connection of a suitably chosen resistor and a suitably chosen inductor in the electrical circuit of a piezoelectrically driven and composed longitudinal oscillator which is to work in pulsed mode, represents an example for that.

3.2 Mechanical Networks for Rotational Motions

For the reference points of mechanical networks concerning translational motions discussed in Sect. 3.1 only motions parallel to a straight line have been allowed. In the following section, only rotations around a fixed axis are allowed. The bearings of this axis are rigidly connected with the surrounding system. The surrounding system (represented by means of a „rigid frame" in the preceding section) is allowed to perform a uniform translational motion at most. Therefore, a surrounding rigid frame is provided with characteristics of an infinitely large mass and an infinitely large rotating mass Θ (see Sect. 3.2.2). With respect to this assumption, a physical structure is available again which is isomorphic in a mathematical sense to the systems discussed before. By means of a suitable coordinate choice, also the topology of the system is consistent with the model representation.
In practice, systems with rotational motions often interact with translational systems by means of transducer elements, e.g. rods connected with axis (see Sect. 5.1). But also torsional oscillation problems, e.g. generated by drive engines and the like, can be addressed by following below-mentioned model approaches. Figure 3.31 shows such a rotational system.
By means of two rods, forces affect an ideal shaft (massless and rigid) in such a way that a torsional moment is induced into the shaft. The torsional

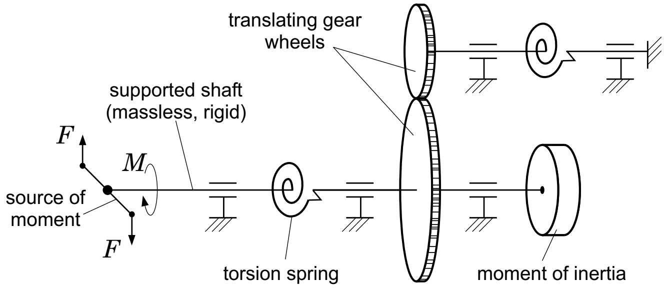


Fig. 3.31. Rotational system

moment results in a torsional twisting of a torsion spring which rests on a rotating mass. In addition, a torsional moment is transmitted to a further torsion spring by means of a gearing. Just like the shaft also the gearing is considered to be massless and rigid. The right connection of this spring is rigidly supported. The torsion angle of the torsion spring is proportional to the torsional moment. The proportionality constant is called *rotational compliance* n_R. The index R will be often omitted, if it concerns a purely rotational system (without transducer elements) and thus it is impossible to mistake it for a translational compliance.

3.2.1 Coordinates

For the description of a uniaxial rotational system it will suffice to use angle coordinates φ_i (cylindrical coordinate system) and torsional moment vectors $\mathbf{M}_i$ or, alternatively, the coordinate torsional moment M in combination with a directional arrow (often drawn as double-headed arrow [43]) and the right-hand rule. However, in order to achieve a system which provides isomorphism to electrical networks while maintaining the topology, it is necessary to apply the torsional moment M and the angular velocity difference

$$\Omega = \frac{\mathrm{d}\varphi_1}{\mathrm{d}t} - \frac{\mathrm{d}\varphi_2}{\mathrm{d}t}$$

(angular velocity across the component) as pair of coordinates. As in case of electrical and mechanical-translational systems, the product of coordinates results in a power quantity. Figure 3.32 shows the complex network coordinates torsional moment $\underline{M}$ and angular velocity $\underline{\Omega}$ using the example of a rotational component. As in case of translational systems, a *directional system arrow* is introduced indicating the positive direction and in order to achieve a uniqueness of signs for a technical circuit representation. The angular velocity $\underline{\Omega}$ will be positive, if both the $\underline{\Omega}$-network arrow and the directional system arrow point to the same direction. A torsional moment along the direction

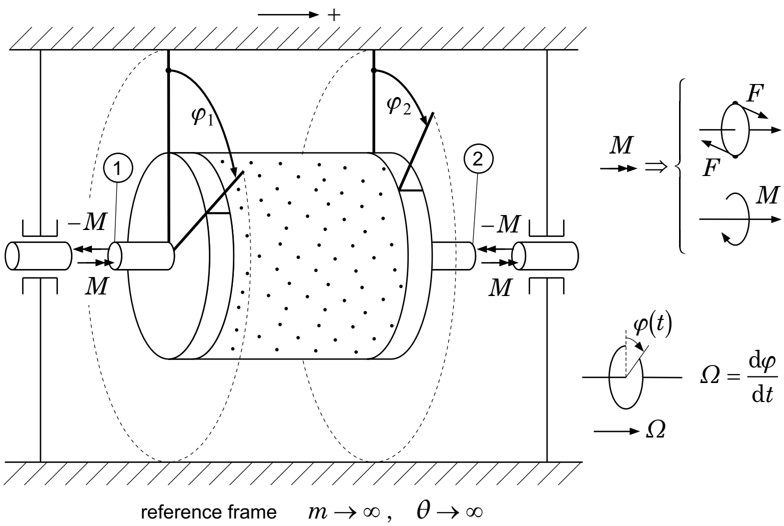


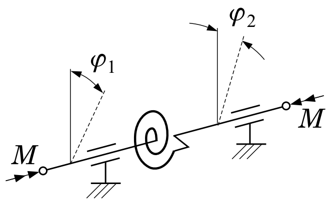
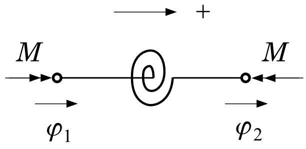
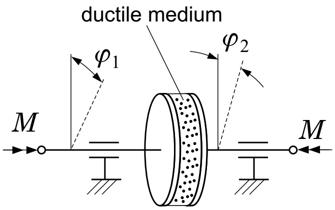
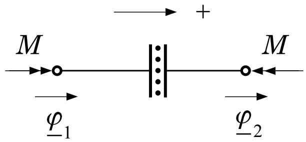
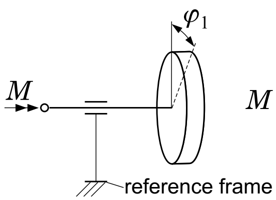
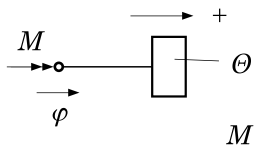
Fig. 3.32. Mechanical coordinates at a general rotational component

of the torsional moment arrow is effective at that side of section, the directional system arrow points to (reference point 1, positive component surface, direction of rotation according to right-hand rule).

3.2.2 Components and System Equations

According to the type of coupling of angular velocity and torsional moment, three different kinds of components can be distinguished within the scope of linear processes. It is referred to the components as torsion spring, torsional friction and rotating mass (moment of inertia). Table 3.4 shows descriptive and schematic representations of rotational components and their system equations. In order to achieve a technical circuit representation, the velocity differences across the elements must be introduced and the rotating mass must get a second reference point. The transition to a technical circuit representation is performed in Table 3.5. In addition to the three types of components, the gearing (analogously to the lever) is specified as „transducer" in the two possible gear wheel configurations.
Analogously to the consideration of translational systems, two source mechanisms are introduced. With the first source mechanism, an angle change is forced by the source (angle source). With the second source mechanism, the source generates a torsional moment. Its value does not depend on the occurring angle change. Table 3.6 shows the mechanical and technical circuit representation of these two source types. Active rotational one-ports (source with source impedance) can be represented by means of these sources. In Sect.

Table 3.4. Rotational components - descriptive and schematic representation

	descriptive representation	schematic representation
torsion spring	 $\varphi = \varphi_1 - \varphi_2 = n_{\mathrm{R}} M$	 $\varphi_1 - \varphi_2 = n_{\mathrm{R}}\ M$
torsional friction	 $M = r_{\mathrm{R}}\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}t} - \frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right) = r_{\mathrm{R}}\ \Omega$	 $M = r_{\mathrm{R}} \frac{\mathrm{d}}{\mathrm{d}t}(\varphi_1 - \varphi_2)$
rotating mass	 $M = \Theta \frac{\mathrm{d}^2\varphi_1}{\mathrm{d}t^2}$ $\varphi_2 = 0$	 $M = \Theta \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$ φ is measured in the inertial frame

3.2.4, it is exemplified that KIRCHHOFF's laws can be formed analogously to translational systems. The interconnection rules of Sect. 3.1.4 can also be applied formally and conceptually to rotational networks.

3.2.3 Isomorphism between Mechanical and Electrical Circuits

By comparing the equations describing completely rotational networks with appropriate equations of electrical networks, the isomorphism of both physical structures existing in a mathematical sense can be distinguished. The identity will also relate to the topological structure, if the electric voltage is assigned to the angular velocity and the electric current is assigned to the torsional

Table 3.5. Technical circuit representation of rotational components

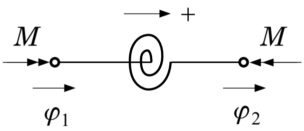
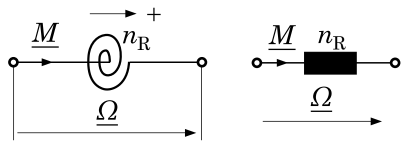
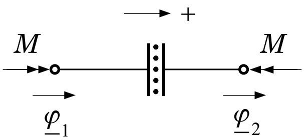
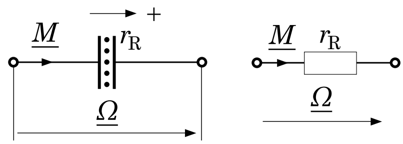
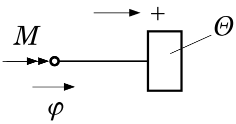
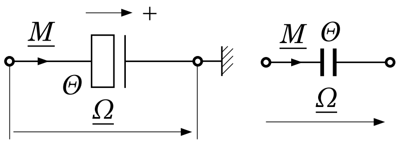
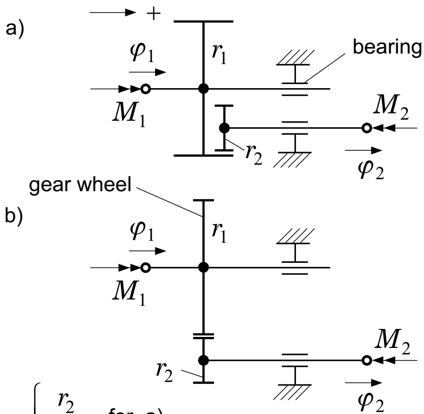
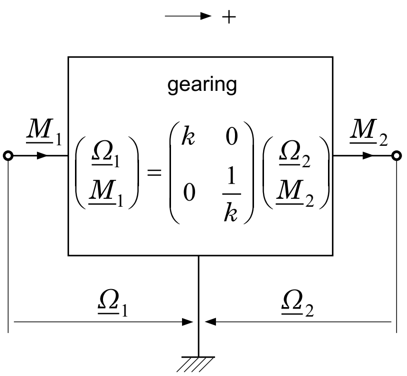
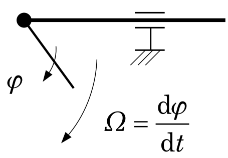
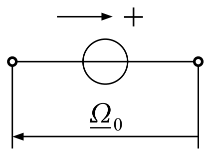
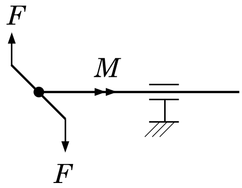
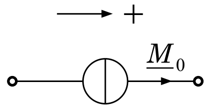
schematic representation	technical circuit representation
 $\varphi_1 - \varphi_2 = n_R\ M$	 $\underline{\Omega} = j\omega\ n_R\ \underline{M}$
 $M = r_R \dfrac{d}{dt}(\varphi_1 - \varphi_2)$	 $\underline{M} = r_R\ \underline{\Omega}$ $\underline{M} = j\omega\,(\varphi_1 - \varphi_2)\cdot r_R$
 $M = \Theta \dfrac{d^2\varphi}{dt^2}$ φ is measured in an inertial frame	 $\underline{M} = j\omega\,\Theta\,\underline{\Omega}$
 $k = \begin{cases} \dfrac{r_2}{r_1} & \text{for a)} \\ -\dfrac{r_2}{r_1} & \text{for b)} \end{cases}$ $\varphi_1 = k\,\varphi_2$ $M_1 = \dfrac{1}{k} M_2$	

Table 3.6. Rotational sources

	mechanical representation	technical circuit representation
angle source		
moment source		

moment. The appropriate relations between electrical and rotational system are summarized in Table 3.7.

In order to provide also quantitative relations, proportionality quantities must be defined between the coordinates. It is to be valid:

$$\underline{u} = G_3 \underline{\Omega} \qquad \text{and} \qquad \underline{i} = \frac{1}{G_4} \underline{M} \tag{3.18}$$

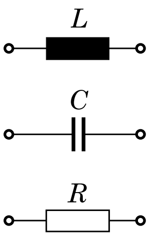
In addition, it could be allowed that the frequencies between the electrical and the mechanical network differ in a constant factor. In practice, no use is made of this possibility. Therefore, it is not used in the following. By means of this definition, it can be written for the components:

$$C = \frac{\Theta}{G_3 G_4}, \qquad L = G_3 G_4 n_{\mathrm{R}} \qquad \text{and} \qquad R = G_3 G_4 h_{\mathrm{R}} \tag{3.19}$$

At first, the porportionality factors G_3 and G_4 are arbitrary with respect to their absolute values. However, with regard to the characteristics of network analysis programs (e.g. pSpice), the sequences of numbers should be maintained. Thus, only the powers of ten are available. If same numerical values are used for G_3 and G_4, a representation with equal power rating will be generated. It is usually made use of this advantage.

Table 3.7. Isomorphic correlations between electrical and rotational network

correlations between coordinates and components				
voltage	$\underline{u}$	○—○	$\underline{\Omega}$	angular velocity
current	$\underline{i}$	○—○	$\underline{M}$	torsional moment
inductance	L	○—○	n_R	rotational compliance
capacitance	C	○—○	Θ	rotating mass (moment of inertia)
resistance	R	○—○	h_R	torsional friction admittance
transformer	$\frac{w_1}{w_2}$	○—○	k	see figure gearing



$\underline{u} = \mathrm{j}\omega L \underline{i}$

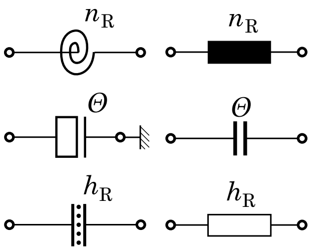
$\underline{u} = \frac{1}{\mathrm{j}\omega C}\underline{i}$

$\underline{u} = R\underline{i}$

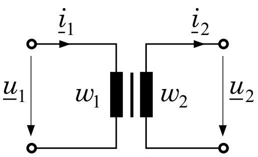
$\underline{\Omega} = \mathrm{j}\omega n_\mathrm{R} \underline{M}$

$\underline{\Omega} = \frac{1}{\mathrm{j}\omega\Theta}\underline{M}$

$\underline{\Omega} = h_\mathrm{R}\underline{M}$

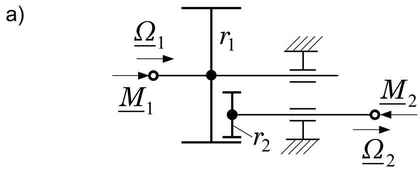


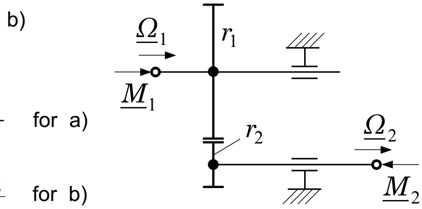
node of electrical circuit structure	$\sum_{*} \underline{i}_\nu = 0$	$\sum_{*} \underline{M}_\nu = 0$	node of mechanical scheme
loop of electrical circuit structure	$\sum_{\circlearrowright} \underline{u}_\nu = 0$	$\sum_{\circlearrowright} \underline{\Omega}_\nu = 0$	loop of mechanical scheme



$$\underline{u}_2 = \frac{w_2}{w_1}\underline{u}_1$$

$$\underline{i}_2 = \frac{w_1}{w_2}\underline{i}_1$$





$$k = \begin{cases} \frac{r_2}{r_1} & \text{for a)} \\ -\frac{r_2}{r_1} & \text{for b)} \end{cases}$$

$$\underline{\Omega}_2 = \frac{1}{k}\underline{\Omega}_1 , \quad \underline{M}_2 = k\,\underline{M}_1$$

3.2.4 Sample Application for a Rotational Network

The deduction of a technical circuit representation from a real rotational configuration should happen again in two steps. While maintaining the geometrical configuration, at first it is appropriate to sketch a representation in which the branching of the torsional moment is distinguishable. One part of the drive torque applied to an axis is absorbed by bearing friction, another part is branched off for acceleration of rotating masses. In a schematic representation this should be demonstrated by means of a moment node. The branch point can be considered to be a gearing fitted on the axis with a gear ratio of one (Fig. 3.33). Then the new output axis supplies the rotational friction or the rotating mass. By means of this separation of the moment flows into several imaginary parallel axis, the separation into network legs is easy to imagine.

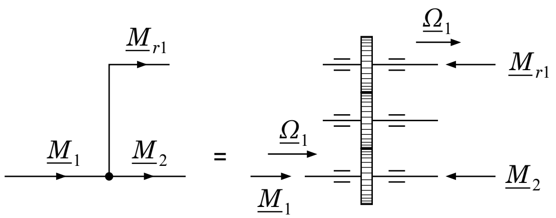


Fig. 3.33. Model of a torsional moment branching (node)

Using the example of a gas turbine, in Fig. 3.34 a system consisting of a source of moment, frictional elements, flexible clutch element and a vibration absorber is represented. A rotational load can be connected to the system output. The moment branchings into the frictional elements and rotating masses specified before are shown in the mechanical scheme. A torsion spring represents the flexible coupling. By means of this spring, the torsional moment is transmitted to the output side. A difference of angular velocity is generated across the clutch. But the flexible clutch comprises also parts with mass. They are represented by the moments of inertia Θ_2 and Θ_3. The vibration absorber consists of a ring with the moment of inertia Θ_4 which is connected to the main axis by means of 4 flexible springs. As a whole the flexible springs represent the torsion spring n_{R2}. The mechanical scheme comprises only rotational components. Thus, a mix-up with translational components is excluded. In this case, the index R can be omitted. Here, a connected load is indicated as a general component with resistance symbol r_L, since the character of this load is unknown.

The mechanical circuit represents the third representation stage. The references to direction and position can be canceled. The representation rules for networks are valid. Now the representation as active source one-port with connected general load is completed.

http://www.springer.com/978-3-642-10805-1

Electromechanical Systems in Microtechnology and Mechatronics
Electrical, Mechanical and Acoustic Networks, their Interactions and Applications
Lenk, A.; Ballas, R.G.; Werthschützky, R.; Pfeifer, G.
2011, XXIV, 472 p. 492 illus., Hardcover
ISBN: 978-3-642-10805-1