Development of a Computational System to Determine the Optimal Bus-stop Spacing in order to Minimize the Travel Time of All Passengers

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1 Introduction

One of the main concerns regarding urban planning nowadays is public transportation. The great number of vehicles in the main cities has been causing many problems, from infrastructure (number of vehicles over street capability), trough safety (high accident rates) and environmental issues (high pollution rates), among others. In infrastructure, one of the problems caused by the large number of vehicles on the streets is the travel time between two locations.

These problems aggravate specially in big cities, where traffic jams have already become part of the urban landscape.

However, one of the main aspects to be considered in a public transportation system is the travel time of the passenger using a bus line. The number of stops affects the total travel time deeply. In this manner, the number stops must be chosen very carefully, in a way that the bus lines become more appealing to the

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users. With a large number of stops, the user walks very little, but it makes the trip too long and unpleasant for those who travel a long distance in that line. On the other hand, too few stops make the trip faster, but the passengers also have to walk more to get to the bus-stop, as well as to the final destination.

Ammons (2001) studied various spacing patterns between bus-stops around the world and concluded that the average spacing is from 200 to 600 m in urban areas. Reilly (1997) noticed that the European traffic departments have different standards to determine the spacing between bus-stops. In Europe, there are 2–3 stops per kilometer, which means that the spacing is from 330 to 500 m, in opposition to United States standards, where the stops are spaced from 160 to 250 m.

These studies show that the distance between stops does not follow a scientific procedure, or even based on predefined methodological studies. According to Kehoe (2004), in many routes along the USA, the bus-stops were defined trough time, as a result of user's requests to authorities and/or bus companies. Because the stops were based on citizen's needs, altering the distance between stops becomes a complicated process, for the population has already grown accustomed to the original spacing.

These remarks lead to the following question:

How will the ideal number of stops be determined in order to optimize the line for the users?

To answer this question we combined the concepts of non-linear programming (Frielander 1994) and Voronoi diagram. Voronoi diagrams has been around for at least four centuries, and many relevant material can be found in many areas, such as anthropology, archeology, astronomy, biology, cartography, chemistry, computational geometry, ecology, geography, geology, marketing, meteorology, operations research, physics, remote sensing, statistics, and urban and regional planning (Novaes 2000).

The concept of Voronoi Diagram is very simple. Given a finite set of distinct, isolated points in a continuous space, we associate all locations in that space with the closest member of the point set. The result is a partitioning of the space in a set of regions where each region is related to only one of the points of the original set.

Since the 1970s, algorithms for computing Voronoi diagrams of geometric primitives have been developed in computational geometry and related areas. There are several ways to construct a Voronoi diagram. One of the most practical is the incremental method, described in Novaes (2000). This method is also one of the most powerful in the subject of numerical robustness. The total time complexity for this method is of $O(n^2)$. However, the average time complexity can be decreased to $O(n)$ by the use of special data structures as described in Novaes (2000), p. 264.

2 Concepts of Voronoi Diagram

In this section we will define the main concepts and some properties of Voronoi Diagrams. The concepts presented in this section were based in Okabe et al. (1992).

2.1 Definition of a Planar Ordinary Voronoi Diagram

Given a set of two or more but a finite number of distinct points in the Euclidian plane, we associate all locations in that space with the closest member(s) of the point set with respect to the Euclidean distance. The result is a tessellation off the plane into a set of regions associated with members off the point set (Okabe et al. 1992).

The mathematical definition is the following:

$$V(p_i) = \left\{x \middle| \|x - x_i\| \leq \left\|x - x_j\right\| \, for \; j \neq i, j \in I_n\right\} \tag{1}$$

where V is the planar ordinary Voronoi diagram associated with p_i and the set given by:

$$V^o = \{V(p_1), \ldots, V(p_n)\} \tag{2}$$

An example of an ordinary Voronoi diagram is presented in Fig. 1.

2.2 Definition of a Multiplicatively Weighted Voronoi Diagram

In the ordinary Voronoi diagram, we assume that all generator points have the same weight. But, in many practical applications, we may have to assume that they have different weights in order to represent, for example the population of a city, or the level of hazardousness that an accident at a point can cause.

Voronoi diagrams for the weighted distance are more complicated to analyze. The sides of the polygons are no longer straight lines but are arcs of circles.

The multiplicative weighted Voronoi diagram is characterized by the weighted distance calculated by

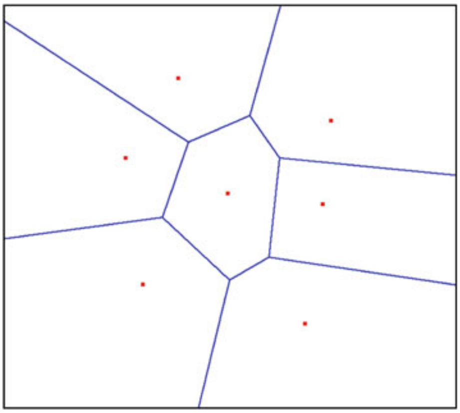


Fig. 1 Example of ordinary Voronoi diagram

$$d_{mw}(p, p_i) = \frac{1}{w_i}||x - x_i||, \quad w_i > 0 \tag{3}$$

where w_i is the weight associated with each point i. After a few steps of calculation, we obtain a bisector that is defined by

$$b(p_i, p_j) = \left\{ x \middle| \left\| x - \frac{w_i^2}{w_i^2 - w_j^2} x_j + \frac{w_j^2}{w_i^2 - w_j^2} x_i \right\| = \frac{w_i w_j}{w_i^2 - w_j^2} ||x_j - x_i|| \right\} \tag{4}$$

This bisector is the set of points that satisfy the condition that the distance from p to the point defined by

$$\frac{w_i^2 x_j}{w_i^2 - w_j^2} - \frac{w_j^2 x_i}{w_i^2 - w_j^2} \tag{5}$$

is constant. The bisector is a circle in $\mathbb{R}^2$. So, the dominance region of p_i over p_j with the weighted distance is written by:

$$\mathrm{Dom}(p_i, p_j) = \left\{ x : \frac{1}{w_i}||x - x_i|| \leq \frac{1}{w_j}||x - x_j|| \right\}, \quad i \neq j \tag{6}$$

Figure 2 is an example of a multiplicatively weighted Voronoi Diagram with the coordinates of the points inside the parenthesis and the weights associated to them outside.

2.3 *Definition of an Additively Weighted Voronoi Diagram*

Similarly to the multiplicative weighted Voronoi diagram, the additively weighted Voronoi diagram (Fig. 3) is characterized by the weighted distance calculated by

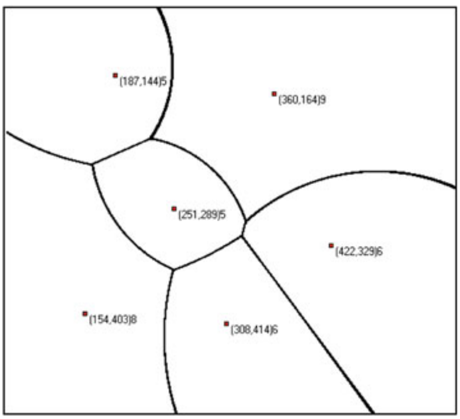


Fig. 2 Example of a multiplicatively weighted Voronoi diagram

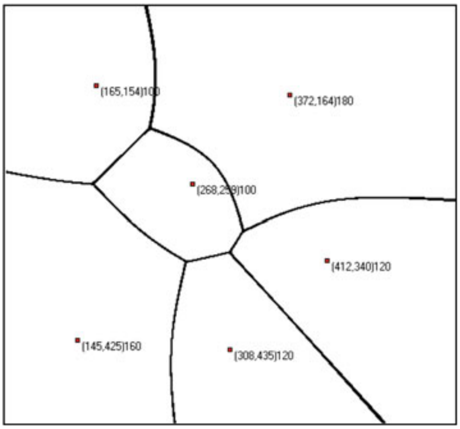


Fig. 3 Example of an additively weighted Voronoi diagram

$$d_{mw}(p, p_i) = ||x - x_i|| - w_i \tag{7}$$

2.4 Definition of a Compoundly Weighted Voronoi Diagram

Similarly to the multiplicative and additively weighted Voronoi diagram, the compoundly weighted Voronoi diagram is characterized by the weighted distance calculated by

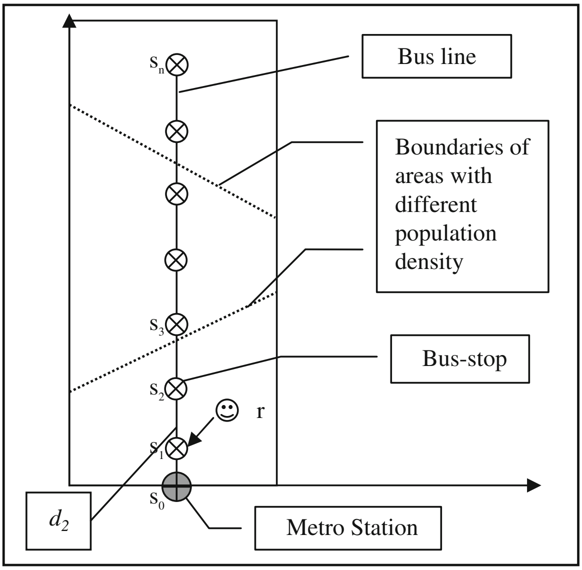
$$d_{mw}(p, p_i) = \frac{1}{y_i}||x - x_i|| - w_i \tag{8}$$

3 Definition of the Problem

This paper presents a model to optimize the bus-stop spacing of a bus line located in the city of São Paulo, Brazil. The city of São Paulo has approximately 10.5 million people and the public transportation system has about six million users daily (SPTrans—São Paulo Transportes S.A—http://www.sptrans.com.br).

The bus line that will be used in this paper is a new line that will be operational after the completion of the metro line number 4. That new metro line will be ready for use in 2010. The bus line was projected to transport passengers from the west region of the city to a metro station. At the station the passengers can switch to the metro line in order to get to their final destination. The purpose of the system is to find the optimal bus-stop spacing in order to minimize the total travel time of the passenger that goes from any point of the region to the metro station. The whole itinerary has 6.4 km.

Fig. 4 The bus line in a Cartesian plane



4 Formulation of the Model

Figure 4 shows the bus line in a Cartesian plan. The pattern on population density $\Phi(x, y)$ will be defined in relation with the variables 'x' and 'y'. The situation presented here is that the users in this area use bus lines to get to a bus terminal.

The area that this specific bus line can reach is called S. The ensemble of bus stops in the lines is represented by s_i where $i \in I_n$ and the number of bus stops is $n + 1$. There is also the distance d_i that represents de spacing between bus stops i and $i - 1$.

Considering that a user is at a specific point r (x_0, y_0) and wishes to go to bus stop s_1 (x_1, y_1), being $r, s_1 = S$. The travel distance D_a will be calculated as shown:

$$D_a = k * \|r - s_1\| = k * \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \tag{9}$$

In this case, k is a correction factor to approximate the Euclidean distance to a walk distance. In this paper the value used is 1.3, as described in (Novaes 2000).

The time T_a that takes to go to the bus stop is calculated dividing the distance D_a by the user's speed on foot V_a. Which means:

$$T_a = \frac{D_a}{V_a} \tag{10}$$

Saka (2001) showed how to calculate how much time the bus takes to reach its destination (the terminal). The time is calculated as shown:

$$T_{\text{bus}} = T_{\text{ad}} + T_{\text{ed}} + T_{\text{c}} + T_{\text{o}} \tag{11}$$

In which:

a) T_{ad} = acceleration and deceleration time;
b) T_{ed} = passenger boarding and disembarkation time;
c) T_{c} = time delay due to traffic control (traffic lights, etc.);
d) T_{o} = travel time in normal traffic speed.

The total time that is lost in each bus stop can be represented as:

$$T_s = T_{\mathrm{ad}} + T_{\mathrm{ed}} \tag{12}$$

Adjusting to this particular case, as it is meant to calculate the ideal spacing between bus-stops, it is possible to eliminate the time delay due to traffic control (traffic lights, etc.) in order to concentrate the calculations in the time spent in the bus stops. So, we can say that the total travel time from stop i until the final destination is:

$$T_{\mathrm{bus}} = (T_s * i) + \frac{D_i}{V_b} \tag{13}$$

where D_i is the distance from bus stop i to the terminal and V_b is the average speed of the bus on the route. And the total time is calculated as follows:

$$T_{\mathrm{tot}} = T_{\mathrm{a}} + T_{\mathrm{bus}} \tag{14}$$

And the total time from bus stop i is

$$T_{\mathrm{tot}} = k\frac{\|r - s_i\|}{V_a} + (T_{\mathrm{s}} * i) + \frac{D_i}{V_b} \tag{15}$$

Assuming that every user will board in the bus-stop that minimizes his/her travel time, he/she will board the bus-stop that meets the following equation:

$$\mathrm{Min}_i\left\{k\frac{\|r - s_i\|}{V_a} + (T_s * i) + \frac{D_i}{V_b}\right\} \tag{16}$$

As a result of this factor, every stop will have its target area defined with:

$$V_i = \left\{r|k\frac{\|r - s_i\|}{V_a} + (T_s * i) + \frac{D_i}{V_b} \leq k\frac{\|r - s_j\|}{V_a} + (T_s * j) + \frac{D_j}{V_b}; i \neq j; i,j \in I_n\right\} \tag{17}$$

which becomes, as described earlier, a additively weighted Voronoi region.

The total travel time from all passengers in the studied area will be calculated by:

$$T = \sum_{i=1}^{n} \int_{Vi} \left\{k\frac{\|r(x,y) - s_i\|}{V_a} + (T_s * i) + \frac{D_i}{V_b}\right\} \phi(x,y)ds \tag{18}$$

This way, the optimization function is:

$$\min_{d_1, d_2, \ldots, d_n} \sum_{i=1}^{n} \int_{Vi} \left\{ k \frac{\| r(x, y) - s_i \|}{V_a} + (T_s * i) + \frac{D_i}{V_b} \right\} \phi(x, y) ds \tag{19}$$

That is a non restricted non-linear programming function.

5 The System

The system that was designed to solve that problem was written in Delphi 6.0 and utilizes some heuristics that are available in the literature.

For the non-linear programming problems we implemented three different heuristics: the Gradient method, the Conjugated Gradient method (Fletcher and Reeves 1964) and the Davidon-Fletcher-Powel method. Those methods are described by Luenberger (2005).

The system also can use three different rules to stop the line search: Armijo, Wolfe and Goldstein.

The gradient of the function is calculated using the Ridders' method of polynomial extrapolation described in Press et al. (2002).

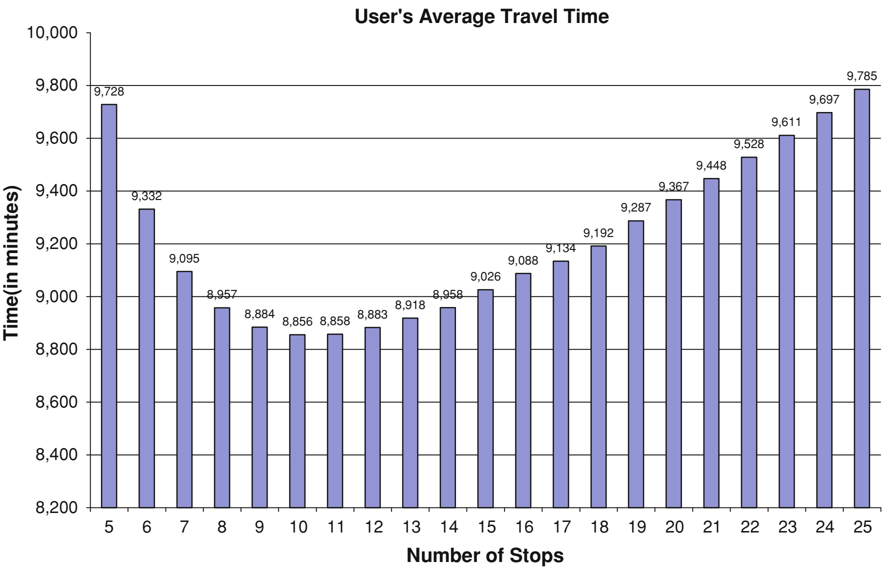


Fig. 5 Result obtained by the system

6 The Result

The system was used to find the optimal solution for the number of stops from $n = 5$ until $n = 25$. The solutions found indicates that the smallest total travel time is obtained with $n = 10$. Looking at the results in Fig. 5, we can see that if you increase the number of stops the user's average travel time will increase.

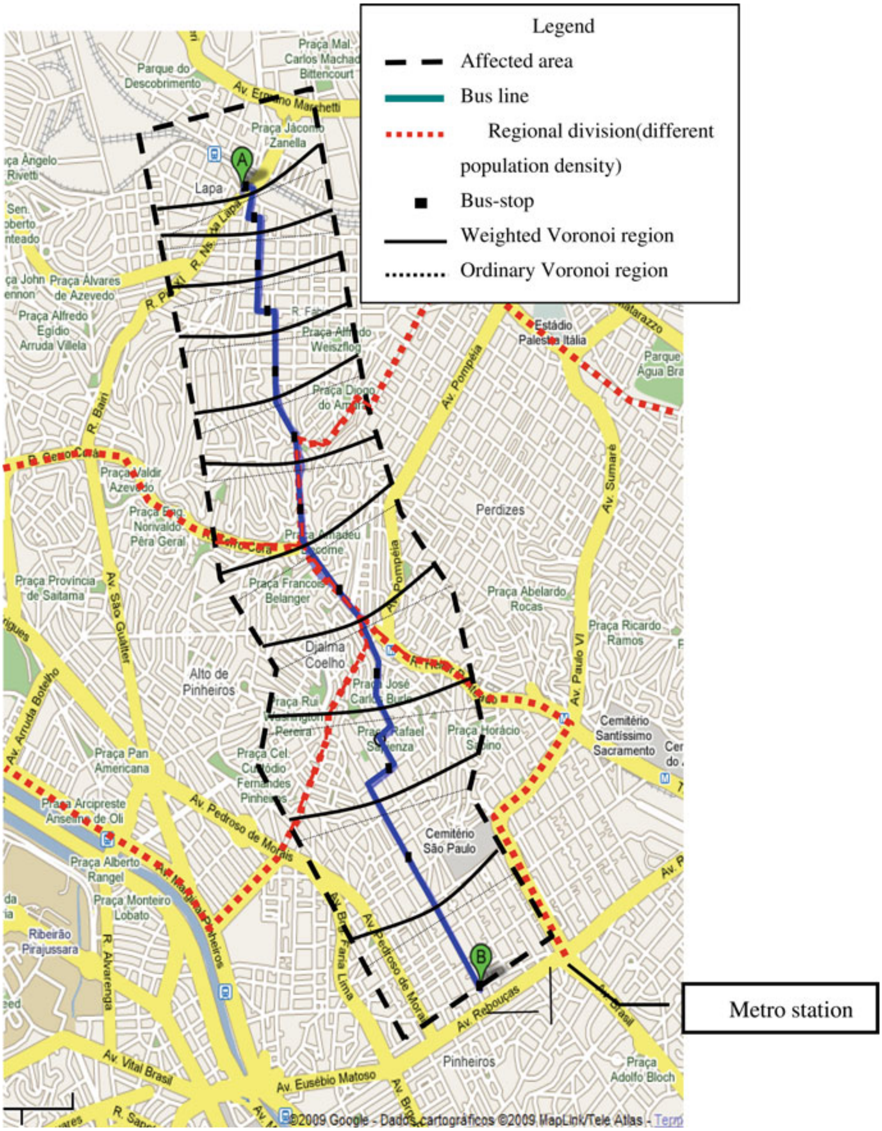


Fig. 6 Result obtained by the system

This result shows an average distance of 620 m from each bus stop. Using the usual bus-stop spacing of 250 m that is the most common in the city of São Paulo, we will have an increase of the travel time in more than 15%. This difference in a universe of six million passengers can be very significant.

Figure 6 shows a map with the location of the bus-stops obtained by the system. The figure shows the bus line and the area affected for it. Is this case we used 600 m for each side of the bus line to be the limit of the area affected by the bus line.

The map also shows the Voronoi regions (ordinary and weighted) associated to each bus-stop. The ordinary Voronoi region is the area where the user will find the nearest bus-stop from his location. The weighted Voronoi region is the area where the user will find the bus-stop that will take him to his destination in the smallest amount of time.

7 Conclusions

In this paper, we designed and implemented a system to find the optimum bus-stop spacing in order to minimize the total travel time of the passengers of a bus line that goes to a final destination. The system was developed using concepts of non-linear programming and Voronoi diagrams. The idea was to use both concepts together to find the optimal solution for the problem. The results showed that there is an optimal number of bus-stops but if this number is increased a little, it will not compromise the solution too much and it will make the user walk less. But if we compare the actual bus-stop spacing with the optimal one found by the system, we can observe that the travel time can be decreased in more than 15%.

The result can also be used as a parameter to be combined with others, like cost, number of vehicles, etc. in order to design a new line or to improve an existing one.

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