

Factorial Conjoint Analysis Based Methodologies

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Abstract Aim of this paper is to underline the main contributions in the context of Factorial Conjoint Analysis. The integration of Conjoint Analysis with the exploratory tools of Multidimensional Data Analysis is the basis of different research strategies, proposed by the authors, combining the common estimation method with its geometrical representation. Here we present a systematic and unitary review of some of these methodologies by taking into account their contribution to several open ended problems.

1 Introduction

Conjoint Analysis [15, 16] is one of the most popular statistical technique used in Marketing to elicit preference functions at both individual and aggregate level. Conjoint Analysis (CA) is a methodology based on several steps starting from designing the experiment, collecting data, estimating the model and, finally, using the results for market segmentation or product positioning.

Since the early 1970s, this technique has known an even wider diffusion in different applicative fields, ranging from Trading to Health, from Agriculture to Food Industry, among others. One of the most recent field is Regulatory Impact Analysis where the aim is to set the *ideal* regulation among alternative policies [24].

In the 1998, Lauro et al. [18] proposed the use of Principal Component Analysis in order to manage dependent and explanatory variables in Conjoint Analysis. In such approach, the traditional interpretative tools of multidimensional techniques enhance the classical CA results. The underlying thought is that individual part-worth coefficients derived for each respondents can be aggregated in a set of common latent utility models, arranged in decreasing importance with respect to their explicative power (see Sect. 3).

Starting from this approach, different methodologies have been then developed and applied in the framework of Multidimensional Data Analysis. Aim of this paper

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is to present a systematic and unitary review of these methodologies by taking into account their contribution to several open ended problems: (i) to obtain homogeneous groups of respondents (ii) to take into account information on the respondents not included in the Conjoint models, and (iii) to consider multiple criteria as response variables.

The paper is structured as follows: in Sect. 2 we refer to the Metric Conjoint Analysis model as the baseline of all successive methods. In Sect. 3 it is shown how the data structure considered in Metric Conjoint Analysis can be analyzed in the framework of exploratory multidimensional data analysis and how to read and interpret the factorial maps as preference map. In Sect. 4, it is considered the opportunity to enrich the original data structure with information about respondents. This new data set is introduced and analyzed by a new specification of the Conjoint model considering the preference system influenced by both the stimuli features and the consumer characteristics.

Thus, in order to derive ex-post cluster of homogeneous set of respondents, a peculiar approach is discussed in Sect. 5. It starts from the results of the factorial decomposition in order to derive global and local utility models. Finally (in Sect. 6), we address the problem of a multi-criteria approach to Conjoint Analysis by introducing a data structure allowing to take into account multiple set of response variables. Some conclusions and future directions are in the Sect. 7.

2 The Metric Approach to Conjoint Analysis

As starting point, we look at the metric approach to Conjoint Analysis. This allows us to consider a well defined data structure that can be analyzed with a multivariate multiple regression model, where the response variable is measured at interval scale and OLS estimation method is applied.

For instance, we consider the role played by two sets of variables: the dependent variables in the matrix \mathbf{Y} ($N \times J$), and the explicative ones in the design matrix \mathbf{X} of size $N \times (K - k)$, where N is the set of Stimuli, J is the number of judges, i.e. the preference responses, and k is the number of experimental factors expanded in K attribute-levels. Let us notice that one dummy category has been dropped out for each attribute to obtain a full rank matrix design.

The Metric Conjoint Analysis model is written as the following multivariate multiple regression model:

$$\mathbf{Y} = \mathbf{XB} + \mathbf{E} \quad (1)$$

where \mathbf{B} is the $(K - k) \times J$ matrix of individual part-worth coefficients and \mathbf{E} is the $(N \times J)$ matrix of error terms for the set of J individual regression models.

Indeed, the simultaneous computation of the elements of the coefficient matrix \mathbf{B} yields the same results as a set of J separate multiple regression models, since the relationships within the multiple responses are not involved in the ordinary least squares method. The OLS is seen here as a decompositive technique because

the classical assumptions on the errors are disregarded in Conjoint Analysis. Typically, some *holdout* runs are used to assess the internal validity of the model. Since researchers often deal with complex stimuli, where a large number of attributes and levels are involved, the use of saturated models is often necessary as a screening study.

Focusing on the quantitative nature of the response variables (preference rating), on the qualitative featuring of the design matrix (dummy variables) and because of the simple reading of the part-worth coefficients computed as average effects, the metric approach to Conjoint Analysis is the most used one. This model is at the basis of the methodologies proposed in the following sections.

3 The Factorial Conjoint Analysis

The Multidimensional Approach to Conjoint Analysis aims at improving the interpretation of the traditional results of this technique by proposing a new reading in the context of Exploratory Data Analysis. The main advantage is to obtain a graphical visualization of the relationships between the preference judgments (dependent variables) and the attribute-levels (independent variables) represented onto a common space.

Different techniques have been proposed in order to take into account the dimension reduction aspect of the model stated in Eq. (1). Among others, we mention the Reduced-Rank Regression Model [1, 17]; the Principal Component of Instrumental Variables [20]; the Simultaneous Linear Prediction Modeling [10]; the Redundancy Analysis [25] and the Principal Component Analysis on a Reference Subspace [5, 6]. The peculiarity of all these techniques is the possibility to link the computational aspects of the regression coefficients with the descriptive and interpretative tools of principal component or canonical variates.

Here we refer to the Principal Component Analysis on a Reference Subspace (PCAR). We consider the asymmetric role played by the two sets of variables (preferences and attributes) involved in multiattribute preference data. Note that in multidimensional data analysis asymmetry refers to the different role played by two or more set of variables when we observe a particular phenomenon. In this context, we highlight the dependence relation between the set of the J preference response variables and the set of the $(K - k)$ attribute-levels described in the design matrix. This technique allows to summarize the multivariate set of preference response variables by performing a Principal Component Analysis of the matrix \mathbf{XB} – stated in model (1) – and equivalent to:

$$\hat{\mathbf{Y}} \equiv \mathbf{XB} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \quad (2)$$

The individual part-worth coefficients are aggregated by means of a suitable weighting system (the PCAR coefficients) reflecting the preference variability:

$$\mathbf{B}'\mathbf{X}'\mathbf{X}\mathbf{B}\mathbf{u}_\alpha = \lambda_\alpha \mathbf{u}_\alpha \quad \alpha=1,\dots,(K-k) \quad (3)$$

It is worth noting, by comparing expression (1) and (3), that the criterion optimized with PCAR (i.e. preference variance accounted by attribute-levels) is fully consistent with the metric Conjoint Analysis data structure. Namely, we define this method as Factorial Conjoint Analysis (FCA).

The PCAR geometrical interpretation allows to enrich even more Conjoint Analysis by joint plots of attribute-levels, judges and stimuli on the first two or three factorial axes. Additional information on judges (e.g. a priori cluster or social-demographic characteristics) can also be shown on the plot.

Traditional interpreting tools of Conjoint Analysis can be read in the context of multidimensional data analysis too. For instance, the relative importance of each attribute are derived by looking at the range of the attribute-level coordinates on each factorial axis. Each factorial axis is a synthesis of the preference variables. They describe the preference of a homogenous subset of respondents towards the attribute levels. The first factorial axis determine the maximum agreement system within judges while the successive ones establish alternative preference patterns of judges subsets.

Considering the expression (2), the principal axes of inertia are obtained as solution of the following characteristic equation under orthonormality constraints:

$$\hat{\mathbf{Y}}'\hat{\mathbf{Y}}\mathbf{u}_\alpha = \lambda_\alpha \mathbf{u}_\alpha \quad \mathbf{u}'_i \mathbf{u}_i = 1; \quad \mathbf{u}'_i \mathbf{u}_j = 0 \quad \{i, j\} \in \alpha=1,\dots,(K-k); \quad (4)$$

which is a Principal Component Analysis of the matrix $\hat{\mathbf{Y}}$.

The eigenvectors \mathbf{u}_α are the weights for the J respondents in the aggregated preference model:

$$\tilde{\mathbf{Y}}_\alpha = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}\mathbf{u}_\alpha = \mathbf{X}\mathbf{B}\mathbf{u}_\alpha \quad (5)$$

Since there are at most $(K - k)$ different weighting systems with decreasing order of importance, we refer to $\alpha = (1, 2)$ as the principal judgment system and define the first factorial plan as a *Preference Scenario*. The (5) is used in computing the coordinates of the N stimuli. The holdouts stimuli can be represented as supplementary points Y^+ :

$$Coor(Y^+) = X^+ \mathbf{B}\mathbf{u}_\alpha \quad (6)$$

where X^+ are the \mathbf{X} rows containing the attribute levels combinations describing any new products. The coordinates of the attribute-levels are:

$$Coor(X) = (\mathbf{X}'\mathbf{X})^{-1/2}\mathbf{X}'\mathbf{Y}\mathbf{u}_\alpha \quad (7)$$

The level coordinates in Eq. (7) are computed as linear combination of the individual part-worth coefficients and assuming the relation between judge and factorial axis as weighting system (i.e. the vector \mathbf{u}), they represent different synthesis of the

estimated part-worth coefficients. In this way, we obtain different synthesis of the individual estimates instead of the unique average which is traditionally considered.

The coordinates of the J respondents are:

$$Coor(Y) = \sqrt{\lambda_\alpha} \mathbf{u}_\alpha \quad (8)$$

which give the directions where pointing out the individual preference models.

The most important feature of this approach is the possibility to synthesize the part-worth coefficients in an optimal way according to hierarchical patterns of preferences.

The advantage of this technique with respect to similar approach, carried out by means of Multidimensional Scaling techniques, is the possibility to recover and interpret the different role of all the *objects* involved in the analysis (e.g. Stimuli, Attributes, Levels and Preference Scores). Furthermore, *holdout* stimuli not involved in the analysis can be represented as supplementary points on the factorial plan which is here interpreted as a *Preference Map*.

4 The FCA with Two Informative Structures

In the multivariate regression model introduced in Eq. (1) we have considered two groups of variables: a set of dependent variables (in \mathbf{Y}), judges' preference, described by a set of explicative variables (in \mathbf{X}) which are the dummy coded attributes of the stimuli.

In Marketing, for example, with the aim of defining a peculiar market strategy, consumers can be a-priori classified on the basis of several socio-demographical characteristics. Starting from this point of view, it is possible to undertake the Taguchi's categorization between controllable versus noise factors proposed in Design of Experiment for Total Quality Control and introduce it in Preference Data Analysis [12]. In particular, the set of a-priori information on judges can be considered as *external* factors and the attribute-levels describing different stimuli as controllable or *internal* factors. Therefore, it is possible to introduce the data matrix \mathbf{Z} of size $(H - h) \times J$, holding socio-demographical characteristics (h nominal variables expanded in full disjunctive binary coding) observed on J judges.

We refer to the design matrix \mathbf{X} as *internal* informative structure or *Inner Array* in Taguchi's notation. while the matrix \mathbf{Z} can be seen as *external* informative structure or *Outer Array*. With the aim of studying the relationships between the two different informative structures, the influence of these two kinds of information on the response variables and, finally, the relationships within each data structure, an extension of the Factorial Conjoint Analysis approach has been applied.

In particular, the two data matrices can be regarded as two different sets of explicative variables in two separated multivariate regression models. The first one is the model (1) above defined and the second one is defined by considering the respondents as statistical units in the model:

$$\mathbf{Y}' = \mathbf{Z}'\mathbf{D} + \mathbf{F} \quad (9)$$

where \mathbf{D} is the $(H - h) \times J$ matrix of coefficients and \mathbf{F} is the $(J \times N)$ matrix of error terms. In order to relate the information of the two designs (\mathbf{X} and \mathbf{Z}' and their own effects on the values in \mathbf{Y} , the coefficients matrices \mathbf{B} and \mathbf{D} have been used in the models (10) and (11) which share common solutions in the matrix Θ .

$$\hat{\mathbf{B}} = \mathbf{Z}'\Theta + \mathbf{V} \quad (10)$$

$$\hat{\mathbf{D}} = \mathbf{X}\Theta' + \mathbf{W} \quad (11)$$

where \mathbf{V} and \mathbf{W} are the corresponding matrices of error terms. The generic elements of $[(H-h) \times (K-k)]$ can be regarded as a coefficient showing the relationship between the two sets of explicative factors. The common OLS solution for Θ is:

$$\hat{\Theta} = (\mathbf{Z}\mathbf{Z}')^{-1}\mathbf{Z}\mathbf{Y}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}. \quad (12)$$

In order to synthesise and represent the information in Θ , the Singular Value Decomposition (SVD) with respect to two different metrics [13] allows to produce a factorial plan where to show the relationships between the users' characteristics and the service features.

This approach allows us to enrich the results of Conjoint Analysis by considering the elements of Θ as inter-reference coefficients while the elements of \mathbf{B} and \mathbf{D} can be regarded as intra-reference coefficients.

The coefficients in \mathbf{B} are useful to answer to questions as: *What happens if we substitute an attribute level with another one?*

The coefficients in \mathbf{D} answer to: *How a category of respondents value a product/service with respect to another category?* The Θ coefficients help the researchers to answer at question as: *What is the effect of changing the attribute level when we target a peculiar category of respondents?*

In this way, for example, we may simulate potential market segments characterised by both consumers and products characteristics.

5 Cluster Based Factorial Conjoint Analysis

The results obtained by Factorial Conjoint Analysis (see Sect. 3) give an aggregate model derived by the total variability within preference judgments. Let us note that judges are represented as variables (judge-vectors) in the FCA subspace.

Starting from FCA results, a strategy which alternates steps of factorial analyses and clustering procedures is proposed in Lauro et al. [19].

With the aim of defining utility models for homogeneous group of judges, we used a method of variables clustering that split the set of judgements into hierarchical clusters. In a Customer Satisfaction strategy, for example, the obtained classes

can be considered as market segments and specialized products are offered to these segments in order to maximize customer satisfaction.

Therefore, for each class a local utility model is derived. Thus, in order to define an aggregate model coherent with the local models and, at the same time, reflecting the preferences of most judges, a weighted PCA is carried out. This analysis aims to synthesize the local preference models in a single model by considering both the number of units in a class and their variability. In this way, the final synthesis put emphasis on more homogenous and larger classes. As a result, this aggregate model is different from the initial aggregate model furnished by the first axis of the FCA; it gets the information from the classes for defining an ideal scenario representative of the preferences of most respondents.

Indeed, each factorial axis obtained through Eq. (3) is a synthesis of the whole set of J preference models. According to standard PCA interpretation rules, vector-variables pointing in the same direction are highly correlated and represent similar preference. Starting from results shown in Eq. (8), a variable clustering (i.e. the VARCLUS procedure of the SAS/Stat system) is performed, so that the individual judgments are aggregated to form homogenous classes. For each class, a local utility model is derived and a standardized scoring coefficient is assigned to each variable to determine the membership to the class. In this step the number of members in each class and a measure of variability explained by each class is also retained. The number of classes C is chosen by exploring the tree structure (dendrogram) or can be set ex ante by the researcher. In a further step we use this information to derive an optimal aggregated utility function, as a synthesis of the local models. The aggregated model is different from the initial PCAR results because it takes into account the relationships among clusters instead of individual judgments. At this aim it is defined the matrix \mathbf{S} ($J \times C$) holding the standardized scoring coefficients for each judge, and the diagonal weighing matrix \mathbf{W} ($C \times C$) of generic term defined as:

$$w_c = \frac{j_c / \sigma_c}{\sum_c j_c / \sigma_c}; \quad c=1, \dots, C; \quad 0 \leq w_c \leq 1; \quad \sum_c w_c = 1 \quad (13)$$

where j_c is th number of judges in the c th class and σ_c is the variability explained by the c th class. By considering the initial coefficients \mathbf{B} , a weighted Principal Component Analysis is defined by the following eigen-equation:

$$\mathbf{W}\mathbf{S}'\mathbf{B}'\mathbf{B}\mathbf{S}\mathbf{v}_\alpha = v_\alpha \mathbf{v}_\alpha \quad (14)$$

where the v_α are the eigenvalues associated to the corresponding eigenvectors v_α . The matrix product $\mathbf{B}\mathbf{S}$ [$(K - k) \times C$] retrieves the importance of each part-worth coefficients in the C segments.

The first principal component obtained by the (14) represents a synthesis of the local preference models. So the new aggregate model can just defined by considering this component. We highlight that the weighing matrix \mathbf{W} used in Eq. (14)

allows to stress on local models with a large number of units (all judges sharing the same model) and allows to give importance to more homogeneous clusters (i.e. market segments).

Therefore, the direction of the new principal component depends on (i) the correlations between the local models; (ii) the size and (iii) the variability within the clusters.

The main results of this strategy are:

- the definition of local preference models;
- the definition of a synthesis model taking into account the local preference models;
- the graphical representations of preferences of both the local and the aggregate models as useful tools for interpreting results.

The local preference models allow to represent utility functions for subset of respondents. For example, in marketing, they could be very useful to establish a marketing specialization strategy. The global preference model is complementary to the average model and to the principal component model. The more homogeneous is the market as a whole, the more the syntheses will tend to the average model. In presence of strong variability among the judgements the principal component model improve the average model. Whereas there exists subsets of consumers that define market segments, the global model is the best suited for a covering market strategy. The graphical representation allows to better visualize the results of the Conjoint experiment and simulation study for product positioning and market simulation.

We warn that internal validity should always achieved by cross-validation techniques, and more important in application fields, the actual use of the methods should produce feedback information to assess external validity and reliability of results.

6 Multi Criteria Factorial Conjoint Analysis

The developments of Conjoint Analysis discussed above are defined on the concept of utility function and applied in the context of Marketing Research, Customer Satisfaction and Customer Relationship Management. Recently, the concept of function of value has taken new meanings. Since one cannot measure utility directly, and attempts to derive it based on preferences (Conjoint Analysis relies on the *Neumann-Morgenstern* theory) could not work because the idea of utility is ambiguous in Social Choice theory where you are speaking about what is useful to society in general. Anyway, *Which are society values?* and *What do you value for society?* In general, different criteria could be taken into account when evaluating some concepts from the socio-political point of view. In this view, we think that Conjoint Analysis can be easily adapted to understand the importance (or value) of different attribute-levels in defining a new product/service, as well as a new policy or politics [24]. All we need, is a value system, different from *Utility*, able to describe

the new concept. Some example are the *Efficacy* or the *Sustainability* in a broader sense. The definition of such analytical functions and their estimates through the use of the Conjoint Analysis approach, will provide a new tool for comparing and evaluate the gaps between what is expected and what is possible to get.

We plan and administer a questionnaire for collecting simultaneously opinions of same judges on a set of stimuli on the basis of m different criteria such as *expected benefits*, *expected utility*, *strategic priority*, and so on.

With this aim, we extend the metric model of Conjoint Analysis (1) by introducing several response matrices:

$$\begin{cases} \mathbf{Y}_1 = \mathbf{X}\mathbf{B}_1 + \mathbf{E}_1 \\ \vdots \\ \mathbf{Y}_m = \mathbf{X}\mathbf{B}_m + \mathbf{E}_m \end{cases} \quad (15)$$

where \mathbf{B}_m is the coefficient matrix related to the m th criterion and \mathbf{E}_m is the corresponding error matrix. Therefore, m sets of OLS part-worth coefficients have been calculated by considering each single criterion separately. Let us notice that the Design Matrix \mathbf{X} is the same in the m criterion.

We are interested in define a similarity measure among the J judges with respect to the m criteria simultaneously. A straightforward approach is to carry out the Factorial Conjoint Analysis on a given criterion and then project (as supplementary points) on the obtained subspace the others $m-1$ criteria. However, this choice has several issues because of the subjectivity of the reference criterion and the absence of a reference subspace where all criteria play an equal (as well as weighted) role.

Different methods allow to face with these issues. They aim at obtaining a synthesis of the multiple criteria directly on a factorial plan. From the others, we refer to STATIS [9], Principal Matrices Analysis [4, 21] and Multiple Factorial Analysis [8] and, from a non-symmetrical point of view, to an extension of PCAR [5] and to a non-symmetrical version of Principal Matrices Analysis [3], which considers the case of multiple observational designs.

The Multi Criteria Factorial Conjoint Analysis deals with a peculiar data structure where the design matrix is the same in the different occasions while the response matrix changes. Therefore, we propose to apply the MFA to the coefficient matrices $\mathbf{B}_i (i = 1, \dots, m)$ and (according to Eq. 2) to interpret it in the frame of a non-symmetrical data analysis.

Multiple factor analysis analyzes observations described by several “blocks” or sets of variables. MFA seeks the common structures present in all of these sets. MFA is performed in two steps. First a principal component analysis (PCA) is performed on each data set which is then “normalized” by dividing all its elements by the square root of the first eigenvalue obtained from each PCA. Second, the normalized data sets are merged to form a unique matrix and a global PCA is performed on this matrix. The individual data sets are then projected onto the global space to analyze communalities and discrepancies.

In analogy with MFA, we carry out m PCA's, one for each separated criterion, and the first eigenvalue is retrieved. So we normalize each $\mathbf{B}_m(i = 1, \dots, m)$ and juxtapose them in order to obtain a unique matrix. A global PCA is performed on this matrix. In this way a synthesis of the coefficients related to all criteria is achieved. On this common plan, we can compare the different criteria and we can project the judges for analyzing their differences and similarities respect to the different criteria. The relationships among the different criteria and between the criteria and the global solution can be analyzed by computing the partial inertia of each analysis for each dimension of the global analysis. In this way we are able to understand the importance of each criterion in the definition of the global solution and we can define the ideal combination of attribute-levels (product, service, policy and so on) by selecting the levels with the larger coordinates on the global plan.

7 Some Conclusions

Different other contributions to Factorial Conjoint Analysis have been proposed by the authors with different aims and from different perspectives [2, 7, 11, 23]. A further development consists in searching a new distance for comparing metric Conjoint Analysis models. In particular, Romano et al. [22] define an inter-models distance which takes into account both the analytical structure of the model (through coefficient deviations) and the information about the model fitting (through the difference between the adjusted R^2 related to each pair of models). The so defined *Model Distance* is parameterized by a trimmer value that allows to take into account the extent to which the model fitting enter in the definition of the distance. This new metric allows to cluster judges in terms of individual models by taking into account both the information on the structural component of the model and on the error term. As Green et al. [14] say:

[...] despite its maturity, conjoint analysis is still far from stagnant, because the methods deal with the pervasive problem of buyer preferences and choices.

In this point of view, we think that new methodological development and applications can be performed in Factorial Conjoint Analysis context. In particular, it will be interesting to investigate the possibility to extend the proposed methods to different structures of data, such as for example, interval value data.

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