Chapter 2
Torsion Stresses in Thin-Walled Multi-Cell Box-Girders

2.1 Torsion of Uniform Thin-Walled Two-Cell Box-Girders

The thin-walled box section with uniform thickness t as shown in Fig. 2.1, is subjected to a torsion moment T.

The shear flow and angle of twist for the thin-walled two cell structure shown in Fig. 2.1 could be determined as follows.

The flexural warping coefficients are given by

$$\delta_{11} = 1/G \cdot \oint_{s_1} ds/t$$

$$= 1/Gt \cdot (AC + CD + DB + BA)$$

$$\delta_{22} = 1/Gt \cdot (DC + CF + FH + HD)$$

$$\delta_{12} = -1/G \cdot CD/t$$

Since the angle of twist is the same for the two cells, then the basic equations are given by

$$\delta_{11} \cdot q_1 + \delta_{12} \cdot q_2 - 2A_1 \cdot \theta = 0 \tag{2.1}$$

$$\delta_{12} \cdot q_1 + \delta_{22} \cdot q_2 - 2A_2 \cdot \theta = 0 \tag{2.2}$$

From equations (2.2.1) and (2.2.2) we get

$$\delta_{11} \cdot \delta_{22} \cdot q_1 + \delta_{12} \cdot \delta_{22} \cdot q_2 = 2\delta_{22} \cdot A_1 \cdot \theta$$

$$\delta_{12}^2 \cdot q_1 + \delta_{12} \cdot \delta_{22} \cdot q_2 = 2\delta_{12} \cdot A_2 \cdot \theta$$

Hence

$$q_1(\delta_{11} \cdot \delta_{22} - \delta_{12}^2) = 2\delta_{22} \cdot A_1 \cdot \theta - 2\delta_{12} \cdot A_2 \cdot \theta$$

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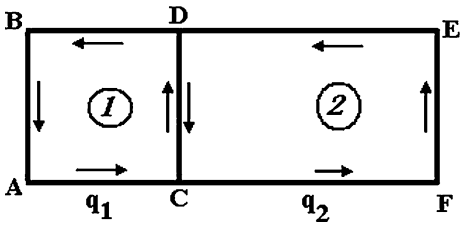


Fig. 2.1 Shear flow due to torsion of a thin-walled box girder with two unequal cells

The solution of equations (2.1) and (2.2) gives

$$\begin{aligned} q_1 &= \theta \cdot (2\delta_{22} \cdot A_1 - 2\delta_{12} \cdot A_2) / \left(\delta_{11} \cdot \delta_{22} - \delta_{12}^2\right) \\ &= \Delta_1 \cdot \theta \end{aligned}$$

$$\begin{aligned} q_2 &= \theta \cdot (2A_1 - \Delta_1 \delta_{11}) / \delta_{12} \\ &= \Delta_2 \cdot \theta \end{aligned}$$

where

$$\Delta_1 = (2\delta_{22} \cdot A_1 - 2\delta_{12} \cdot A_2) / \left(\delta_{11} \cdot \delta_{22} - \delta_{12}^2\right)$$

$$\Delta_2 = (2A_1 - \Delta_1 \delta_{11}) / \delta_{12}$$

The equilibrium condition gives

$$\begin{aligned} T &= 2A_1 \cdot q_1 + 2A_2 \cdot q_2 \\ &= \Delta_3 \cdot \theta \end{aligned}$$

where

$$\Delta_3 = (2A_1 \cdot q_1 + 2A_2 \cdot q_2)$$

Hence

$$\theta = 1/\Delta_3 \cdot T$$

$$q_1 = \Delta_1/\Delta_3 \cdot T$$

$$q_2 = \Delta_2/\Delta_3 \cdot T$$

$$q_{12} = q_1 - q_2 = (\Delta_1 - \Delta_2)/\Delta_3 \cdot T.$$

Example 2.1 Determine the torsion shear stress and angle of twist for the two uniform thickness thin-walled box-girder shown in Fig. 2.2.

Solution The shear flow and angle of twist for the thin-walled two cell structure shown in Fig. 2.2 could be determined as follows.

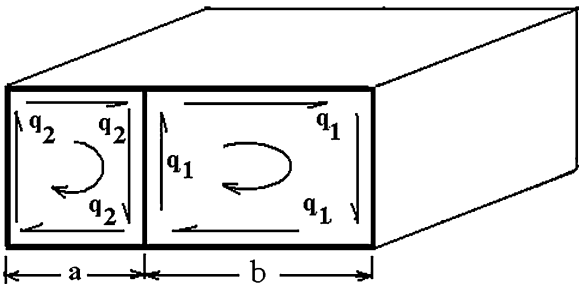


Fig. 2.2 A uniform thin-walled box-girder with two cells

The torsional moment is given by

$$T = 2(q_1 A_1 + q_2 A_2) \tag{2.3}$$

The angle of twist for cells ***1*** and ***2*** are given by

$$\theta_1 = \frac{1}{2GA_1} \cdot \left(q_1 \oint_1 ds/t - q_2 \int_{12} ds/t \right) \tag{2.4}$$

$$\theta_2 = \frac{1}{2GA_2} \left(q_2 \oint_2 ds/t \oint - q_1 \int_{21} ds/t \right) \tag{2.5}$$

Since the angle of twist is the same for the two cells, then we have

$$\theta_1 = \theta_2 = \theta_3$$

Reformulating equations (2.4) and (2.5), we get

$$\frac{1}{G} \left(q_1 \oint_1 ds/t - q_2 \int_{12} ds/t \right) = 2A_1\theta \tag{2.6}$$

$$\frac{1}{G} \left(q_2 \oint_2 ds/t - q_1 \oint_{21} ds/t \right) = 2A_2\theta \tag{2.7}$$

Let

$$\delta = \text{warping flexibility}$$

$$\delta_{11} = \frac{1}{G} \oint_1 ds/t$$

$$\delta_{22} = \frac{1}{G} \oint_2 ds/t$$

$$\delta_{12} = \delta_{21} = -\frac{1}{G}\int_{12} ds/t$$

Substituting in equations (2.2.4) and (2.2.5), we get

$$\delta_{11}q_1 + \delta_{12}q_2 - 2A_1\theta = 0 \tag{2.8}$$

$$\delta_{12}q_1 + \delta_{22}q_2 - 2A_2\theta = 0 \tag{2.9}$$

Solving equations (2.2.3), (2.2.8) and (2.2.9), q_1 and q_2 could be determined.

Example 2.2 Determine the torsion shear stresses and the rate of twist for the thin-walled ***2***-cell box-girder shown in Fig. 2.3. The girder is subjected to a constant torque T.

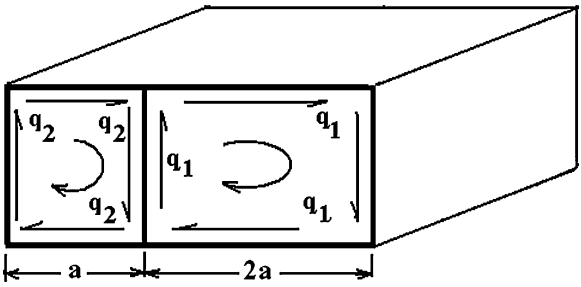


Fig. 2.3 A thin-walled box-girder with two unequal cells

Solution Area of cell (***1***) is given by

$$A_1 = 2a^2$$

Area of cell (***2***) is given by

$$A_2 = a^2$$

Let

$$\delta = \text{warping flexibility}$$

$$\delta_{11} = 1/G \cdot \oint ds/t = 6a/Gt$$

$$\delta_{22} = 4a/Gt$$

$$\delta_{12} = -a/Gt$$

The basic equations are

$$\delta_{11}q_1 + \delta_{12}q_2 - 2A_1\theta = 0 \tag{2.10}$$

$$\delta_{12}q_1 + \delta_{22}q_2 - 2A_2\theta = 0 \tag{2.11}$$

The equilibrium equation gives

$$T = 2(q_1 A_1 + q_2 A_2) = 2a^2(2q_1 + q_2) \tag{2.12}$$

From equations (2.10) and (2.11), we get

$$\delta_{11} q_1 A_2 + \delta_{12} q_2 A_2 - \delta_{12} q_1 A_1 - \delta_{22} q_2 A_1 = 0$$

$$q_1(\delta_{11} A_2 - \delta_{12} A_1) + q_2(\delta_{12} A_2 - \delta_{22} A_1) = 0$$

Hence

$$q_1 = q_2 \frac{\delta_{22} A_1 - \delta_{12} A_2}{\delta_{11} A_2 - \delta_{12} A_1} = q_2 \frac{2\delta_{22} - \delta_{12}}{\delta_{11} - 2\delta_{12}}$$

From equation (2.12), we get

$$T = 2a^2 q_2 \left[2\frac{2\delta_{22} - \delta_{12}}{\delta_{11} - 2\delta_{12}} + 1 \right]$$

$$T = 2a^2 q_2 \left[\frac{4\delta_{22} - 4\delta_{12} + \delta_{11}}{\delta_{11} - 2\delta_{12}} \right]$$

From which q_2 is given by

$$q_2 = \frac{T}{2a^2} \left(\frac{\delta_{11} - 2\delta_{12}}{4\delta_{22} - 4\delta_{12} + \delta_{11}} \right)$$

$$q_1 = \frac{T}{2a^2} \left(\frac{2\delta_{22} - \delta_{12}}{4\delta_{22} - 4\delta_{12} + \delta_{11}} \right)$$

Substituting in equation (2.10), we get

$$\begin{aligned} \theta &= \frac{1}{2A_1} \cdot (\delta_{11} q_1 + \delta_{12} q_2) \\ &= \frac{1}{4a^2} \cdot \frac{T}{2a^2} \cdot \left[\frac{2\delta_{11}\delta_{22} - \delta_{11}\delta_{12} + \delta_{12}\delta_{11} - 2\delta_{12}^2}{4\delta_{22} - 4\delta_{12} + \delta_{11}} \right] \\ &= \frac{T}{4a^4} \cdot \left(\frac{\delta_{11}\delta_{22} - \delta_{12}^2}{4\delta_{22} - 4\delta_{12} + \delta_{11}} \right) \end{aligned}$$

Substituting for δ_{12}, δ_{11} and δ_{22}, we get

$$q_1 = \frac{T}{2a^2 G}[(8 \cdot a/t + a/t)/(16 \cdot a/t + 4 \cdot a/t + 6 \cdot a/t)] = \frac{T}{2a^2 G} \times \frac{9}{26}$$

$$q_2 = \frac{T}{2a^2 G} \cdot [(6 \cdot a/t + 2 \cdot a/t)/(26a/t)] = \frac{T}{2a^2 G} \times \frac{8}{26}$$

$$\theta = \frac{T}{4a^4 G}\left[\left(24 \cdot a^2/t^2 - a^2/t^2\right)/(26 \cdot a/t)\right] = \frac{T}{4a^4 G} \cdot 23/26 \cdot a/t$$

But

$$\theta = \mathrm{T}/\mathrm{GJ}$$

Hence

$$\mathrm{J} = \mathrm{T}/\mathrm{G}\theta = 104/23 \cdot \mathrm{a}^3 \cdot \mathrm{t}.$$

2.2 The General Case of a Uniform Two-Cell Box Girder

This is an indeterminate structural problem and its solution is based on the assumption that the rate of twist for each cell is the same as for the whole section, see Fig. 2.4.

i.e.,

$$\theta_1 = \theta_2 = 0 \quad \text{and} \quad (\theta = \mathrm{d}\varphi/\mathrm{d}z)$$

$$\theta = \mathrm{T}/\mathrm{GJ}$$

The torque $\boldsymbol{T}$ is given by

$$\mathrm{T} = 2\mathrm{q}_1\mathrm{A}_1 + 2\mathrm{q}_2\mathrm{A}_2$$

$$\theta_1 = \frac{1}{2\mathrm{GA}_1} \oint_1 \mathrm{q}/\mathrm{t} \cdot \mathrm{ds}$$

and

$$\theta_2 = \frac{1}{2\mathrm{GA}_2} \oint_2 \mathrm{q}/\mathrm{t} \cdot \mathrm{ds}$$

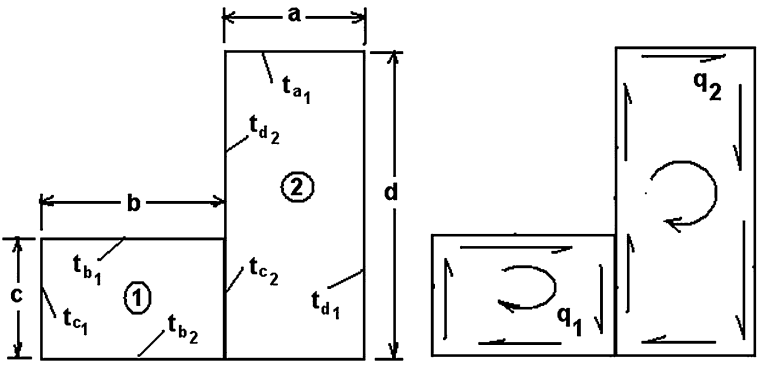


Fig. 2.4 Idealized section and torsion shear flow of a thin-walled two cell structure

i.e.,

$$\frac{1}{G}\left[q_1 \oint_1 q/t \cdot ds - q_2 \cdot (ds/t)_{12}\right] = 2A_1\theta_1 \tag{2.13}$$

$$\frac{1}{G}\left[q_2 \oint_2 ds/t - q_1 (ds/t)_{21}\right] = 2A_2\theta_2 \tag{2.14}$$

Equations (2.13) and (2.14) are simplified to

$$\delta_{11}q_1 + \delta_{12}q_2 = 2A_1\theta$$

$$\delta_{21}q_1 + \delta_{22}q_2 = 2A_2\theta$$

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} 2\theta$$

or

$$[\delta]\{q\} = 2\theta\{A\}$$

The shear flow in each cell is given by

$$\{q\} = [\delta]^{-1} \cdot 2\theta \cdot \{A\}$$

i.e.,

$$\begin{aligned} \{q\} &= 2\theta \cdot [\delta]^{-1} \cdot \{A\} \\ \delta_{11} &= \frac{1}{G}\oint_1 ds/t \\ \delta_{22} &= \frac{1}{G}\oint_2 ds/t \\ \delta_{12} &= \delta_{21} \\ &= -\frac{1}{G}[ds/t]_{12} \end{aligned} \tag{2.15}$$

The torque is given by

$$T = (2q_1A_1 + 2q_2A_2) \tag{2.16}$$

Solving equations (2.15) and (2.16), we get q_1, q_2 and θ

The torque T is given by

$$T = G \cdot J \cdot \theta$$

Hence J is given by

$$J = T/G\theta.$$

Example 2.3 Determine the torsion shear stresses and angle of twist for the thin-walled box section having uniform thickness t as shown in Fig. 2.5. The section is subjected to a torsion moment ***T***.

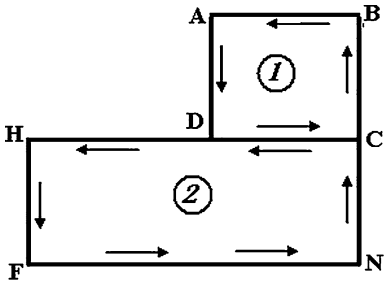


Fig. 2.5 Shear flow due to torsion of a box-girder with two cells

Solution Condition for Compatibility (Consistent Deformation). The warping flexibilities are given by

$$\delta_{11} = 1/Gt \cdot (AB + BC + CD + DA)$$
$$\delta_{22} = 1/Gt \cdot (CH + HF + FN + NC)$$
$$\delta_{12} = -1/G \cdot CD/t \delta_{12} = -1/G \cdot CD/t$$

The basic equations of consistent deformation are given by

$$\delta_{11} \cdot q_1 + \delta_{12} \cdot q_2 = 2A_1 \cdot \theta \tag{2.17}$$

$$\delta_{12} \cdot q_1 + \delta_{22} \cdot q_2 = 2A_2 \cdot \theta \tag{2.18}$$

Solving equations (2.17) and (2.18) we get

$$q_1 = \frac{2\delta_{22} \cdot A_1 - 2\delta_{12} \cdot A_2}{\delta_{11} \cdot \delta_{22} - \delta_{12}^2} \cdot \theta = \Delta_1 \cdot \theta$$
$$q_2 = \frac{2A_1}{\delta_{12}}\theta - \Delta_1 \frac{\delta_{11}}{\delta_{12}}\theta = \Delta_2 \cdot \theta$$

Equation for equilibrium condition is given by

$$T = (2A_1 \cdot q_1 + 2A_2 \cdot q_2) \cdot \theta = \Delta_3 \cdot \theta \tag{2.19}$$

The solution of equations (2.17), (2.18) and (2.19) gives

$$\theta = T/\Delta_3$$

$$q_1 = T \cdot \Delta_1/\Delta_3$$

$$q_2 = T \cdot \Delta_2/\Delta_3$$

$$q_{12} = q_1 - q_2 = T \cdot (\Delta_1 - \Delta_2)/\Delta_3.$$

2.3 Torsion Stresses in a Two Identical Cells Box-Girder

The two identical thin-walled cells box girder, see Fig. 2.6, behaves exactly as a single cell box girder.

Since the two cells are identical we have

$$q_I = q_{II} = q.$$

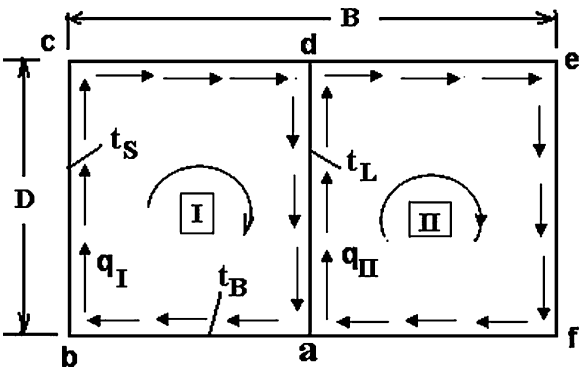


Fig. 2.6 A thin-walled box girder with two identical cells

2.3.1 Shear Flow q

The shear flow q is the same for the two cells and is given by

$$q = T/2A$$

where

$$A = B \cdot D.$$

2.3.2 Shear Stress τ

The shear stress in the sides, top, bottom and the internal partition plating are given by

$$\tau_S = q/t_s, \quad \tau_D = q/t_D, \quad \tau_B = q/t_B, \quad \tau_L = 0.$$

2.3.3 *Rate of Twist θ*

The rate of twist is given by

$$\theta = T/GJ$$

where

$$J = 4A^2 \Big/ \oint ds/t$$

$$\oint ds/t = 2B/t_B + 2D/t_S$$

2.4 Torsion of Three-Cell Box-Girder

Following the same principle that the angle of twist is the same for the three cell box-girder shown in Fig. 2.7.

Then

$$\theta_1 = \theta_2 = \theta_3 = \theta$$

The equations of consistent deformation are given by

$$\delta_{11}q_1 + \delta_{12}q_2 - 2A_1\theta_1 = 0 \tag{2.20}$$

$$\delta_{12}q_1 + \delta_{22}q_2 - 2A_2\theta_2 = 0 \tag{2.21}$$

$$\delta_{32}q_1 + \delta_{33}q_2 - 2A_3\theta_3 = 0 \tag{2.22}$$

The torque is given by

$$T = (2A_1 \cdot q_1 + 2A_2 \cdot q_2 + 2A_3 \cdot q_3)\theta \tag{2.23}$$

Solving equations (2.20)–(2.23), we get q_1, q_2, q_3, q_4 and θ.

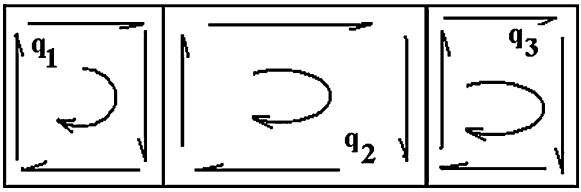


Fig. 2.7 Torsion of a three-cell box-girder

Hence

$$\begin{bmatrix} \delta_{11} & \delta_{12} & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} \\ 0 & \delta_{32} & \delta_{33} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = 2\theta \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} \tag{2.24}$$

$$[\delta]\{q\} = 2\theta\{A\}$$

Hence the shear flow in each cell is given by

$$\{q\} = \left[\delta^{-1}\right]\{A\}2\theta$$

and

$$T = 2\theta \cdot \sum A_i q_i$$

where

$$\delta_{11} = \oint_1 ds/t$$

$$\delta_{22} = \oint_2 ds/t$$

$$\delta_{33} = \oint_3 ds/t$$

$$\delta_{12} = \delta_{21} = -[ds/t]_{12}$$

$$\delta_{23} = \delta_{32} = -[ds/t]_{23}.$$

Example 2.4 Determine the shear flow, shear stress and rate of twist for the three-cell box girder shown in Fig. 2.8.

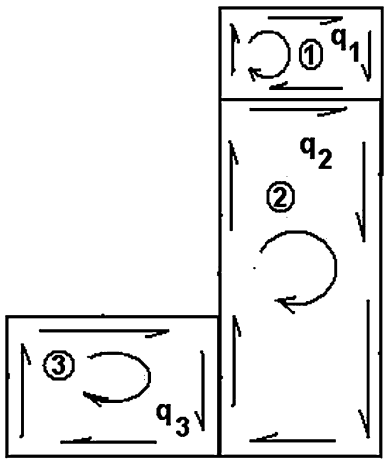


Fig. 2.8 Shear flow due to torsion of a 3-cell thin-walled box girder

Solution Following the same principle that the angle of twist is the same for all cells, see Fig. 2.8, we get

$$\begin{aligned}\delta_{11}q_1 + \delta_{12}q_2 &= 2A_1\theta \\ \delta_{21}q_1 + \delta_{22}q_2 + \delta_{23}q_3 &= 2A_2\theta_2 \\ \delta_{32}q_2 + \delta_{33}q_3 &= 2A\theta_3\end{aligned} \tag{2.25}$$

but

$$\theta_1 = \theta_2 = \theta_3 = \theta$$

Then

$$\begin{bmatrix}\delta_{11} & \delta_{12} & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} \\ 0 & \delta_{32} & \delta_{33}\end{bmatrix}\begin{Bmatrix}q_1 \\ q_2 \\ q_3\end{Bmatrix} = 2\theta\begin{Bmatrix}A_1 \\ A_2 \\ A_3\end{Bmatrix}$$

$$[\delta]\{q\} = 2\theta\{A\} \tag{2.26}$$

$$\{q\} = \left[\delta^{-1}\right]\{A\}2\theta \tag{2.27}$$

The torque is given by

$$T = (2q_1A_1 + 2q_2A_2 + 2q_3A_3) \tag{2.28}$$

Solving equations (2.26) and (2.28) we get

$$q_1 = 2\theta\theta^{-1}A_1$$

$$q_2 = 2\theta\theta^{-1}A_2$$

$$q_3 = 2\theta\theta^{-1}A_3$$

Substituting in equation (2.25), we get

$$\theta = 1/2A_1 \cdot (\delta_{11}q_1 + \delta_{12}q_2)$$

where

$$\delta_{11} = 1/G \cdot \oint_1 ds/t$$

$$\delta_{22} = 1/G \cdot \oint_2 ds/t$$

$$\delta_{33} = 1/G \cdot \oint_3 ds/t$$

$$\delta_{12} = \delta_{21} = -1/G \cdot [ds/t]_{12}$$

$$\delta_{23} = \delta_{32} = -1/G \cdot [ds/t]_{23.}$$

2.5 Torsion of Uniform Thin-Walled Multi-Cell Box-Girder

The multi-cell thin-walled structure when subjected to pure torsion is a statically indeterminate problem; see Fig. 2.9.

The torque T is given by

$$T = \sum_{i=1}^{n} 2A_i q_i = GJ\theta J$$

where T = applied uniform torque; A_i = enclosed area of the ***i***th cell; J = torsion constant

$$J = 4\sum_{i=1}^{n} A_i \delta^{-1} A_i$$

The angle of twist per unit length

$$\theta = d\varphi/dz$$
$$\theta_i = \theta_j = \theta_{ij} = \theta_{jn}$$

where

$$\theta_i = 1/2GA_i \cdot \left(q_i \oint ds/t \right)$$

The angle of twist for cell ***i*** is given by

$$\theta_i = 1/2GA_i \cdot \left(q_i \oint ds/t - q_{i-1} \int ds/t - q_{i+1} \int ds/t \right) \tag{2.29}$$

Equation (2.29) represents a series of simultaneous equations which gives $q_1, q_2, q_3, \ldots, q_n$.

The set of equations of consistent deformation is given by

$$\delta_{11} q_1 + \delta_{12} q_2 - 2A_1 \theta = 0 \tag{2.30}$$

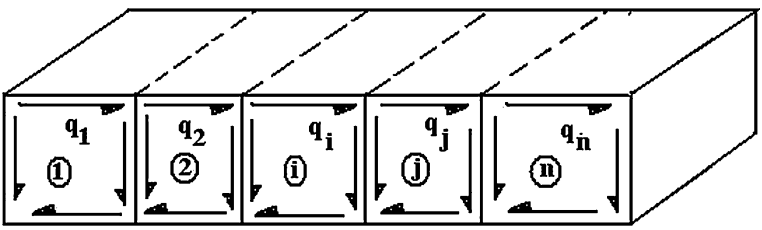


Fig. 2.9 Torsion of a multi-cell thin-walled box-girder

$$\delta_{12}q_1 + \delta_{22}q_2 - 2A_2\theta = 0 \tag{2.31}$$

$$\delta_{32}q_1 + \delta_{33}q_2 - 2A_3\theta = 0 \tag{2.32}$$

This set of equations could be put in the following form

$$[\delta]\{q\} = 2\theta\{A\}$$

Hence, for a multi cell box girder, the shear flow in each cell is given by

$$\{q\} = [\delta]^{-1} \cdot 2\theta \cdot \{A\} \qquad (i = 1, 2, \ldots, n)$$

where

$$\delta = \begin{pmatrix} \delta_{11} & \delta_{12} & 0 & 0 & 0 & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} & 0 & 0 & 0 \\ 0 & \delta_{32} & \delta_{33} & \delta_{34} & 0 & 0 \\ 0 & 0 & \delta_{43} & \delta_{44} & \delta_{45} & 0 \\ 0 & 0 & 0 & \delta_{54} & \delta_{55} & \delta_{56} \\ 0 & 0 & 0 & 0 & \delta_{65} & \delta_{66} \end{pmatrix}$$

The torsion shear stresses are given by

$$\tau_1 = q_1/t_1, \quad \tau_2 = q_2/t_2, \quad \tau_3 = q_3/t_3.$$

2.6 Combined Open and Closed Thin-Walled Sections

For the combined open and closed section, see Fig. 3.1, the angle of twist is the same for the whole section whether it is an open or closed section.

2.6.1 Combined Open Section with One Closed Cell

The total torque T for the thin-walled section shown in Fig. 2.10 is given by

$$T = \sum_{I=1}^{2} T_i = G \cdot J \cdot \theta$$

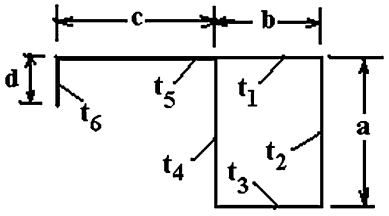


Fig. 2.10 Combined open and closed one-cell thin-walled section

Hence

$$\theta = T/GJ$$

where

$$T_1 = GJ_1 \cdot \theta$$

J_1 = torsion constant of the open section; J_2 = Torsion constant of the closed section.

For the open part of the structure, the shear flow q_1 is given by

$$q_1 = T_1 \cdot t_1^2/J_1$$

$$T_1 = G \cdot J_1 \cdot \theta$$

For the closed section of the structure, the shear flow q_2 is given by

$$q_1 = T_2/2A_2$$
$$T_2 = 2A_2 \cdot q_2 = G \cdot J_2 \cdot \theta.$$

2.6.2 Combined Open Section with Two Closed Cells

The applied torque T for the thin-walled structure shown in Fig. 2.11 is given by

$$T = \sum_{j=1}^{n} 2q_j A_j + \sum_{i=1}^{m} GJ_i\theta$$

In the above particular example

$$T = 2q_1A_1 + 2q_2A_2 + GJ_3\theta$$

where J_3 = the torsion constant of the open section part of the structure and is given by $\mathbf{J}_3$

$$T = 1/3 \sum_{i=1}^{n} b_i t_i^3$$

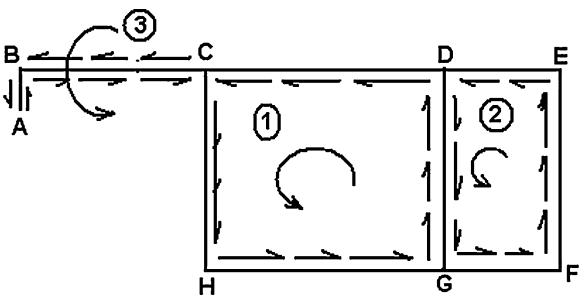


Fig. 2.11 Combined thin-walled open and closed two-cell structure

The shear flow in the two cells is given by

$$\delta_{11}q_1 + \delta_{12}q_2 = 2A_1\theta$$

$$\delta_{21}q_1 + \delta_{22}q_2 = 2A_2\theta$$

$$\delta_{11} = 1/G \cdot \oint_1 ds/t$$

$$\delta_{22} = 1/G \cdot \oint_2 ds/t$$

$$\delta_{12} = \delta_{21} = -1/G \cdot [ds/t]_{12}$$

In the general case, for combined open and closed sections, the shear flow in each cell is given by

$$q_i = \delta^{-1}2\theta\theta_i \quad (i = 1, 2, \ldots, n)$$

And in each open member the shear flow is given by

$$q_i = T/J \cdot t_i^2$$

And the angle of twist is given by

$$\theta = T/GJ$$

where T = the torque and is given by

$$T = 4\sum_{i=1}^{n} A_i'\delta^{-1}A_i + \frac{1}{3}\sum_{j=1}^{m} b_j t_j^3.$$

Example 2.5 Determine the shear flow distribution and rate of twist for the idealized ship section shown in Fig. 2.12. The ship section is subjected to a torque T.

Solution The torque T is distributed among the thin-walled structural members of the ship section as follows

$$T = \sum_{i=1}^{4} T_i$$

where

$$T_1 = 2A_1 \cdot q_1, \quad T_2 = 2A_2 \cdot q_2, \quad T_3 = 2A_3 \cdot q_3,$$

$$T_4 = GJ_4 \cdot \theta = \frac{q_4 \cdot J_4}{t_4^2}, \qquad J_4 = 1/3 \cdot St_4^3$$

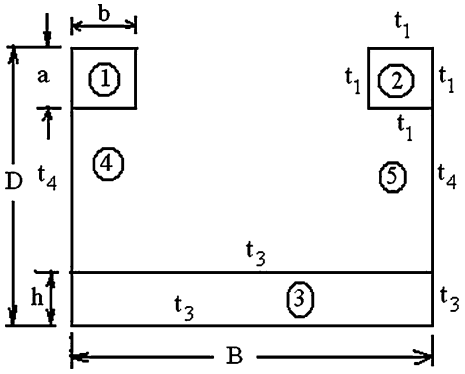


Fig. 2.12 Idealized ship section

because of symmetry of the ship section, we have

$$T_1 = T_2, \quad T_4 = T_5$$

Hence

$$T_1 = T_2 = 2A_1 \cdot q_1 = G \cdot J_1 \cdot \theta$$

Thus

$$T = 2GJ_1 \cdot \theta + 2GJ_4 \cdot \theta + GJ_3 \cdot \theta = \sum_{j=1}^{3} G \cdot J \cdot \theta$$

where

$$G = \frac{E}{2(1+\upsilon)} = \frac{E}{2 \cdot 6}$$

$$J = 2J_1 + 2J_4 + J_3$$

where

$$J_1 = 4A_1^2 \Big/ \oint \frac{ds}{t} = 4(a \cdot b)^2 / (2b/t + 2a/t)$$

$$J_3 = A(B \cdot h)^2 / (2B/t_3 + 2h/t_3)$$

$$J_4 = 1/3 \cdot S_4 \cdot t_4^3 = 1/3 \cdot (D - a - h) \cdot t_4^3$$

Hence

$$\theta = T \Big/ \sum GJ$$

Substituting, we get the torque carried by each structural element.

Hence

$$T_1 = GJ_1 \cdot \theta$$

$$T_3 = GJ_3 \cdot \theta$$

$$T_4 = GJ_4 \cdot \theta$$

Substituting, we get the shear flow in each structural element as follows

$$q_1 = T_1/2A_1, \quad q_3 = T_3/2A_3, \quad \text{and} \quad q_4 = T_4/2A_4.$$

Example 2.6 Determine the shear flow and rate of twist for the ship section of bulk carrier shown in Fig. 2.13.

Solution The torque T is given by

$$T = 2[2A_1q_1 + 2q_2A_2 + GJ_6\theta + 2A_3q_3 + 2A_4q_4 + 2A_5q_5]$$

The torsion constant J is given by

$$J = 2\left\{\sum_{i=1}^{5} 4A_i'\delta^{-1}A_i + 1/3 \cdot \lambda_6 \cdot t_6^3\right\}$$

The rate of twist θ is given by

$$\theta = T/GJ$$

The set of equations of consistent deformation for cells (***1***) and (***2***) is given by

$$\delta_{11}q_1 + \delta_{12}q_2 = 2A_1\theta = 2T/J \cdot A_1$$

$$\delta_{21}q_1 + \delta_{22}q_2 = 2A_2\theta = 2T/J \cdot A_2$$

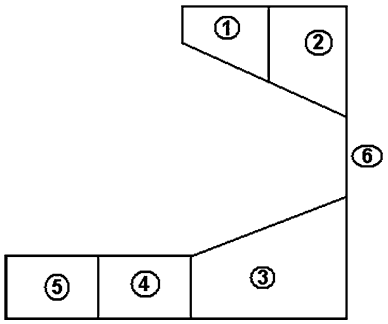


Fig. 2.13 An idealized section of a bulk carrier

This set of equations can be put in the matrix form as follows

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 2\theta \cdot \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix}$$

i.e.,

$$(\delta)\{q\} = 2T/J \cdot \{A\}$$

Hence, the torsion shear flow in cells (***1***) and (***2***) are given by

$$q_1 = \delta^{-1} \cdot 2T/J \cdot A_1$$

$$q_2 = \delta^{-1} \cdot 2T/J \cdot A_2$$

Similarly, the torsion shear flow in cells (***3***), (***4***) and (***5***) are given by

$$q_3 = \delta^{-1} \cdot 2T/J \cdot A_3$$

$$q_4 = \delta^{-1} \cdot 2T/J \cdot A_4$$

$$q_5 = \delta^{-1} \cdot 2T/J \cdot A_5$$

where

$$\delta = \begin{bmatrix} \delta_{33} & \delta_{34} & 0 \\ \delta_{43} & \delta_{44} & \delta_{45} \\ 0 & \delta_{54} & \delta_{55} \end{bmatrix}$$

$$\delta_{ii} = \left[1/G \cdot \sum_{J=1}^{m} \lambda_j / t_j \right], \quad i = 1, 2, \ldots, n = N \text{ of cells}$$

$$\delta_{rj} = 1/G \cdot \left[\lambda_{rj} / t_{rj} \right]_i$$

where r, and j are cells, having a common boundary; i = cell No. i.



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