Preface

The real revolution in mathematical physics in the second half of twentieth century (and in pure mathematics itself) was algebraic topology and algebraic geometry. Meanwhile there is the Course in Mathematical Physics by W. Thirring, a large body of monographs and textbooks for mathematicians and of monographs for physicists on the subject, and field theorists in high-energy and particle physics are among the experts in the field, notably E. Witten. Nevertheless, I feel it still not to be easy for the average theoretical physicist to penetrate into the field in an effective manner. Textbooks and monographs for mathematicians are nowadays not easily accessible for physicists because of their purely deductive style of presentation and often also because of their level of abstraction, and they do not really introduce into physics applications even if they mention a number of them. Special texts addressed to physicists, written both by mathematicians or physicists in most cases lack a systematic introduction into the mathematical tools and rather present them as a patchwork of recipes. This text tries an intermediate approach. Written by a physicist, it still tries a rather systematic but more inductive introduction into the mathematics by avoiding the minimalistic deductive style of a sequence of theorems and proofs without much of commentary or even motivating text. Although theorems are highlighted by using italics, the text in between is considered equally important, while proofs are sketched to be spelled out as exercises in this branch of mathematics. The text also mainly addresses students in solid state and statistical physics rather than particle physicists by the focusses and the choice of examples of application.

Classical analysis was largely physics driven, and mathematical physics of the nineteens century was essentially the classical theory of ordinary and partial differential equations. Variational calculus, since the very beginning of theoretical mechanics a standard tool of physicists, was seen with great reservation by mathematicians until D. Hilbert initiated its rigorous foundation by pushing forward functional analysis. This marked the transition into the first half of twentieth century, where under the influence of quantum mechanics and relativity mathematical physics turned mainly into functional analysis (as for instance witnessed by the textbooks of M. Reed and B. Simon), complemented by the theory of Lie groups and by tensor analysis. Physicists, nowadays more or less familiar with these branches, still are on average mainly analytically and very little algebraically educated, to say nothing of topology. So it could happen that for nearly sixty years it was overlooked that not every quantum mechanical observable may be represented by an operator in Hilbert space, and only in the middle of the eighties of last century with Berry's phase, which is such an observable, it was realized how polarization in an infinitely extended crystal is correctly described and that textbooks even by most renowned authors contained meaningless statements about this question.

This author feels that all branches of theoretical physics still can expect the strongest impacts from use of the unprecedented wealth of results of algebraic topology and algebraic geometry of the second half of twentieth century, and to introduce theoretical physics students into its basics is the purpose of this text. It is still basically a text in mathematics, physics applications are included for illustration and are chosen mainly from the fields the author is familiar with. There are many important examples of application in physics left out of course. Also the cited literature is chosen just to give some sources for further study both in mathematics and physics. Unfortunately, this author did not find an English translation of the marvelous Analyse Mathématique by L. Schwartz,[1] which he considers (from the Russian edition) as one of the best textbooks of modern analysis. A rather encyclopedic text addressed to physicists is that by Choquet-Bruhat et al.,[2] however, a compromise between the wide scope and limitations in space made it in places somewhat sketchy.

The order of the material in the present text is chosen such that physics applications could be treated as early as possible without doing too much violence to the inner logic of the mathematical building. As already said, central results are highlighted in italics but purposely avoiding the structure of a sequence of theorems. Sketches of proofs are given, if they help understanding the matter. They are understood as exercises for the reader to spell them out in more detail. Purely technical proofs are omitted even if they prove central issues of the theory. A compendium is appended to the basic text for reference also of some concepts (for instance of general algebra) used in the text but not treated. This appendix is meant as an expanded glossary and, apart form very few exceptions, not covered by the index.

Finally, I would like to acknowledge many suggestions for improvement and corrections by people from the Springer-Verlag.

Dresden, May 2010 Helmut Eschrig

[1] Schwartz, L.: Analyse Mathématique. Hermann, Paris (1967).

[2] Choquet-Bruhat, Y., de Witt-Morette, C., Dillard-Bleick, M.: Analysis, Manifolds and Physics, Elsevier, Amsterdam, vol. I (1982), vol. II (1989).

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