Chapter 2
Fluid Turbulence

Abstract In this chapter we present characteristics of turbulent flows in general and see how a turbulent flow is so much different from laminar flow. We briefly refer to some engineering applications where turbulent fluid flow and heat transfer are encountered. We study how an already complex turbulent flow becomes much more complex due to some features such as the presence of a wall and buoyancy. We also characterize turbulent flows. We present the need to numerically study turbulent fluid flows and heat transfer and consequently what is the job of a turbulence model.

2.1 Physical Description

We encounter turbulent flows almost everywhere, for example, atmospheric and ocean currents, discharge of pollutants, oil transport in pipelines, flow through pumps and turbines, and the flow in boat wakes and around aircraft wing tips. Turbulent flows are characterized mainly by unsteadiness, vorticity, three-dimensionality, dissipation, wide spectrum of scales, and large mixing rates. In contrast, laminar flows are realized mostly in laboratories or in special situations. Fluid turbulence remains one of the biggest challenges to scientists and engineers and therefore it continues to be an active research topic.

Let us first look at the significance of some of the above-mentioned characteristics. In a turbulent flow, a flow property at a point continuously undergoes changes in magnitude. Figure 2.1 shows time traces of a typical flow property (a velocity component in the present case) that is usually obtained at a point within a turbulent flow. Turbulent flows are irregular in nature (Tennekes and Lumley 1997). This makes a deterministic approach to turbulence problems impossible and therefore statistical methods are used for treating turbulent flows. The diffusivity of turbulence causes good mixing and consequently increased rates of momentum, heat and mass transfers. Turbulent-mixing, due to the movement of eddies, is

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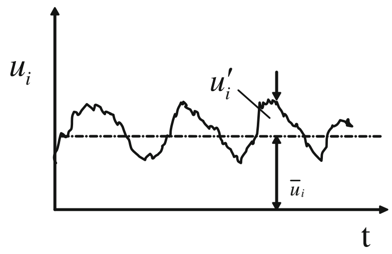

Fig. 2.1 Mean and fluctuating components of a flow property in a turbulent flow

much stronger than that of laminar flows which is due molecular action only. We must, however, not forget that mixing due to molecular effects is always present in a turbulent flow. Nevertheless, molecular mixing is negligible compared to turbulent mixing, except in certain regions (for example in the vicinity of a solid wall). Right at the solid wall, however, turbulence is zero because of the no-slip condition and therefore in the close vicinity of a solid wall the molecular effects become important. A turbulent flow is always three-dimensional and rotational.

Turbulence is characterized by high levels of fluctuating vorticity. A turbulent flow contains a wide range of eddies interacting with each other. Vortex stretching is an important mechanism in a turbulent flow. A continuous transport of energy from mean flow to large eddies, from these large eddies to a series of increasingly smaller eddies takes place and this is termed as the cascade process. The smaller eddies are influenced by the strain rate imposed by the large eddies and are continually stretched. The smallest eddies dissipate the kinetic energy into thermal energy due to viscous effects. The smallest eddy size, termed as the Kolmogorov scale, decreases with increasing Reynolds number. The largest eddy size depends on the flow configuration and is usually approximately as large as the size of the domain.

Turbulence is a continuum phenomenon, governed by the equations of fluid mechanics already described in the previous chapter. Even the smallest scales occurring in a turbulent flow are normally larger than any molecular length scale. Turbulence is not a feature of fluids but of flows. Most of the dynamics of turbulence may be similar in all fluids, whether gases or liquids or in other words independent of fluid properties.

A quantity of interest in a turbulent flow is the energy distribution over different wave numbers (or eddy sizes). If κ denotes the wave number (inversely proportional to the size of the eddy) and $E(\kappa)d\kappa$ the turbulence kinetic energy between a change of wave number $d\kappa$, then total turbulence kinetic energy may be given as

$$k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) = \int_0^\infty E(\kappa)d\kappa \tag{2.1}$$

where $\overline{u'^2}, \overline{v'^2}$, and $\overline{w'^2}$ denote mean squared velocity fluctuations in three directions, respectively. $E(\kappa)d\kappa$ depends on molecular viscosity, rate of dissipation of turbulence kinetic energy (ε), integral length scale, wave number and mean strain rate. Kolmogorov (1942) proposed that for fully turbulent flows an intermediate size of eddies can be found for which the cascade process is independent of the details of the energy containing eddies and molecular viscosity. This range of eddies is termed as the inertial subrange (Fig. 2.2). Kolmogorov (1942) showed that in the inertial subrange

$$E(k) = C_k \varepsilon^{2/3} \kappa^{-5/3} \tag{2.2}$$

Here C_k denotes the Kolmogorov constant.

No general solution to the governing Navier–Stokes equations for turbulent flows is known and no general solutions to problems in turbulent flow can be found. Every turbulent flow is different, even though different turbulent flows may have many common characteristics.

Intermittency (γ) is an important parameter for turbulent flows and is defined as the fraction of time at a point for which the flow is turbulent. Obviously in a laminar flow intermittency is zero everywhere. In a turbulent flow, intermittency is also zero right at the wall because of the no slip condition. However, it rises extremely sharply close to the wall and attains the maximum value of 1.0 almost next to the wall (Fig. 2.3). The intermittency approaches zero as one moves normal to the wall and towards the free-stream.

Entrainment is another important parameter and is defined as the rate of increase of mass in the downstream direction. Entrainment and consequently the growth rate is large in a turbulent flow compared to that in a laminar flow because eddies in a turbulent flow cause outer irrotational flow to move with the main flow direction. As a result the mass flow rate of a turbulent flow increases. In almost all cases turbulence is a stable state and once the flow reaches this state, it maintains itself be supplying energy from the mean flow to replace that lost at the smallest Kolmogorov scale due to viscous effects.

We must never forget that a raw turbulent flow is always unsteady and three-dimensional. Engineers are mostly interested in time averaged flow and such flow may have many simplifications such as steady or two-dimensional and so on. Before we move on to next section, we must pause for a few minutes and try to

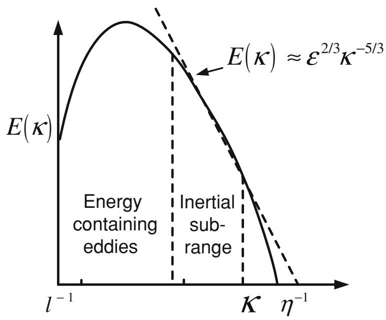

Fig. 2.2 Energy spectrum for a fully turbulent flow

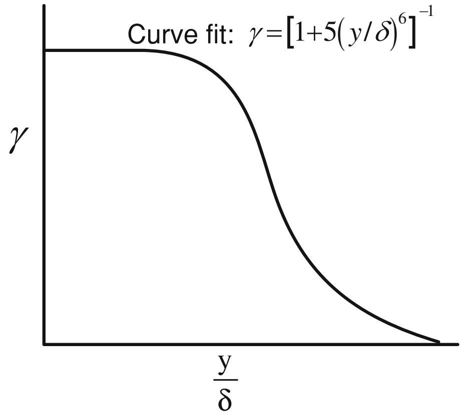

Fig. 2.3 Intermittency profile for turbulent flow past a flat plate. γ is zero right at the wall ($y = 0$), but it rises sharply to 1.0 and therefore this behaviour cannot be shown in a linear plot

understand main differences between laminar flows and turbulent flows and appreciate the complexities and salient features of turbulent flows.

2.2 Stability of Laminar Flows

A flow becomes turbulent when a laminar flow becomes unstable at a high value of the governing parameter, for example, Reynolds number in viscous flows and Grashof number in buoyancy driven flows. The instabilities are related to a complex interaction of viscous and nonlinear terms in the governing momentum equations. Linear hydrodynamic stability theory deals with conditions that lead to the amplification of disturbances in a flow. A basic treatment of stability of a flow involves selection of a basic flow, adding a disturbance to the flow to arrive at governing equations for disturbance, linearizing the governing equations and solving for the eigenvalues to arrive at the conditions for the instability of flow. The resulting governing equations for the disturbance are termed as the Orr–Sommerfeld equations. A study of stability of laminar flows is a complex subject and separate books have been dedicated to this subject (Chandrasekhar 1961; Drazin and Reid 1996; Drazin 2002). Typically free-shear flows are quite unstable even at low values of Reynolds number (typically about 10) and readily undergo transition. Wall bounded flows are known to be more stable.

2.3 Transition and Onset of Turbulence

Engineers are primarily interested in the prediction of the values of the governing parameter, for example critical Reynolds number Re_{crit} at which disturbances occurring in the flow become unstable and transitional Reynolds number Re_{tr} at

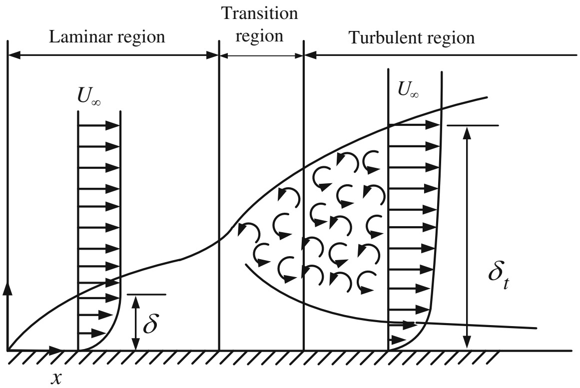

Fig. 2.4 Turbulent boundary-layer flow past a flat plate

which disturbances amplify to such an extent that they begin to influence the mean flow. The former can be predicted by the linear stability theory. However no theory exists that can predict the value of Re_{tr} for a particular flow. Some values have been proposed in the literature based on experimental data or correlations.

In a zero pressure gradient flow past a flat plate the value of $\mathrm{Re}_{x,cr}$ approximately equals 91,000 and different researchers have proposed different values of $\mathrm{Re}_{x,tr}$ with a range from 2.0×10^6 to 4.7×10^6 (Fig. 2.4). In a flow through a circular pipe, flow is always laminar for $\mathrm{Re}_D < 2{,}000$ and flow becomes turbulent for $\mathrm{Re}_D > 4{,}000$ depending on inlet conditions. Reynolds (1883) performed the famous dye experiment to study transition to turbulent flows in a circular pipe with several interesting outcomes.

The values of Reynolds number encountered in practical applications are such that flows are almost always turbulent. Most free-shear flows become unstable at very small values of Reynolds number and therefore for all practical purposes may be assumed turbulent under all conditions. The process of change in flow behaviour from laminar to turbulent flow is termed as the transition. A sequence of events in flow past a flat plate undergoing laminar to turbulent transition can be described as (White 2007): first the stable laminar flow experiences unstable two-dimensional Tollmein–Schlichting waves followed by the development of three-dimensional waves. Subsequently there is vortex breakdown locally where high shear occurs followed by three-dimensional fluctuations. This is followed by the local formation of turbulent spots and coalescence of these spots into fully turbulent flow.

2.4 Types of Turbulent Flows

- *Wall bounded and free-shear flows*. Turbulence in the presence of walls (termed as wall bounded turbulence) is quite complex compared to that far away from a wall. In Sect.3.1 in Chap.3 we will present a detailed treatment of turbulence in the presence of a solid wall.

- *Homogeneous turbulence.* If fluid turbulence has the same behaviour in all parts of the flow field, then it is termed as the homogenous turbulence.
- *Isotropic turbulence*. If the statistical features of turbulence are same in all flow directions, then it is termed as the isotropic turbulence.
- *Stationary turbulence*. If the time-average of all flow properties in a turbulent flow does not vary with time, then it is termed as the stationary turbulence.

It may be noted that in general no turbulent flow is homogenous or isotropic.

2.5 Significance of Turbulent Flows and Heat Transfer

We need to device ways to model/simulate turbulent flows that are encountered in a wide range of engineering applications. With the emergence of computers and different computational techniques, the computational fluid dynamics (CFD) has become an important tool for investigating turbulent flows. In CFD a set of appropriate governing equations are solved numerically within a computational domain using appropriate boundary conditions to obtain flow behaviour. The present book the fluctuations in the density are not considered, or in other words the treatment of the present book is applicable only for incompressible flows. Turbulence is usually specified as a percentage of mean flow properties $T_i = \sqrt{(2/3)k}/U_{ref}$ where k denotes turbulence kinetic energy and U_{ref} a reference velocity. This expression is based on the assumption of isotropic turbulence, i.e., $\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$.

Let us consider the complexity involved in computing a turbulent flow. A typical domain of 0.1 m by 1.0 m with a high Reynolds number turbulent flow might contain eddies down to 10–100 μm sizes and therefore the computing meshes of 10^9 to 10^{12} points may be required to accurately describe the processes at all length scales using the numerical solution of unsteady, three-dimensional Navier–Stokes equations. The fastest events may take place with a very large frequency of the order of 10 kHz and therefore we need to march in time with extremely small steps (of the order of hundreds of micro seconds). Thus we require extremely large grid points in both three dimensions and time. This approach of numerically solving unsteady, three-dimensional Navier–Stokes equations without any approximations or assumptions and resolving all scales in space and time is termed as the direct numerical simulation (DNS). Since a direct solution of the time-dependent Navier–Stokes equations of turbulent flows is computationally expensive, it is restricted to low Reynolds numbers and simple geometries. Fortunately, in many situations only time-averaged flow properties are required by engineers and thus there may not be a need to obtain an instantaneous numerical solution.

We will devote a large part of the present book on ways to compute time averaged flow properties such as velocity and temperature profiles, heat transfer coefficient, pumping power, and drag, etc. Subsequently, we will treat issues related to DNS of turbulent flows. We will also consider large eddy simulation of turbulent flows which is somewhat similar to DNS. In LES, three-dimensional, Navier–Stokes equations are numerically solved for instantaneous flows. However, one does not need to go down to the smallest Kolmogorov scale as in a DNS. The effect of smallest scales is modeled by means of subgrid scales and such models are termed as the subgrid scale models. The computations requirements for LES are somewhat less than those for DNS. LES has good potential for the design applications.

Now we will briefly consider three practical situations and see how turbulence is important in these situations. The flow within a stirred vessel that is used in several industries is highly turbulent, three-dimensional and unsteady. CFD requires that the computational grid match the shape of the vessel and all its components, which are often geometrically complex. The grid chosen should be fine enough to capture the smallest turbulent flow scales. A very fine grid will result in large computational requirements.

The design procedure of heat exchangers is quite complicated, as it involves an analysis of heat transfer rate and pressure drop. The major challenges to the design of a heat exchanger are to make it compact, which is, to achieve a high heat transfer rate and at the same time to allow its operation with a small power loss (or pumping power). The flow is usually turbulent in a heat exchanger and modeling of turbulent fluid flow and heat transfer becomes an important issue in the design of heat exchangers.

A plume is the simplest example of buoyancy driven flows, and it has important applications in environmental studies, atmospheric science and many engineering applications such as the cooling of electronic equipment, waste disposal in the atmosphere and oceans. An understanding of the plume behaviour is important for modeling more complicated buoyancy driven flows such as buoyant jets. In addition, plume provides a good example for illustrating the influence of buoyancy on turbulence. A turbulent plume is significantly more complex compared to its non-buoyant counterpart (a jet) because of interplay between buoyancy and turbulence and as we will see in Sect.9.4 in Chap. 9 that the modeling of a turbulent plume is a challenging task.

2.6 Turbulence in the Vicinity of a Solid Wall

Turbulence in the presence of solid walls exhibits interesting flow features. The region near the wall can be divided in three distinct layers: (a) viscous sublayer, where molecular effects dominate; (b) buffer layer, where the molecular and turbulent properties of the flow are both important and (c) fully turbulent layer, where the turbulent properties of the flow play the major role and molecular effects may be ignored. What is the influence of wall in a turbulent flow? The no-slip condition at the wall means the fluid velocity at the wall is equal to the wall velocity that in most cases is zero. As a result the viscous damping and kinematic blocking near the wall reduce the velocity fluctuations to zero. However, extremely large gradients in flow properties occur near the wall and therefore it can be said that walls cannot resist turbulence.

Figure 2.5 shows the profile of average streamwise velocity in the wall co-ordinates in the vicinity of a solid wall. Here $u^+ = u/u_\tau$ and $y^+ = yu_\tau/v$ denote the non-dimensional velocity and normal distance from the wall, respectively. u_τ denotes the friction velocity and is given as

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \tag{2.3}$$

Here τ_{w} denotes the wall shear stress. In the innermost viscous layer from $y^+ = 0$ to $y^+ = 5$ the non-dimensional velocity profile varies linearly

$$u^+ = y^+ \tag{2.4}$$

From y^+ equal to 30 to nearly $y/\delta = 0.1$ termed as the overlap layer or logarithmic layer the profile follows the logarithmic behaviour

$$u^+ = \frac{1}{\kappa}\ln y^+ + B \tag{2.5}$$

Here κ is the von Karman constant = 0.41. The profile in the outer region y/δ from 0.1 to 1.0 depends on the pressure gradient and Reynolds number. It may be noted that for y^+ from 5 to 30 a smooth transition between the linear behaviour and logarithmic behaviour is observed.

In the separation region the skin friction (and therefore the friction velocity) approaches zero and therefore u^+ approaches infinity and y^+ approaches zero. For a

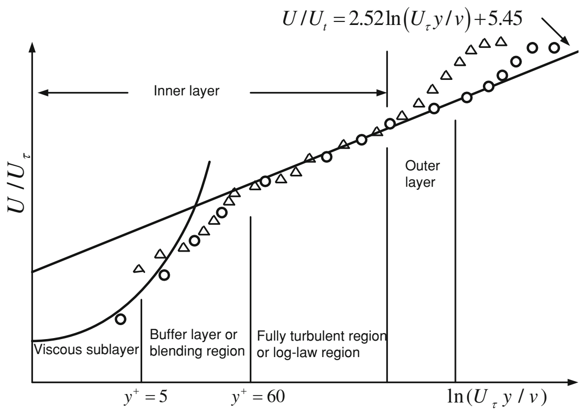

Fig. 2.5 Velocity profile in the wall coordinates in the presence of a solid wall

separating flow, the logarithmic behaviour does not hold and no overlap layer is observed.

The nature of the logarithmic overlap layer for a turbulent flow depends on the roughness of the solid wall. A small amount of roughness can disturb an extremely thin viscous sublayer and significantly increase the wall friction. The intercept B in Eq. 2.5 moves downward with an increasing roughness. The amount of shift is different at different non-dimensional values of roughness. The consequence of this shift will be discussed when we consider friction factor using the Moody's diagram in Sect. 3.5 in Chap.3.

Mean temperature in the wall co-ordinates also behaves similar to the mean velocity (Fig. 2.6). Right in the vicinity of the solid wall the mean temperature profile is linear in the thermal sublayer

$$T^+ = \Pr y^+, \tag{2.6}$$

where Pr denotes the Prandtl number and T^+ denotes the mean temperature in wall coordinates and is defined as

$$T^+ = \frac{T_w - \overline{T}}{T^*} \text{ where } T^* = \frac{q_w}{\rho C_p u_\tau} \tag{2.7}$$

Further away a logarithmic behaviour is observed.

$$T^+ = \frac{\Pr_t}{k} \ln y^+ + A(\Pr) \tag{2.8}$$

Here T^* denotes the wall conduction temperature and is equivalent of the friction velocity and $\overline{T}$ denotes the mean temperature. The last term in Eq. 2.8 is a strong function of Prandtl number (Pr).

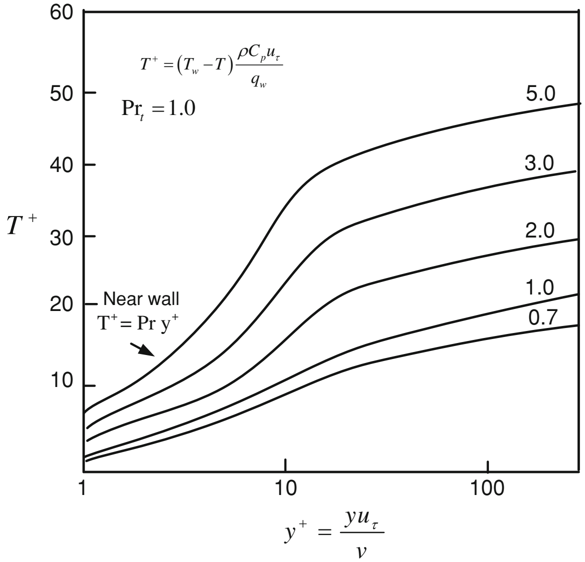

Fig. 2.6 Mean temperature profile in the wall coordinates in the presence of a solid wall

Kader (1981) provides a good curve fit to $A(\mathrm{Pr})$ that works well for a wide range of Prandtl numbers.

$$A(\mathrm{Pr}) = (3.85\,\mathrm{Pr}^{1/3} - 1.3)^2 + 2.12\ln(\mathrm{Pr}) \tag{2.9}$$

Some researchers have proposed curve fits for complete temperature profile covering sublayer, overlap layer and outer layer for a range of pressure gradients. Recently Cruz and Silva-Freire (2002) have proposed a thermal law of the wall for separating and reattaching flows.

The turbulent Prandtl number is defined as

$$\mathrm{Pr}_t = \frac{C_p \mu_t}{k_t} \tag{2.10}$$

where C_p denotes the specific heat at constant pressure and k_t thermal diffusivity of fluid due to turbulence.

The first question that arises is, "what value of turbulent Prandtl number Pr_t should be used?". For $\mathrm{Pr} < 0.7$ the value of Pr_t decreases from approximately 1.5 in the viscous sublayer and the adjoining region to approximately 1.0 in the logarithmic layer (Blackwell 1973). Therefore a constant value of $\mathrm{Pr}_t \sim 1.0$ can be assumed in the computations. A large value of Pr_t in close vicinity of a solid wall is not of much significance because turbulence is anyway negligible in this region. For low values of Prandtl number (Pr) the turbulent Prandtl number seems to have a higher value (between 2 and 4). For turbulent free-shear flows a constant value of Pr_t equal to 0.7 may be used everywhere.

2.7 Task of a Turbulence Model

Having seen complexities associated with fluid turbulence we may wonder how one can model such flows. Fortunately, there are ways to predict such flows using different turbulence models. However, no turbulence model seems to be universally applicable under a wide variety of applications and therefore it is important to choose a suitable turbulence model for a particular situation. In the subsequent chapters we will see salient features of important turbulent flows. We will also see that one needs to make several assumptions to mathematically model and compute complex turbulent flows. Therefore to gain confidence in a particular turbulence model, the predictions need to be first validated for a flow configuration by comparing the predictions with experimental data or DNS data. Once the model is validated it can be applied to a new flow configuration with the hope that it will work well in a new flow configuration as well. One must realize that due to inherent several complexities in a turbulent flow, it is quite likely that a single turbulence model will not be able to handle reasonably well a wide variety of turbulent flows. We will study a wide variety of turbulence models and it will not be easy to conclude which of the models studied will be suitable for a particular flow situation.

2.8 Concluding Remarks

In this chapter we have introduced a reader to turbulence and have seen key features of turbulent flows. We have seen how a laminar flow is different from a turbulent flow and some factors that lead to further complexity in a turbulent flow. Engineers need to understand turbulent flows in order to control them. Therefore prediction is an important component of engineering calculations. In the next five chapters we will examine different ways that can be adopted to predict/simulate turbulent flows.

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