Preface

The books by Athreya and Ney (1972), Harris (1963) and Jagers (1975) contain a lot about finite-dimensional branching processes and their applications. Measure-valued branching processes with abstract underlying spaces were constructed in Watanabe (1968), who showed those processes arose as high-density limits of branching particle systems. The connection of measure-valued branching processes with stochastic evolution equations was investigated in Dawson (1975). A special class of measure-valued branching processes are known as Dawson–Watanabe superprocesses, which have been undergoing rapid development thanks to the contributions of a great number of researchers. The developments have been stimulated from different subjects including classical branching processes, interacting particle systems, stochastic partial differential equations and nonlinear partial differential equations. The study of superprocesses leads to better understanding of results in those subjects as well. We refer the reader to Dawson (1992, 1993), Dynkin (1994, 2002), Etheridge (2000), Le Gall (1999) and Perkins (1995, 2002) for detailed treatments of different aspects of the developments in the past decades. Branching processes give the mathematical modeling for populations evolving randomly in isolated environments. A useful and realistic modification of the branching model is the addition of immigration from outside sources. From the viewpoint of applications, branching models allowing immigration are clearly of great importance and physical appeal; see, e.g., Athreya and Ney (1972). This modification is also familiar in the setting of measure-valued processes; see, e.g., Dawson (1993), Dawson and Ivanoff (1978) and Dynkin (1991a).

The main purpose of this book is to give a compact and rigorous treatment of the basic theory of measure-valued branching processes and immigration processes. In the first part of the book, we give an analytic construction of Dawson–Watanabe superprocesses with general branching mechanisms. The spatial motions of those processes can be general Borel right processes in Lusin topological spaces. We show that the superprocesses arise as high-density limits of branching particle systems, giving the intuitive interpretations of the former. Under natural assumptions, it is shown that the superprocesses have Borel right realizations. From the general model, we use transformations to derive the existence and regularity of several different forms of the superprocesses including those in spaces of tempered measures, multitype models, age-structured models and time-inhomogeneous models. This unified treatment of the different models simplifies their constructions and gives useful perspectives for their properties. When the underlying space shrinks to a single point, the superprocess reduces to a one-dimensional continuous-state branching process. We discuss briefly extinction probabilities and limit theorems related to the latter. The theory of the one-dimensional processes requires much less prerequisite knowledge and is helpful for the reader in developing their intuitions for superprocesses. Under Feller type assumptions, several martingale problems for superprocesses are formulated and their equivalence are established. The martingale measures induced by those martingale problems are not necessarily orthogonal, but they are still worthy. To make the book essentially self-contained, overlaps of the first part with Dawson (1993) and Dynkin (1994) cannot be avoided completely, but we have made them as little as possible.

In the second part of the book we investigate the immigration structures associated with measure-valued branching processes. For that purpose, we first give some characterizations of entrance laws for those processes. We define immigration processes in an axiomatic way using skew convolution semigroups as in Li (1995/6). It is then proved that the skew convolution semigroups associated with a given measure-valued branching process are in one-to-one correspondence with its infinitely divisible probability entrance laws. The immigration superprocess has regularities similar to those of the Dawson–Watanabe superprocess if the corresponding probability entrance law is closable. Instead of establishing the results by repeating the techniques in the first part, we concentrate on the genuinely new or different aspects of the immigration processes and develop the theory on the bases of the processes without immigration. In this way, we hope to give the book a more compact and unified form.

The concept of skew convolution semigroups can actually be introduced in an abstract setting. Roughly speaking, such a semigroup gives the law of evolution of a system with branching structure under the perturbation of random extra forces. The immigration process is only a special case of this formulation. There is another special case investigated by Bogachev and Röckner (1995) and Bogachev et al. (1996), who formulated Ornstein–Uhlenbeck type processes on Hilbert spaces using generalized Mehler semigroups. Skew convolution semigroups were also used in Dawson and Li (2006) to study the affine Markov processes introduced in mathematical finance. In the last part of the book, we discuss briefly characterizations of generalized Mehler semigroups and properties of the corresponding Ornstein–Uhlenbeck type processes. We also show that a typical class of those processes arise as fluctuation limits of immigration superprocesses.

The main theory of Dawson–Watanabe superprocesses and immigration superprocesses is developed for general branching mechanisms that are not necessarily decomposable into local and non-local parts. Most of the results were obtained before only for specific classes of branching mechanisms. The emphasis here is the basic structures and regularities, rather than intensive properties of specific models. The setting of Borel right processes we have chosen is very convenient for the development of the theory. The title of the book stresses the applications of techniques from the theory of general Markov processes. Our main references for those are Ethier and Kurtz (1986) and Sharpe (1988). In the appendix we give a summary of the basic concepts and results that are frequently used. We hope the summary will help the reader in a quick start of the main parts of the book. In the last section of each chapter, comments on the history and recent development are given. This book can be used as a reference of the basics of Dawson–Watanabe superprocesses and immigration superprocesses. It can also be used in a course for graduate students specialized in probability and stochastic processes.

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