

# Modeling the Out-of-Stock Risk and the EOQ–JIT Cost Indifference Point

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**Abstract** The most important advantage of an economic order quantity (EOQ) system is its ability to handle the unexpected demand. A model for comparing the inventory costs of purchasing under the EOQ system and the just-in-time (JIT) order purchasing system in existing literature concluded that JIT purchasing was virtually always the preferable inventory ordering system. By expanding the classical EOQ model, taking into account out-of-stock risk, which was not considered by previous researchers, this paper shows that it is possible for an EOQ system to be more cost effective than a JIT system when the out-of-stock risk associated with the JIT purchasing system is high or the annual demand is either too low or too high.

**Keywords** Cost indifference point · EOQ · JIT · Out-of-stock · Risk

## 1 Introduction

The successful implementation of just-in-time (JIT) purchasing policy in various industries has prompted many companies that still use the economic order quantity (EOQ) purchasing system to ponder whether they should switch to the JIT purchasing policy. This decision is, however, difficult to be made.

Fazel (1997) and Schniederjans and Cao (2001) made significant contributions in developing EOQ–JIT cost indifferent point functions. Fazel (1997) suggested that JIT was only preferable when demand was low. The “fixed costs” such as rental, utilities and personnel salaries were omitted from the EOQ–JIT cost difference function by Fazel (1997).

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Schniederjans and Cao (2001) argued that those “fixed costs” items were not fixed and thus should not be left out from the EOQ–JIT cost difference function. Schniederjans and Cao (2001) further argued that in situations where plants adopting the JIT operations experienced or could take advantage of physical plant space square meter reduction, to include a single cost item, namely, the physical plant space factor into the EOQ–JIT cost difference function would substantially increase the EOQ–JIT indifference point. Hence, the existing physical plant space could not hold the revised indifference point’s amount of inventory. Consequently, additional physical plant space has to be purchased. This would again force “. . . a new round of additional facility space costs favoring a JIT system . . .” (Schniederjans and Cao 2001, p.117). Schniederjans and Cao (2001) further suggested that saving space and using it to house additional increasing amounts of inventory to meet larger annual demand were juxtaposed issues. Schniederjans and Cao (2001) then concluded that the inclusion of a single cost item that was omitted by Fazel (1997) would prove that the JIT system was always preferable to an EOQ system (Schniederjans and Cao 2001). However, Schniederjans and Cao (2001) had difficulty to either scientifically or empirically ascertain the capability of an inventory facility to hold the EOQ–JIT cost indifference point’s amount of inventory.

The most important advantage of an EOQ system is its ability to handle the unexpected demand. This paper expands the classical EOQ model, takes into account out-of-stock risk, which was not considered by previous researchers, and shows that it is possible for an EOQ system to be more cost effective than a JIT system when the out-of-stock risk associated with the JIT purchasing system is high or the annual demand is either too low or too high.

## 2 Harris’ (1915) EOQ Model

Both Schniederjans and Cao’s (2001) and Fazel’s (1997) EOQ–JIT cost difference functions were based on Harris’s (1915) EOQ model, namely, the classical EOQ model. The classical EOQ model aims to minimize the total of ordering and holding costs, while assuming some inventory operating costs such as rental, utilities, and personnel salary, etc are “fixed” costs. The total annual cost of the classical EOQ system,  $TC_E$ , is the sum of the inventory ordering cost, inventory holding cost, and the cost of the actual purchased units, or:

$$TC_E = \frac{kD}{Q} + \frac{Qh}{2} + P_ED \quad (1)$$

where  $Q$  is the fixed order quantity,  $h$  is the annual cost of holding one unit of inventory in stock,  $k$  is the cost of placing an order,  $D$  is the annual demand for the item,  $P_E$  is the purchase price per unit,  $\frac{D}{Q}$  is the annual ordering frequency,  $\frac{Q}{2}$  is the annual average inventory level in the inventory facility. The first ratio is the inventory ordering cost item. The second ratio is the inventory holding cost item.

The last item is the annual purchasing cost component. Suggested that  $k$  and  $h$  are the most subjective components in (1). Nevertheless,  $k$  usually includes the inventory delivery charges and transaction costs of clerical paperwork.  $h$  often includes opportunity cost of the working capital tied up in purchased goods, taxes and insurance paid on inventory items, inventory spoilage cost and inventory obsolescence cost. The classical EOQ model provides appropriate inventory ordering decisions only when its assumptions can be met. These assumptions are: (1) the inventory operating costs, rental, utilities and personnel salary, etc are constant; and (2)  $h$  the annual cost of holding one unit of inventory in stock and  $k$  the cost of placing an order are constant. It should be noted that although the term “the total annual cost of an inventory item under an EOQ system” is widely used to refer to “ $TC_E$ ” in (1), “ $TC_E$ ” is not the actual total annual cost of an inventory item under an EOQ system. The actual total annual cost of an inventory item under an EOQ system should be the sum of “ $TC_E$ ” and the “fixed costs”.

As mentioned earlier, that the so called “fixed costs”, including “rental, utilities, and personnel salary” were excluded from the inventory holding cost item in (1) was also an important assumption made by Fazel (1997) and Schniederjans and Cao (2001) when they derived their EOQ–JIT cost indifference points. However, since (a) It is agreed that the so called “fixed costs” were left out from the so called “total annual cost of the EOQ system”, and (b) Gaither (1996) suggested that the annual inventory holding cost should include the opportunity cost of the working capital tied up in purchased goods, taxes and insurance paid on inventory items, inventory spoilage cost and inventory obsolescence cost, together with the cost of physical storage, and (c) Schonberger (1982) and Wantuck (1989) etc proved that the so called “fixed costs” would no longer be constant during JIT operations, and (d) Schniederjans and Olsen (1999) and Schniederjans and Cao (2001) observed that the saved inventory facilities can be rented out when the annual average inventory level dropped, then there is a reason to include all components of inventory holding costs into the holding cost item, when comparing an EOQ system with a JIT system. To sum up, one of the assumptions of the classical EOQ model, namely, the so called “fixed” costs are excluded from the holding cost item need to be revised, and the traditional EOQ model need to be expanded when comparing an EOQ purchasing system with a JIT purchasing system.

### 3 Revised EOQ Model

The revised EOQ model was identified from the ready mixed concrete (RMC) industry in land-scarce Singapore. The expensive land rental promoted the RMC suppliers to reduce the size of their inventory facilities to save on inventory holding costs of the raw materials for mixing RMC. “An inventory facility”, in this study, is defined as a physical plant place where raw materials, goods or merchandise are stored. An inventory facility can be a storehouse, a warehouse, an aggregates

depot, a cement terminal, or a sand yard. The total cost under the revised EOQ model is thus:

$$TC_{Er} = \frac{kD}{Q} + \frac{HQ}{2} + P_E D \quad (2)$$

$TC_{Er}$  is the sum of the inventory ordering cost, the expanded inventory holding cost, and the cost of the actual purchased units.  $TC_{Er}$ , with the inclusion of the so called “fixed costs”, is the actual total cost of the EOQ ordering system and is thus greater than  $TC_E$ , in (1).  $H$  is the expanded annual cost of holding one unit of inventory in stock. “ $H$ ”, with the inclusion of the additional inventory holding costs, is thus significantly greater than “ $h$ ” in (1). The revised EOQ model assumes that the so called “fixed costs”, including rental, utilities and personnel salary are proportion to the annual average inventory level. This assumption is possible, particularly when the square meter area of an inventory facility is designed in proportion to its annual average inventory level and the rental, utilities and personnel salary are in proportional to the size of the inventory facility. The revised EOQ model is particularly suitable for the scenarios in which the so called “fixed costs” are not fixed, for example during the feasibility study stage and design stage of an inventory facility, or the excess inventory facility space can be rented out when the annual average inventory level drops, as observed by Schniederjans and Olsen (1999) and Schniederjans and Cao (2001).

By taking the first order derivative with respect to  $Q$  of (1) and setting it to equal to zero, the optimum order quantity of the classical EOQ model,  $Q^*$ , can be derived as:

$$Q^* = \sqrt{\frac{2kD}{h}} \quad (3)$$

By taking the first order derivative with respect to  $Q$  of (2) and setting it to equal to zero, the optimum order quantity of the revised EOQ model,  $Q_r^*$ , can be derived as:

$$Q_r^* = \sqrt{\frac{2kD}{H}} \quad (4)$$

The optimum order quantity of the revised EOQ model,  $Q_r^*$  is significantly less than the optimum order quantity of the classical EOQ model,  $Q^*$ , as  $H$  the annual cost of holding one unit of inventory in the revised EOQ model is substantially greater than  $h$  the annual cost of holding one unit of inventory in the classical EOQ model, supposing the values of the other parameters, namely,  $D$  and  $k$  are the same.

To sum up, the revised EOQ model is different from the classical EOQ model on four counts. Firstly, the so called “fixed costs”, such as rental, utilities, personnel salaries, etc, are considered in the inventory holding cost item in the revised EOQ

model, thus  $H \succ h$ . Secondly, the so called “fixed costs” are also included into the total annual inventory costs in the revised EOQ model, thus  $TC_{Er} \succ TC_E$ . Thirdly, the revised EOQ model prefers small lot sizes and frequent deliveries. Last, but not least, the revised EOQ model aims to reduce the actual total inventory ordering and holding cost, while the classical EOQ model aims to reduce the sum of the inventory ordering cost and a part of the inventory holding cost. The last point makes it very clear that the revised EOQ model is more suitable than the classical EOQ model to represent the total cost under the EOQ system when comparing the EOQ system with the JIT system.

## 4 Revised EOQ–JIT Cost Indifference Point

Equation (4) results in a total annual optimal cost under the EOQ purchasing approach of:

$$TC_{Er} = \sqrt{2kDH} + P_ED \quad (5)$$

It is essential to note that (5) is valid only when two conditions are concurrently satisfied. Firstly,  $k$  the cost of placing one order and  $H$  the annual cost of holding one unit of inventory are constant. Secondly, the inventory is ordered at its economic order quantity.

Under the JIT system, the ordering cost and holding cost, including the so called “fix costs” are mainly transferred to the supplier. The total annual cost under the JIT system,  $TC_J$ , suggested by Fazel (1997) is therefore given by:

$$TC_J = P_J D \quad (6)$$

where  $P_J$  is the unit price under the JIT system.  $P_J$  is greater than  $P_E$ . This is to partially reflect the holding costs and ordering costs that have been transferred to the materials suppliers (Fazel 1997; Schniederjans and Cao 2001). However, JIT purchasing systems are time sensitive. JIT purchasing requires precise level schedules and rely on frequent transportation, as they are generally unable to cope with significant fluctuation in demand. This can be seen in situations arising from the Kobe earthquake in Japan (Low and Choong 2001), the strike on the West Coast of American and the 2003 Iraqi War (Singh 2003). Thus, the risk parameter, namely the out-of-stock costs, should be considered. Let  $\gamma\beta$  represents the additional out-of-stock costs under a JIT purchasing system comparing to that under an EOQ purchasing system, where  $\gamma$  represents the number of additional working hours that may be affected in a JIT system than that in an EOQ system,  $\beta$  represents the value created in one working hour.  $\gamma\beta$  is a penalty for using JIT purchasing instead of EOQ purchasing. The revised total annual cost under the JIT system,  $TC_{Jr}$ , is therefore given by:

$$TC_{Jr} = P_J D + \gamma \beta \quad (7)$$

To make a comparison between the total costs under the EOQ system and the JIT system, a  $Z_r$  model that combines the total annual optimal cost under the EOQ system in (5) and the revised total annual cost under the JIT system in (7) can be presented as:

$$Z_r = \sqrt{2kDH} + P_E D - P_J D - \gamma \beta \quad (8)$$

$Z_r$  represents the cost difference between an EOQ purchasing system and a JIT purchasing system.

When setting  $Z_r$  equal to zero, the roots of (8) are the revised EOQ–JIT cost indifference points,  $D_{indr1}$  and  $D_{indr2}$ . The values of  $D_{indr1}$  and  $D_{indr2}$  are given by:

$$D_{indr1} = \frac{kH - (P_J - P_E)\gamma\beta - \sqrt{k^2H^2 - 2kH\gamma\beta(P_J - P_E)}}{(P_J - P_E)^2} \quad (9)$$

$$D_{indr2} = \frac{kH - (P_J - P_E)\gamma\beta + \sqrt{k^2H^2 - 2kH\gamma\beta(P_J - P_E)}}{(P_J - P_E)^2} \quad (10)$$

## 5 Conclusion

JIT purchasing of raw materials is one important technique of the JIT philosophy. However, JIT purchasing is not always more cost effective than the EOQ purchasing system. By expanding the classical EOQ model, this study suggests that it is possible for an EOQ purchasing system to be more cost effective than a JIT purchasing system in the scenarios where demand is too low, or where demand is extremely large and the order costs cannot be economically split, or the out-of-stock costs associated with the JIT purchasing system is high.

## References

- Fazel, F. (1997), "A comparative analysis of inventory cost of JIT and EOQ", *International Journal of Physical Distribution and Logistics Management*, Vol. 27 No. 8, 496–505.
- Gaither, N. (1996), *Production and Operations Management*, Duxbury Press, Belmont, CA.
- Harris, F.W. (1915), *Operations and Cost – Factory Management Series*, A.W. Shaw Co, Chicago.
- Low, S.P. and Choong, J.C. (2001), "Just-In-Time management of precast concrete components", *Journal of Construction Engineering and Management*, Vol. 127 No. 6, pp. 494–501.
- Schniederjans, M.J. and Cao, Q. (2001), "An alternative analysis of inventory costs of JIT and EOQ purchasing", *International Journal of Physical Distribution & Logistics Management*, Vol. 31 No. 2, pp. 109–123.

- Schniederjans, M.J. and Olsen, R.J. (1999), *Advanced Topics in Just-in-Time Management*, Westport, Conn: Quorum Books.
- Schonberger, R.J. (1982), "A revolutionary way to streamline the factory", *The Wall Street Journal*, 15 November, pp. 24.
- Singh, K. (2003), "Just-In-Time systems thrown into chaos suppliers rush to meet orders", *The Straits Time*, 20 March, Singapore.
- Wantuck, K.A. (1989), *Just-In-Time for America: A Common Sense Production Strategy*, The Forum Ltd., Milwaukee, WI.

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