Preface

This volume is based on the themes of, and records advances achieved as a result of, the *Heidelberg Knot Theory Semester*, held in winter 2008/09 at Heidelberg University under the sponsorship of the Mathematics Center Heidelberg (MATCH), organized by M. Banagl and D. Vogel. In the preceding summer semester an introductory seminar on knots aimed at providing non-experts and young mathematicians with some of the foundational knowledge required to participate in the events of the winter semester. These comprised expository lecture series by several leading experts, representing rather diverse aspects of knot theory and its applications, and a concluding workshop held December 15 to 19, 2008.

Knots seem to be a deep structure, whose peculiar feature it is to surface unexpectedly in many different and a priori unrelated areas of mathematics and the natural sciences, such as algebra and number theory, topology and geometry, analysis, mathematical physics (in particular statistical mechanics), and molecular biology. Its relevance in topology, apart from its intrinsic interest, is partly due to the fact that every closed, oriented 3-manifold can be obtained by surgery on a framed link in the 3-sphere. Modern topology has also obtained information on high-dimensional knots, that is, embeddings of an n-sphere in an $(n+2)$-sphere with n larger than one. In algebra, representations of quantum groups lead to a multitude of knot invariants. Based on ideas of B. Mazur in number theory, one can assign to two prime ideals of a number field a linking number in analogy with classical knot theory. This number-theoretic linking number plays a role in studying the structure of Galois groups of certain extensions of the number field. Analysis touches on knot theory by means of operator algebras and their connection to the Jones polynomial. As far as geometry is concerned, results by Fenchel on the curvature of a closed space curve date back to the 1920s. Milnor showed in 1949 that the curvature must exceed 4π if the curve is knotted. One also considers "real" knots as physical objects in 3-space and studies various natural energy functionals on them. Sums taken over all states of suitable models originating in statistical mechanics, describing large ensembles of particles, can express knot invariants such as L. Kauffman's bracket polynomial. The discovery of the Jones polynomial entailed ties with mathematical physics based on a curious congruity of five relations, namely the Artin-relation in braid groups, a fundamental relation in certain operator algebras due to Hecke, the third Reidemeister move, the classical Yang-Baxter equation, and its quantum version. This lead to the construction of topological quantum field theories by Witten and Atiyah. Cellular DNA is a long molecule, which may be closed (as e.g. the genome of certain bacteria) and knotted or linked with other DNA strands. Enzymes such as topoisomerase or recombinase operate on DNA changing the topological knot or link type.

The objective of the Heidelberg Knot Theory Semester was to do justice to this diversity by bringing together representatives of most of the above research avenues, accompanied by the hope that such a meeting might foster inspiration and synergy across the various questions and approaches. Certainly, a fairly comprehensive portrait of the current state-of-the-art in knot theory and its applications emerged as a result.

Four lecture series were given: DeWitt Sumners gave 5 lectures on scientific applications of knot theory, discussing DNA topology, a tangle model for DNA site-specific recombination, random knotting, topoisomerase, spiral waves and viral DNA packing. Kent Orr's 3 lectures explained knot concordance and surgery techniques, while Louis Kauffman's 2 lectures introduced virtual knots and detailed parallels to elementary particles. The topic of Masanori Morishita's 6 lectures were the aforementioned analogies between knot theory and number theory.

The 21 speakers of the final workshop "The Mathematics of Knots" reported on a variety of interesting current developments. Many of these accounts are mirrored in the papers of the present volume. Among the low-dimensional topics were virtual knots and associated invariants such as arrow and Jones polynomials, the HOMFLY polynomial, questions about Dehn filling, Legendrian knots, Khovanov homology, surface knots, slice knots, fibered knots and property R, colorings by metabelian groups, singular knots, Gram determinants of planar curves, as well as geometric structures such as surfaces associated with knots, and the fibering of 3-manifolds when the product of the manifold with a circle is known to be symplectic. High-dimensional topics concerned the Cohn noncommutative localization of rings and its application to knots via algebraic K- and L-theory, as well as high-dimensional non-locally flat embeddings and the role of knot theory vis-à-vis transformation groups. Scientific talks discussed random knotting, viral DNA packing, and the topology of DNA-protein interactions.

We wish to extend our sincere thanks to the contributors of this volume and to all participants of the Heidelberg Knot Theory Semester, especially to the lecturers giving mini-courses, for the energy and time they have devoted to this event and the preparation of the present collection. Paul Seyfert receives the editors' thanks for technical help in typesetting this volume. Furthermore, we are grateful to Dorothea Heukäufer for her efficient handling of numerous logistical issues. Finally, we would like to express our gratitude to Willi Jäger and MATCH, whose financial support made the Heidelberg Knot Theory Semester possible.

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