Chapter 1
Basic Description of Laser Diode Dynamics by Spatially Averaged Rate Equations: Conditions of Validity

A laser diode is a device in which an electric current input is converted to an output of photons. The time-dependent relation between the input electric current and the output photons is commonly described by a pair of equations describing the time evolution of photon and carrier densities inside the laser medium. This pair of equations, known as the laser rate equations, is used extensively in the following chapters. It is, therefore, appropriate, in this first chapter, to summarize the results of Moreno [2] regarding the conditions under which the rate equations are applicable.

1.1 The "Local" Rate Equations

The starting point for the analysis of laser kinetics involves the coupled rate equations, which are basically *local* photon and injected carrier conservation equations [3]:

$$\frac{\partial X^+}{\partial t} + c\frac{\partial X^+}{\partial z} = A(N - N_{\mathrm{tr}})X^+ + \beta\frac{N}{\tau_s}, \tag{1.1a}$$

$$\frac{\partial X^-}{\partial t} - c\frac{\partial X^-}{\partial z} = A(N - N_{\mathrm{tr}})X^- + \beta\frac{N}{\tau_s}, \tag{1.1b}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{J}{ed} - \frac{N}{\tau_s} - A(N - N_{\mathrm{tr}})(X^+ + X^-), \tag{1.1c}$$

where z is the spatial dimension along the length of the laser, with reflectors of (power) reflectivities R placed at $z = \pm L/2$, X^+ and X^- are the forward and backward propagating photon densities (which are proportional to the light intensities), N is the local carrier density, N_{tr} is the electron density where the semiconductor medium becomes transparent, c is the group velocity of the waveguide mode, A is the gain constant in s^{-1}/(unit carrier density), β is the fraction of spontaneous emission entering the lasing mode, τ_s is the spontaneous recombination lifetime of the carriers, z is the distance along the active medium with $z = 0$ at the center of the laser, J is the pump current density, e is the electronic charge, and d the thickness

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of the active region in which the carriers are confined. For the remaining of this chapter, it is assumed that $N_{\mathrm{tr}} = 0$, the only consequence of which is a DC shift in the electron density, which is of significance only in considering lasing threshold. In addition, the following simplifying assumptions are made in writing down (1.1):

(a) The quantities X^* describe the local photon number densities of a longitudinal mode of the passive laser cavity at a given (longitudinal) position in the laser cavity, at time t, integrated over the lasing linewidth of the longitudinal mode, which is assumed to be much narrower than the homogeneously broadened laser gain spectrum.
(b) The gain coefficient (AN) is a linear function of the injected carrier density N (A is popularly known as the "differential optical gain coefficient" and is shown in later chapters to play a key role in determining direct modulation bandwidth of laser diodes).
(c) Variations of the carrier and photon densities in the lateral dimensions are neglected.
(d) Diffusion of carriers is ignored.

Assumptions 1 and 2 are very reasonable assumptions that can be derived from detailed analysis [4–6]. The representation of the semiconductor laser as a homogeneously broadened system can also be derived from basic considerations [7]. Transverse modal and carrier diffusion effects, ignored in assumptions 3 and 4, can lead to modifications of the dynamic behavior of lasers [8, 9].

Equations (1.1) are to be solved subject to the boundary conditions

$$X^-\left(\frac{L}{2}\right) = RX^+\left(\frac{L}{2}\right), \tag{1.2a}$$

$$X^+\left(\frac{-L}{2}\right) = RX^-\left(\frac{-L}{2}\right). \tag{1.2b}$$

The steady-state solution of (1.1) gives the static photon and electron distributions inside the laser medium and has been solved analytically in [4]. The solution is summarized as follows, where the zero subscript denotes steady-state quantities:

$$X_0^+(z) = \frac{a\mathrm{e}^{u(z)} - \beta}{Ac\tau_s}, \tag{1.3a}$$

$$X_0^-(z) = \frac{a\mathrm{e}^{-u(z)} - \beta}{Ac\tau_s}, \tag{1.3b}$$

where a is a quantity given by the following transcendental equation:

$$(1 - 2\beta)\xi + 2a\sinh\xi = \frac{gL}{2}, \tag{1.4}$$

where

$$\xi = \frac{1}{2}\sqrt{\frac{(R-1)^2\beta^2}{(Ra)^2} + \frac{4}{R}} + (R-1)\frac{\beta}{Ra} \tag{1.5}$$

and $g = AJ_0\dfrac{\tau_s}{ed}$ is the unsaturated gain, and $u(z)$ is given transcendentally by

$$(1-2\beta)u(z) + 2a\sinh u(z) = gz. \tag{1.6}$$

The electron density $N_0(z)$ is given by

$$AcN_0(z) = \frac{g}{1 + 2a\cosh u(z) - 2\beta}. \tag{1.7}$$

Figure 1.1 shows plots of $X_0^+(z)$, $X_0^-(z)$, and $g_0(z) = AcN_0(z)$ for a 300−μm laser with three values of end-mirror reflectivities. (a) 0.3, (b) 0.1, and (c) 0.9. The high nonuniformity in the distributions becomes apparent at low reflectivities.

1.2 Spatially Averaged Rate Equations and their Range of Validity

Equation (1.1) constitute a set of three coupled nonlinear differential equations in two variables that do not lend themselves to easy solutions. Considerable simplification can be made if the longitudinal spatial variable (z) is integrated over the length of the laser. Such simplification is valid only when the end-mirror reflectivity is "sufficiently large", A more precise definition of the range of validity of such an assumption is given in the following, summarizing the approach of [2].

To begin, (1.1a) and (1.1b) are integrated in the z variable, resulting in

$$\frac{\mathrm{d}X^{+*}}{\mathrm{d}t} + c\left[X^+\left(\frac{L}{2}\right) - X^+\left(\frac{-L}{2}\right)\right] = A(NX^+)^* + \beta\frac{N^*}{\tau_s}, \tag{1.8a}$$

$$\frac{\mathrm{d}X^{-*}}{\mathrm{d}t} - c\left[X^-\left(\frac{L}{2}\right) - X^-\left(\frac{-L}{2}\right)\right] = A(NX^-)^* + \beta\frac{N^*}{\tau_s}, \tag{1.8b}$$

where * denotes the spatial average $\displaystyle\int_{-L/2}^{L/2}\frac{\mathrm{d}z}{L}$. Adding (1.8a) and (1.8b),

$$\frac{\mathrm{d}P^*}{\mathrm{d}t} + \frac{2c(1-R)P(L/2)}{L(1+R)} = A(NP)^* + 2\beta\frac{N^*}{\tau_s}, \tag{1.9}$$

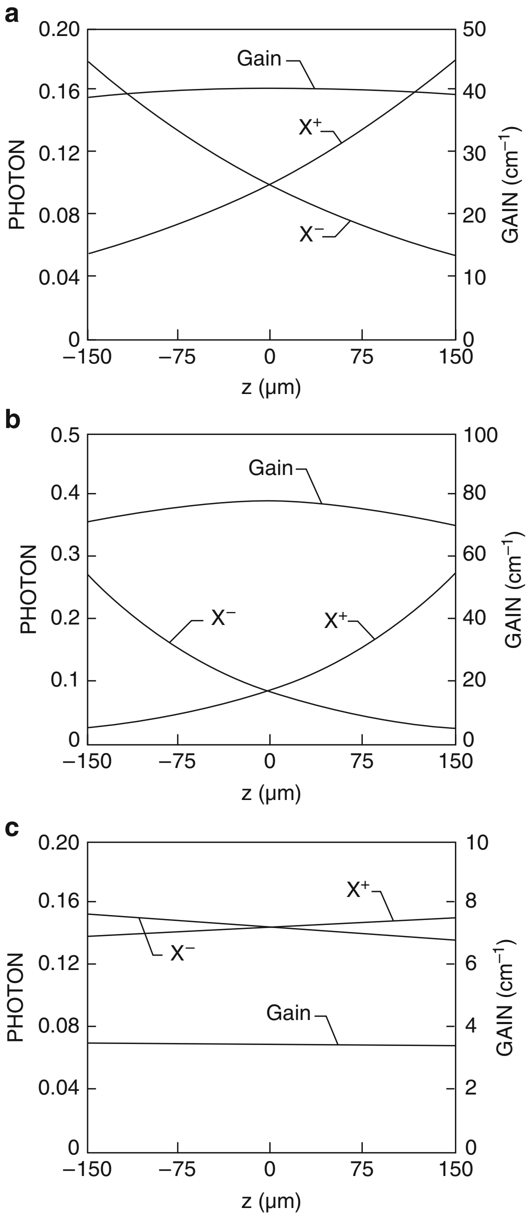

Fig. 1.1 Steady-state photon and electron-density distributions inside laser diodes with mirror reflectivities of (**a**) 0.3, (**b**) 0.1, and (**c**) 0.9

where $P = X^+ + X^-$ is the total local photon density and the boundary conditions (1.2) have been used. Equation (1.1c) integrates straightforwardly to

$$\frac{\mathrm{d}N^*}{\mathrm{d}t} = \frac{J}{ed} - \frac{N^*}{\tau_s} - A(NP)^* \tag{1.10}$$

where a uniform pump current of density J is assumed. A is known as the "differential optical gain". It is shown in later chapters to play a key role in determining direct modulation bandwidth of laser diodes. Introducing factors f_1 and f_2 as follows:

$$f_1 = \frac{(NP)^*}{N^* P^*}, \tag{1.11}$$

$$f_2 = \frac{P(L/2)}{P^*(1+R)}, \tag{1.12}$$

one can write the spatially averaged rate equations (1.9) and (1.10) in the following form:

$$\frac{\mathrm{d}P^*}{\mathrm{d}t} = Af_1 N^* P^* - 2c(1-R) f_2 \frac{P^*}{L} + 2\beta \frac{N^*}{\tau_s}, \tag{1.13}$$

$$\frac{\mathrm{d}N^*}{\mathrm{d}t} = \frac{J}{ed} - \frac{N^*}{\tau_s} - Af_1 N^* P^*, \tag{1.14}$$

which are recognized as the commonly used rate equations [10, 11] if the conditions

$$f_1 = 1, \tag{1.15}$$

$$f_2 = -\frac{1}{2}\frac{\ln R}{1-R} \tag{1.16}$$

are satisfied. The first of these conditions requires, for the quantities N and P, that the spatial average of the product equals the product of the spatial averages. This condition is not satisfied in general, but it will be true if the electron density N is uniform, as in the case when R approaches unity, which is apparent from Fig. 1.1c. The second condition requires the photon loss rate in (1.13) to be inversely proportional to the conventional photon lifetime. It will also be satisfied if R is very close to unity, since both (1.12) and (1.16) converge to 0.5 at this limit.

A more precise delineation of the range of the applicability of conditions (1.15) and (1.16) is obtained by calculating f_1 and f_2 from exact steady-state solutions (1.3)–(1.7), and comparing them with (1.15) and (1.16). From (1.3) and (1.7),

$$f_1 = \frac{L \displaystyle\int \frac{P}{1 + A\tau_s P}\,\mathrm{d}z}{\displaystyle\int \frac{\mathrm{d}z}{1 + A\tau_s P} \int P\,\mathrm{d}z}, \tag{1.17}$$

$$f_2 = \frac{LX^+(L/2)}{\int P\,\mathrm{d}z}, \tag{1.18}$$

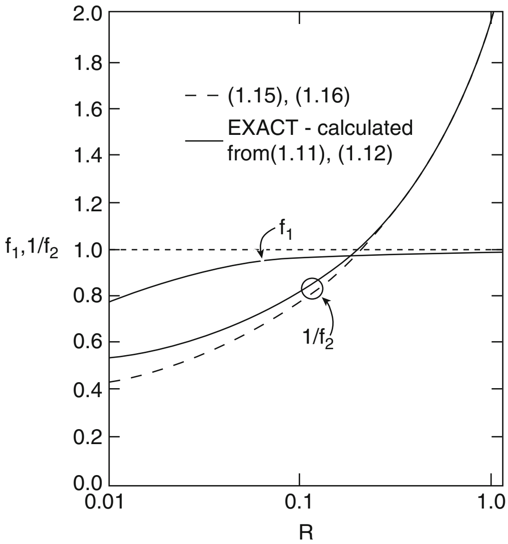

Fig. 1.2 Variations of f_1 and $\frac{1}{f_2}$ with R when $\beta \leq 10^{-3}$ and $gL > 10$

where the integrals are evaluated over the length of the laser. These integrals can be numerically evaluated using (1.3)–(1.7), and the results are shown in Fig. 1.2. Figure 1.2 shows numerically computed plots (solid lines) of f_1 and $1/f_2$ as a function of end-mirror reflectivity R; the calculation was done with the laser biased above threshold. The dotted lines are the "ideal" values of f_1 and f_2 given by (1.15) and (1.16). The figure indicates that the usual rate equations are reasonably accurate for R larger than approximately 0.2 – valid for laser diodes constructed from III–V materials, which have facet reflectivities of ≈ 0.3.

The above results lead to the conclusion that the simple rate equations, expressed in (1.19) and (1.20) (where the N and P now denote *averaged* quantities, in the *longitudinal spatial dimension*):

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{J}{ed} - \frac{N}{\tau_s} - ANP \tag{1.19}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = ANP - \frac{P}{\tau_p} + \beta\frac{N}{\tau_s} \tag{1.20}$$

($1/\tau_p = c/(2L)\ln(1/R)$ is the classical photon lifetime and $A = \kappa c$) are reasonable representations if the end-mirror reflectivity is above 0.2 and the laser is above threshold. The spontaneous emission factor β in (1.20) is a factor of two higher than that defined in (1.1) due to the inclusion of photons propagating in both directions. Common GaAs or quaternary lasers, with the mirrors formed by the cleaved crystal facets, have a reflectivity of ~ 0.3 and are thus well within the scope of (1.19) and (1.20). In Appendix D, the exact small signal version of (1.1)) is solved numerically, and it is found that (1.19) and (1.20) can very accurately describe the small signal frequency response of the laser for end-mirror reflectivities as low as 10^{-3}. This is certainly not expected from a physical standpoint and serves as a surprise bonus for this simplification.

Another factor that can render the spatially uniform assumption invalid is when "fast" phenomena, occurring on the time scale of a cavity transit time, are being considered. It is obvious that the concept of "cavity lifetime" and that of cavity modes, appearing in (1.20), are no longer applicable on that time scale. In common semiconductor lasers where the cavity length is approximately 300 μm, the cavity transit time is about 3.5 ps. The usual rate equations are, therefore, not applicable in describing phenomena shorter than about 5 ps, or at modulation frequencies higher than 60 GHz. Modulation regimes in the millimeter wave frequencies can take advantage of this cavity round-trip effect and is known as "resonant modulation", discussed in detail in Part II of this book.

In the following chapters, (1.19) and (1.20) are used extensively and serve as the basis for most of the analysis of the direct modulation characteristic of lasers.

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