

Chapter 2

Light Propagation in Nonlinear Periodic Media

This chapter describes the fundamentals of light propagation in nonlinear periodic media and provides the basis for all further results presented in this thesis. Starting from general considerations of nonlinear light-matter interaction, the photo-refractive nonlinearity as an ideal mechanism for the combination of periodic refractive index structures with a strong nonlinear material response at moderate laser powers is discussed in detail. Subsequently, the concept of photonic band gaps as a result of periodicity is introduced and the influence of the band structure on linear and nonlinear light propagation is presented.

2.1 Basic Equations of Wave Propagation in Nonlinear Optical Media

We start our discussion by considering Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad (2.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}. \quad (2.4)$$

In a nonmagnetic material, the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ are related to the electric displacement field $\mathbf{D}$ and the magnetic induction $\mathbf{B}$ by the following material equations:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (2.5)$$

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (2.6)$$

where ϵ_0 denotes the free space permittivity and $\mathbf{P}$ the induced polarization of the material. Since we are interested in solutions to these equations in regions containing no free charges or currents, we set $\rho = 0$ and $\mathbf{j} = 0$. Introducing the speed of light in vacuum $c = 1/\sqrt{\mu_0 \epsilon_0}$ and combining (2.3)–(2.6) then leads to the general wave equation

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = - \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}. \quad (2.7)$$

In this equation, the material's response to the electric field associated with an incident light wave is determined by the polarization $\mathbf{P}$. For linear dielectric media, it can be written as

$$\mathbf{P} = \epsilon_0 \chi^{(1)} \mathbf{E} \quad (2.8)$$

where $\chi^{(1)}$ is the (linear) electric susceptibility of the medium. It should be noted that although it can often be treated as a scalar constant, in the general case of propagation in anisotropic media, $\chi^{(1)}$ is a tensor of rank 2. Substituting (2.8) into (2.5) gives

$$\mathbf{D} = \epsilon_0 (1 + \chi^{(1)}) \mathbf{E} = \epsilon_0 \epsilon \mathbf{E} \quad (2.9)$$

with the dielectric permittivity ϵ . If the electric field of the incident light wave becomes comparable to the intra-atomic field, the linear relation (2.8) is no longer valid. In this case, the polarization is often written as

$$\mathbf{P} = \epsilon_0 \left(\chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{E} \mathbf{E} + \chi^{(3)} \mathbf{E} \mathbf{E} \mathbf{E} + \dots \right) \quad (2.10)$$

where $\chi^{(i)}$ with $i \geq 2$ represents the i -th order nonlinear susceptibility. The nonlinear terms in (2.10) give rise to a number of interesting phenomena [1]. For instance, the second-order term is responsible for second harmonic generation or sum- and difference frequency generation while the third-order term causes phenomena like third harmonic generation or optical phase conjugation.

For further analysis, it is convenient to rewrite (2.10) as

$$\mathbf{P} = \epsilon_0 \chi_{\text{eff}} \mathbf{E} \quad (2.11)$$

with an effective, intensity-dependent susceptibility $\chi_{\text{eff}}(I \equiv |\mathbf{E}|^2)$. Using this relation and introducing the effective refractive index $n(I) = \sqrt{1 + \chi_{\text{eff}}}$, the Helmholtz equation can be obtained from (2.7):

$$-\nabla^2 \mathbf{E} + \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \quad (2.12)$$

In this derivation, we have used the relation $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ together with the fact that the term $\nabla(\nabla \cdot \mathbf{E})$ is small for most cases of interest [1]. As a next step, the refractive index is divided into its value in the absence of light n_0^2 and the light induced refractive index shift $\Delta n^2(I)$:

$$n^2 = n_0^2 + \Delta n^2(I). \quad (2.13)$$

Moreover, we consider a wave that is linearly polarized in x -direction and propagating in z -direction, i.e.

$$\mathbf{E}(\mathbf{r}, t) = A(\mathbf{r})e^{i(k_z z - \omega t)} \cdot \mathbf{e}_x \quad (2.14)$$

with $\mathbf{r} = (x, y, z)$ and $k_z = n_0 \omega / c$. In the standard paraxial approximation, the envelope $A(x, y, z)$ is assumed to vary with z on a scale much longer than the wavelength. In this case, its second derivative with respect to z can be neglected such that inserting (2.14) into (2.12) results in the paraxial wave equation

$$i \frac{\partial A}{\partial z} + \frac{1}{2k_z} \nabla_{\perp}^2 A + \frac{k_z}{2n_0^2} \Delta n^2(I) A = 0 \quad (2.15)$$

with $\nabla_{\perp}^2 = (\partial^2 / \partial x^2 + \partial^2 / \partial y^2)$. Introducing the dimensionless variables $x' = x/x_0$, $y' = y/y_0$ and $z' = z/k_z x_0^2$ with a transverse scaling factor x_0 gives

$$2i \frac{\partial A}{\partial z'} + \nabla_{\perp}^2 A + \frac{k_z^2 x_0^2}{n_0^2} \Delta n^2(I) A = 0 \quad (2.16)$$

with $\nabla_{\perp}^2 = (\partial^2 / \partial x'^2 + \partial^2 / \partial y'^2)$. Throughout the rest of this work, we will use this dimensionless form unless otherwise noted. Therefore, we will drop the primes for convenience.

2.2 The Photorefractive Nonlinearity

The photorefractive effect describes the change in the local refractive index of a medium as a result of an optically induced redistribution of charge carriers. It was first observed in 1966 as wavefront distortions of a coherent light beam propagating through a Lithium Niobate (LiNbO_3) crystal [2]. Since then, it has been observed in many different materials such as Barium Titanate (BaTiO_3), Lithium Tantalate (LiTaO_3), Potassium Niobate (KNbO_3), and Strontium Barium Niobate (SBN) which will be the material of choice in all experiments presented in this thesis.

The origin of the photorefractive effect is illustrated schematically in Fig. 2.1. Typically, the crystals are doped with acceptors as well as donors. Illuminating them with light of appropriate wavelength lifts up electrons or holes from donors or traps into the conduction band via photoionization. The charges in the

conduction band then move under the influence of diffusion, drift in an externally applied field E_0 or the photovoltaic effect and finally recombine with empty donors or traps. This recombination process builds up a static space charge field E_{sc} that modulates the refractive index via the linear electrooptic effect which will be described in the next section.

2.2.1 The Linear Electrooptic Effect

In electrooptic crystals, the presence of a dc electric field leads to a change in the dielectric permittivity ϵ or equivalently a change in dimension and direction of the optical indicatrix [3]. In photorefractive materials, this change depends linearly on the strength of the induced space charge field and is therefore described by the linear electrooptic effect (Pockels effect). The mathematical description of this effect is traditionally given as a change in the impermeability tensor

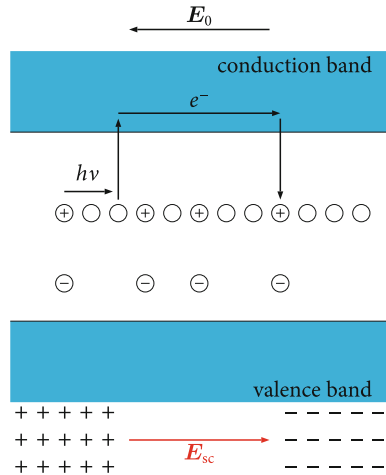
$$\Delta\eta_{ij} = \Delta\left(\frac{1}{n^2}\right) = \sum_k r_{ijk} E_k. \quad (2.17)$$

In this notation the constants r_{ijk} are the elements of the electrooptic tensor and E_k represents the k -th component of the total electric field $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_{sc}$ as the sum of the externally applied field $\mathbf{E}_0$ and the space charge field $\mathbf{E}_{sc}$ generated inside the crystal. The corresponding change in the dielectric permittivity is given by

$$\Delta\epsilon_{ij} = -\epsilon_0 n_i^2 n_j^2 \Delta\eta_{ij}. \quad (2.18)$$

To abbreviate notations, the coefficients r_{ijk} are typically written in a contracted notation defined by the following scheme for $k = 1, 2, 3$:

Fig. 2.1 Schematic illustration of charge carrier transport in photorefractive crystals



$$\begin{array}{lll}
r_{1k} = r_{11k} & r_{2k} = r_{22k} & r_{3k} = r_{33k} \\
r_{4k} = r_{23k} = r_{32k} & r_{5k} = r_{13k} = r_{31k} & r_{6k} = r_{12k} = r_{21k}.
\end{array}$$

Throughout this thesis, all experiments will be carried out in Strontium Barium Niobate (SBN) which belongs to the point group 4 mm. In contracted notation, the electrooptic coefficients for this particular symmetry class are given by¹

$$r_{\text{SBN}} = \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{42} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (2.19)$$

Typically, SBN-crystals obey the relation $r_{33} \gg r_{13}, r_{42}$ such that the light induced changes of the dielectric permittivity ϵ can be described by an effective electrooptic coefficient $r_{\text{eff}} = r_{33} = r_{333}$. If the incident light is extraordinarily polarized, the effective refractive index change is thus determined by

$$\Delta n^2 = -n_0^4 r_{\text{eff}} \mathbf{E} \cdot \mathbf{e}_c \quad (2.20)$$

where $\mathbf{e}_c$ is the unit vector along the direction of the c -axis.

2.2.2 The Band Transport Model

According to (2.20), the light induced refractive index change depends on the effective electro-optic coefficient r_{eff} and the electric field $\mathbf{E}$. Therefore, it is crucial to know how the space charge field generated by the incident light looks like. This can be achieved by a simple band transport model which has been developed by Kukhtarev et al. in 1979 to describe the dynamics of the charge carriers inside the crystal [4].

In the following derivation, a photorefractive material of electron-conductivity type is considered and the density of holes is neglected. Similar equations can be derived for materials of the hole-conducting type quite easily, then neglecting the density of electrons.

¹ It is assumed that the c -axis of the crystal is oriented along the z -direction of the chosen coordinate system and the space charge field is parallel to this direction. This is the typical choice for the description of electro optic effects. Therefore, it has been adopted here although it is not the common situation throughout the rest of thesis, where propagation along the z -direction is considered. In this case, the crystal is rotated by 90° and the c -axis points in the x -direction.

First, the effective generation rate of electrons is given by the rate of generation of ionized donors minus the rate of recapture of electrons

$$G = (s\tilde{I} + \beta)(N_D - N_D^+) - \alpha_R N_D^+ n_e = \beta(\tilde{I}/I_{\text{sat}} + 1)(N_D - N_D^+) - \alpha_R N_D^+ n_e \quad (2.21)$$

where s is proportional to the probability that an incoming photon lifts up an electron into the conduction band (photo excitation) and β describes the excitation of electrons in the absence of light (thermal excitation). N_D , N_D^+ and n_e are the density of donors, ionized donors and free electrons, respectively. Finally, α_R represents the probability for an electron in the conduction band to recombine with an ionized donor.

The saturation intensity I_{sat} determines the ratio between the thermal and the photoionization coefficient which is expressed by a characteristic intensity. This notation is useful because the nonlinear response of a photorefractive material is not only determined by the optical beam intensity $\tilde{I}$, but also by an intensity-equivalent from thermal excitation. Moreover, an artificial contribution to the thermal excitation created by an incoherent background illumination is usually used in experiments to control the behavior of the photorefractive crystal.

The electrons in the conduction band are influenced by three different transport processes: drift in an electric field, diffusion, and the photovoltaic effect. Hence, the resulting current density is given by

$$\begin{aligned} \mathbf{j} &= \mathbf{j}_{\text{drift}} + \mathbf{j}_{\text{diff}} + \mathbf{j}_{\text{pv}} \\ &= e\mu n_e \mathbf{E} + eD\nabla n_e + \beta_{\text{ph}}(N_D - N_D^+) \mathbf{e}_c \tilde{I}. \end{aligned} \quad (2.22)$$

Here, $D = \mu k_B T / e$ is the diffusion constant containing the Boltzmann constant k_B , the absolute temperature T , the elementary charge e and the electron mobility μ . The strength of the photovoltaic effect is given by the photovoltaic tensor β_{ph} which has its largest component along the direction of the c -axis, whereas all the other components can be neglected. Finally, $\mathbf{e}_c$ gives the unit vector along the c -axis.

Assuming that all acceptors N_A are ionized, the space charge density can be written as

$$\rho = e(N_D^+ - N_A - n_e) \quad (2.23)$$

and Gauß's law [cf. (2.1) and (2.9)] for the present situation becomes

$$\nabla(\epsilon_0 e \mathbf{E}) = \rho = e(N_D^+ - N_A - n_e). \quad (2.24)$$

Since only electrons are considered here as mobile charge carriers and the ionized donor or acceptor atoms are assumed to be stationary within the crystal lattice, (2.23) transforms into the following continuity equation:

$$\frac{\partial \rho}{\partial t} = e \left(\frac{\partial N_D^+}{\partial t} - \frac{\partial n_e}{\partial t} \right) = -\nabla \mathbf{j}. \quad (2.25)$$

Furthermore, with this assumption, the change of the density of ionized donors is given by the generation of electrons

$$\frac{\partial N_D^+}{\partial t} = G = \beta(I + 1)(N_D - N_D^+) - \alpha_R N_D^+ n_e \quad (2.26)$$

where I denotes the normalized intensity $I = \tilde{I}/I_{\text{sat}}$.

The equations (2.22)–(2.26) form the fundamental set of equations of the Kukhtarev model. To get a simplified expression for the induced space charge field, several approximations that are well justified in most experimental situations can be made. First, a quasi-homogeneous illumination of the crystal is assumed and the system is regarded to be in a steady state ($\partial_t = 0$). Furthermore, the photovoltaic term in (2.22) is neglected.

Based on these assumptions, two major approaches exist which will be described in the following sections. The first one was introduced by Christodoulides and Carvalho [5] and is denoted as *isotropic approximation*. The second one was proposed by Zozulya and Anderson [6] and is referred to as *anisotropic model*.

2.2.3 Isotropic Approximation

In photorefractive materials, the density of impurity ions is typically much larger than the electron density, i.e.

$$N_D^+, N_A \gg n_e. \quad (2.27)$$

Using this relation, the density of ionized donor impurities and the free electron density can be derived from (2.24) and (2.26), respectively:

$$N_D^+ = N_A \left(1 + \frac{\epsilon \epsilon_0}{e N_A} \nabla E \right) \quad (2.28)$$

$$n_e = \frac{\beta(N_D - N_D^+)}{\alpha_R N_D^+} (1 + I). \quad (2.29)$$

Assuming that the intensity I asymptotically attains a constant value I_∞ at the crystal border and the space charge field in this region is also independent of the transverse coordinates, i.e. $|\mathbf{E}(x \rightarrow \pm\infty)| = |\mathbf{E}_0|$, the free electron density $n_e^{(0)}$ at the border can be derived from (2.29):

$$n_e^{(0)} = \frac{\beta(N_D - N_A)}{\alpha_R N_A} (1 + I_\infty). \quad (2.30)$$

Since the system is assumed to be in a steady state, the current density (2.22) has to be constant everywhere, i.e. $n_e^{(0)} \mathbf{E}_0 = n_e \mathbf{E} + D \mu^{-1} \nabla n_e$ or equivalently

$$\mathbf{E} = \frac{n_e^{(0)} \mathbf{E}_0}{n_e} - \frac{D}{\mu n_e} \nabla n_e. \quad (2.31)$$

Considering drift-dominated charge carrier migration, the diffusion term can be neglected. Further on, assuming that the beam intensity varies slowly with respect to the spatial coordinates ($\nabla E_{sc} \approx 0$) and setting $I_\infty = 0$, the combination of (2.29), (2.30) and (2.31) results in

$$\mathbf{E} = \mathbf{E}_0 \frac{1}{1 + I}. \quad (2.32)$$

It is striking that the electric field in (2.32) does not depend on any spatial coordinate. This is why the model is denoted as *isotropic approximation*. It is quite appropriate for all one-dimensional configurations and the study of saturable nonlinear media in general, but it does not take into account the specific anisotropic properties of biased photorefractive media. To get a more realistic model of the physical process, it is therefore necessary to use the anisotropic model which will be described in the next section.

2.2.4 Anisotropic Model

Applying an external electric field in one transverse dimension inherently causes a symmetry breaking and destroys the isotropic situation. As a consequence, the isotropic approximation is no longer applicable and a model which includes the distinct anisotropic properties of biased photorefractive crystals has to be used. Such a model was first introduced by Zozulya and Anderson [6] in 1994.

The basic idea is to express the electric field in terms of its electrostatic potential:

$$\mathbf{E} = -\nabla \tilde{\phi}. \quad (2.33)$$

Considering an external field $\mathbf{E}_0$ applied across the transverse x -direction, the potential $\tilde{\phi}$ consists of a light induced term ϕ and an external bias term $-|\mathbf{E}_0|x$, thus $\tilde{\phi} = \phi - |\mathbf{E}_0|x$. As before, it is assumed that the intensity varies slowly with respect to the spatial coordinates. With this assumption, (2.29) transforms into

$$n_e = \frac{\beta(N_D - N_A)}{z'N_A} (1 + I). \quad (2.34)$$

Inserting (2.34) into (2.22) and neglecting the photovoltaic term for a steady state system ($\nabla \mathbf{j} = 0$) then yields

$$\begin{aligned} \nabla^2 \phi + \nabla \ln(1 + I) \nabla \phi = |\mathbf{E}_0| \frac{\partial \ln(1 + I)}{\partial x} \\ - D \left[\nabla^2 \ln(1 + I) + (\nabla \ln(1 + I))^2 \right]. \end{aligned} \quad (2.35)$$

For the case of two transverse dimensions, this equation has no analytical solutions and thus has to be solved numerically.² The resulting potential can then be used to obtain the total electric field $\mathbf{E} = \mathbf{E}_0 - \nabla\phi$ and the refractive index change according to (2.20). However, when solving the propagation equation (2.16), the refractive index change $\Delta n^2(I)$ is often determined directly from the light-induced space charge field $\mathbf{E}_{sc} = -\nabla\phi$. This can be done because the contribution of $\mathbf{E}_0$ only gives an additional phase factor and neglecting this term is therefore equivalent to rescaling n_0 .

In the transversally one-dimensional limit (all y-derivatives equal to zero), an analytical solution of (2.35) can be found:

$$\mathbf{E}_{sc} = \left(-|\mathbf{E}_0| \frac{I}{1+I} - \frac{D}{1+I} \frac{\partial I}{\partial x} \right) \cdot \mathbf{e}_x. \quad (2.36)$$

In fact, this one-dimensional solution is often taken as an approximation in the two-dimensional case. Neglecting the diffusion term ($D = 0$) then results in the same expression for the total electric field $\mathbf{E} = \mathbf{E}_{sc} + \mathbf{E}_0$ as obtained from the isotropic approximation presented in the previous section [cf. (2.32)]. Such an isotropic approach can obviously not describe the anisotropic features of the photorefractive nonlinearity which have already been observed in several experiments [7–9]. In the following, we will therefore use the anisotropic model (2.35) and again neglect the diffusion term such that

$$\nabla^2\phi + \nabla \ln(1+I) \nabla\phi = |\mathbf{E}_0| \frac{\partial \ln(1+I)}{\partial x}. \quad (2.37)$$

The propagation equation (2.16) is finally written as

$$2i \frac{\partial A}{\partial z} + \nabla_{\perp}^2 A - \Gamma E_{sc} (|A|^2) A = 0 \quad (2.38)$$

where we have introduced the coupling constant $\Gamma = k_z^2 x_0^2 n_0^2 r_{\text{eff}}$. The space charge field $\mathbf{E}_{sc}$ is treated as a scalar quantity, since only the component parallel to the c-axis contributes to the refractive index change.

2.3 Periodic Photonic Structures

In a periodic photonic structure, the refractive index n is periodically modulated along one or more spatial dimensions. The simplest examples of such structures are one-dimensional Bragg gratings which have been known for more than 100 years now and are widely used as filters that reflect optical waves incident at

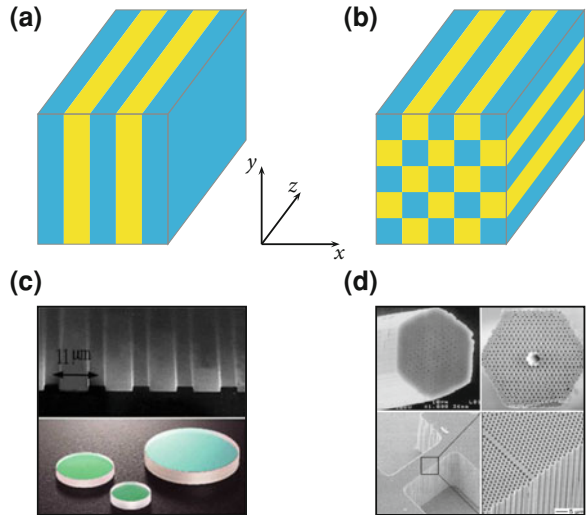
² A suitable algorithm is described in appendix A.

certain angles or waves at certain frequencies [10]. Following the pioneering works of Yablonovitch [11] and John [12], dielectric structures with a spatially modulated refractive index in one, two or even three dimensions are nowadays commonly denoted as photonic crystals. This notation is due to the existence of photonic band gaps (see below), representing an optical analog of electronic band gaps in crystalline solids (e.g. semiconductors).

Throughout the rest of this work, we consider transversally periodic structures being homogeneous in the direction of propagation and the coordinate system is chosen such that the light propagates in z -direction (Fig. 2.2). Typical examples for the one-dimensional case include dielectric mirrors and arrays of optical waveguides [13], whereas two-dimensional structures are commonly represented by photonic crystal fibers [14, 15] and planar photonic crystals [16] as shown in Fig. 2.2c, d.

Currently, there exists a number of different approaches for the fabrication of photonic crystals in one or more spatial dimensions, e.g. focused ion beam milling, e-beam lithography combined with reactive ion etching, or two-photon polymerization. However, all these techniques are resource demanding and cost ineffective and thus not well suited for our fundamental investigations relying on flexible modification of structural parameters. Furthermore, the access to nonlinear effects is often hampered by high power requirements. In Chap. 3, we will demonstrate how optically induced photonic lattices can be employed to overcome these limitations, providing highly reconfigurable photonic structures with a strong nonlinear response at very low power levels.

Fig. 2.2 Schematic illustration (*top*) and real fabricated examples (*bottom*) of transversally periodic photonic structures being homogeneous in propagation direction. (a), (c) One-dimensional case; (b), (d) two-dimensional case; pictures in (c) and (d) taken from [14–17]



2.3.1 Band Gap Spectrum

To introduce the basic concepts of wave propagation in transversally periodic structures, we first assume a static, prefabricated refractive index modulation. The special case of optically induced photonic lattices will be discussed in [Chap. 3](#).

Including a transversally periodic, static refractive index modulation Δn_{pot}^2 results in an additive potential term in the propagation equation (2.38) which then reads as

$$2i \frac{\partial A}{\partial z} + \nabla_{\perp}^2 A + V(x, y)A - \Gamma E_{\text{sc}}(|A|^2)A = 0 \quad (2.39)$$

with $V(x, y) = k_z^2 x_0^2 n_0^{-2} \Delta n_{\text{pot}}^2$. It is important to note that this equation shows strong formal analogies to the description of matter waves in optical lattices. The reason for this is that by choosing $E_{\text{sc}}(|A|^2) = \pm |A|^2$ (Kerr-type nonlinearity) and substituting $z \rightarrow t$, (2.39) transforms into a two-dimensional form of the Gross-Pitaevskii equation describing a Bose-Einstein condensate in a two-dimensional optical lattice [18].

Furthermore, the linearized version of (2.39) (i.e. $\Gamma = 0$) is the standard equation for the description of electrons in periodic electronic potentials in solid state physics.

Inserting the ansatz $A(\mathbf{r}) = \tilde{A}(\mathbf{r}_{\perp}, \mathbf{k}_{\perp}) \cdot e^{i\beta z}$ with $\tilde{A}(\mathbf{r}_{\perp}, \mathbf{k}_{\perp}) = a(\mathbf{r}_{\perp}, \mathbf{k}_{\perp}) \cdot e^{i\mathbf{k}_{\perp} \cdot \mathbf{r}_{\perp}}$, this linearized equation transforms into the eigenvalue problem

$$\frac{1}{2} \left[(\nabla_{\perp} + i\mathbf{k}_{\perp})^2 + V(\mathbf{r}_{\perp}) \right] a(\mathbf{r}_{\perp}, \mathbf{k}_{\perp}) = \beta(\mathbf{k}_{\perp}) a(\mathbf{r}_{\perp}, \mathbf{k}_{\perp}) \quad (2.40)$$

with $\mathbf{r}_{\perp} = (x, y)$ and $\mathbf{k}_{\perp} = (k_x, k_y)$. β is the propagation constant which can be seen as an offset to the wave vector component k_z [cf. (2.14)].

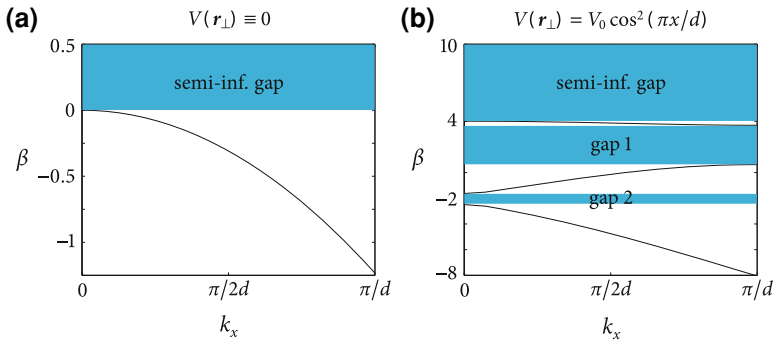


Fig. 2.3 Calculated linear dispersion relations. **a** Bulk homogeneous medium with $V(\mathbf{r}_{\perp}) \equiv 0$; **b** one-dimensional lattice with $V(\mathbf{r}_{\perp}) = V_0 \cdot \cos^2(\pi x/d)$, $V_0 = 0.0009$, $d = 2x_0$

In homogeneous bulk media ($V(\mathbf{r}_\perp) \equiv 0$), the eigenmode solutions of the system are plane waves $\tilde{A}(\mathbf{r}) = a_0 \cdot e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}$ with $a_0 = \text{const.}$ and we obtain the dispersion relation $\beta(\mathbf{k}_\perp) = -(k_x^2 + k_y^2)/2$. This parabolic relation is plotted in Fig. 2.3a. It shows a semi-infinite gap of forbidden propagation constants which may be seen as an optical analog of the forbidden energy regime below the ground state of an electron in a solid state. Note that the absolute value of β is only determined up to an arbitrary additional constant, since the total wave vector component in z -direction is given by $k_z + \beta$ and there is no fixed rule for the choice of k_z in the initial ansatz (2.14). Throughout this thesis, we adopt a common choice in which the semi-infinite gap reaches from 0 to $+\infty$ for $V_0 \rightarrow 0$.

In the presence of a periodic modulation $V(\mathbf{r}_\perp)$, the dispersion gets strongly modified due to multiple Bragg-reflections within the structure. As a result, the dispersion curve $\beta(\mathbf{k}_\perp)$ is divided into several transmission bands separated by forbidden gaps of finite size. The eigenmode solutions to the linearized version of (2.39) are in this case given by the well-known Bloch waves

$$\tilde{A}_m(\mathbf{r}_\perp, \mathbf{k}_\perp) = a_m(\mathbf{r}_\perp, \mathbf{k}_\perp) \cdot e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \quad (2.41)$$

where the functions $a_m(\mathbf{r}_\perp, \mathbf{k}_\perp)$ have the periodicity of the underlying lattice and the index $m = 1, 2, \dots$ denotes the band number. If we are not referring to a specific band, we will drop this index for convenience in the following. Figure 2.3b shows the resulting dispersion relation for a one-dimensional modulation $V(x, y) = V_0 \cdot \cos^2(\pi x/d)$ with effective modulation depth V_0 and lattice constant d . Note that in general the dispersion relation shows the translation invariance $k_{x,y} \rightarrow k_{x,y} \pm 2\pi/d$. Therefore, it is fully defined by its values in the first Brillouin zone $k_{x,y} \in [-\pi/d, \pi/d]$. Moreover, the first Brillouin zone itself is often redundant since periodic structures often possess additional symmetries. By eliminating these redundant regions, one obtains the so-called irreducible Brillouin zone. For the one-dimensional structures represented in Fig. 2.3, this irreducible Brillouin zone is given by $k_x \in [0, \pi/d]$.

2.3.2 Linear Propagation

The dispersion curve $\beta(\mathbf{k}_\perp)$ fully describes the linear propagation of light through a given photonic structure. Each point along the curve belongs to an extended Bloch wave with its own propagation constant β and propagation direction defined by the normal to the dispersion curve at the particular point. Any wave or wavepacket entering the lattice is decomposed into these Bloch waves which acquire different phases as they propagate. Such an accumulation of different phases during propagation certainly affects the waveform and results in an output that may considerably differ from the input waveform.

As an example, we consider the diffraction of a spatially finite beam covering a certain spectral range $[\Delta k_x, \Delta \beta]$. The diffractive properties of such a wavepacket

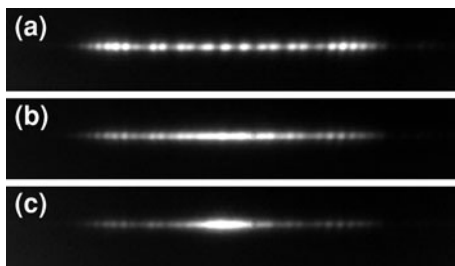
are governed by the relative spread or convergence of adjacent waves which can be determined from the curvature of the dispersion curve. In regions of convex curvature, the beam acquires a convex phasefront, resulting in normal diffraction with wave behavior similar to that in homogeneous media. In contrast, exciting a group of waves in a region of concave band curvature leads to anomalous diffraction, i.e. the input beam acquires a concave wavefront during propagation. For the dispersion relation shown in Fig. 2.3b, this means that beams centered at the top of the first band and propagating along the lattice will experience normal diffraction ($\partial^2 \beta / \partial k_x^2 < 0$) while beams corresponding to the bottom of the first band will experience anomalous diffraction ($\partial^2 \beta / \partial k_x^2 > 0$). Between the two regions of normal and anomalous diffraction, a region of zero curvature exists where $\partial^2 \beta / \partial k_x^2 = 0$. Waves belonging to this zone will therefore experience no linear diffraction broadening. As a result, the diffraction properties can be controlled by changing the incident angle. This feature has been denoted as *diffraction management* in analogy to dispersion management of optical pulses in fibers [19].

In addition, the effect of the lattice on the wave propagation also depends on the transverse size of the input beam relative to the lattice spacing. For example, a broad Gaussian beam launched at zero angle into the lattice excites modes from different bands and stays mostly Gaussian as it propagates. In contrast, coupling a narrow beam into the fundamental guided mode of a single period of the lattice excites Bloch modes primarily from the first band. In this case, due to coupling between lattice sites and multiple interference effects, the beam undergoes *discrete diffraction* characterized by intense side lobes with little or no light in the central region [20, 21]. The corresponding intensity distribution is shown in Fig. 2.4a for propagation in an AlGaAs waveguide array [20].

2.3.3 Nonlinear Propagation

As demonstrated in Sect. 2.1, the refractive index and thus the wave vector of light propagating in a nonlinear medium depends on the intensity. Therefore, in a nonlinear periodic structure, the wave vector component in propagation direction can be shifted by the nonlinearity such that, compared to the linear regime, it can

Fig. 2.4 First experimental observation of a discrete soliton in an AlGaAs waveguide array (reprinted from [20]). **a** Discrete diffraction at low power; **b** partial self-trapping at intermediate power; **c** formation of a discrete soliton at high power



be located in a gap and nonlinear propagation inside the linear gaps becomes possible. In particular, these nonlinear gap waves can be localized and form a *discrete or gap soliton* if diffraction is compensated by nonlinearity. In 1998, Eisenberg et al. [20] realized the first experimental observation of optical discrete solitons by focusing femtosecond laser pulses into a one-dimensional AlGaAs waveguide array. While at low peak power, the already discussed discrete diffraction pattern was observed (Fig. 2.4a), increasing the peak powers led to a narrowing of the transverse intensity distribution and finally the formation of a discrete soliton as shown in Fig. 2.4b, c, respectively.

Similar to the case of linear propagation, the influence of nonlinearity again depends on the curvature of the dispersion relation. In regions of normal diffraction, the resulting convex curvature of the phase front can be compensated by a focusing nonlinearity ($E_{sc} > 0$) while regions of anomalous diffraction require a defocusing nonlinearity ($E_{sc} < 0$). Consequently, such a dependence enables manipulation of the nonlinear dynamics by engineering the band structure in much the same way as linear diffraction management [22].

Gap solitons can be considered as self-guided modes of nonlinearity induced positive or negative defects in the periodic structure (Fig. 2.5a, b). Such defects naturally have localized modes (rather than the extended Bloch modes of the ideal lattice), whose propagation constant now lies off the linear transmission band, i.e. in a gap. For a focusing nonlinearity, the locally increased refractive index and thus the increased propagation constant causes a positive defect as sketched in Fig. 2.5a. The negative defect resulting from a defocusing nonlinearity is illustrated in Fig. 2.5b.

Depending on the gap, the self guiding property of the gap soliton modes has two different origins. On the one hand, it can arise from the locally increased refractive index in the presence of a focusing nonlinearity, similar to homogeneous

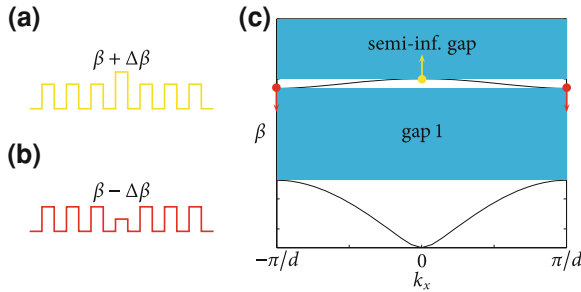


Fig. 2.5 Schematic illustration of discrete and gap soliton formation originating from the first band in a one-dimensional periodic photonic structure. **a** Positive defect caused by the locally increased refractive index in case of a focusing nonlinearity; **b** negative defect for a defocusing nonlinearity; **c** calculated dispersion relation with arrows indicating the nonlinear shift of the propagation constant: (yellow) positive shift, formation of a discrete soliton; (red) negative shift, formation of a gap soliton in the presence of a defocusing nonlinearity

media. This is the case for solitons in the semi-infinite gap which are commonly named *discrete solitons*. In this case, the propagation constant originating from the top of the first band is shifted up into the gap (yellow arrow in Fig. 2.5c). On the other hand, multiple transverse Bragg reflections may also enable self-guided modes. This is the dominant mechanism in all finite gaps and the corresponding solitons are often denoted as *gap solitons*. Unfortunately, the use of these terms is not consistent in literature. However, throughout the rest of this thesis, we will keep the most common choice of notation and refer to the solitons in the semi-infinite gap as discrete solitons while using the term gap solitons for localization in all finite gaps.

To give an example of a gap soliton, we consider Bloch waves from the first band at the edge of the Brillouin zone of a one-dimensional periodic structure at $k_x = \pm\pi/d$. The corresponding soliton has a propagation constant lying between the first and the second band [23]. For modes originating from the first band, a defocusing nonlinearity is necessary for the soliton formation and the propagation constant goes down into the gap with increasing nonlinearity as indicated by the red arrows in Fig. 2.5c.

In the context of this thesis, the term soliton describes a spatial amplitude profile which does not change during propagation, i.e.

$$A(\mathbf{r}) = \psi(\mathbf{r}_\perp) \cdot e^{i\beta z}. \quad (2.42)$$

Substituting this expression into the propagation equation (2.39) results in the following equation for the soliton profile $\psi(\mathbf{r}_\perp)$ in a static, prefabricated photonic structure:

$$-2\beta\psi + \nabla_\perp^2\psi + V(x, y)\psi - \Gamma E_{sc}(|\psi|^2)\psi = 0. \quad (2.43)$$

The corresponding equation for the case of optically induced photonic lattices will be given in Chap. 3. Due to the complex structure of the photorefractive nonlinearity, analytical solutions to (2.43) do not exist and thus it has to be solved numerically. A suitable algorithm to achieve this is described in [24].

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