Chapter 4
Crack Propagation Study Using Double-*K* and Double-*G* Fracture Parameters

4.1 Introduction

This chapter is based on the works of Kumar and Barai (2008a–c, 2009a–f, 2010a, b) and Kumar (2010). In this chapter the application of universal form of weight function is introduced for determining the double-*K* fracture parameters using various test geometries. The definition of brittleness of concrete is mentioned using the double-*K* fracture parameters. An empirical formula for determining directly the effective crack length for CT specimen is highlighted. Furthermore, the size-effect prediction from the double-*K* fracture model is investigated. Finally, comprehensive numerical studies on the double-*K* and double-*G* fracture criteria are represented.

4.2 Weight Function Method

For a linear elastic cracked body subjected to any symmetrical mode I loading, it was proved (Bueckner 1970, Rice 1972) that if the SIF and the corresponding crack face displacement are known as functions of crack length, then the SIF for any other symmetrical loading acting on the same body can be directly determined. Assume that the K_r and K_s are the SIFs of a cracked body corresponding to two cases of mode I symmetrical loadings r and s respectively, in which r corresponds to the known reference value and s corresponds to any arbitrary loading case. According to weight function method value of K_s may directly be determined for the known value of K_r and the respective crack face displacement u_r as given below:

$$K_\mathrm{S} = \int_0^a \sigma_\mathrm{S}(x).m(x,a)\mathrm{d}x_\mathrm{S} \tag{4.1}$$

The term $m(x,a)$ in Eq. (4.1) is known as weight function and expressed as

$$m(x,a) = \frac{E^{'}}{2K_\mathrm{r}}\frac{\partial u_r}{\partial a} \tag{4.2}$$

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$$K_{rk} = \int_0^a \sigma_{rk}(x) \frac{2}{\sqrt{2\pi(a-x)}} \begin{bmatrix} 1 + M_1(1-x/a)^{1/2} + M_2(1-x/a) \\ + M_3(1-x/a)^{3/2} \end{bmatrix} \mathrm{d}x \tag{4.4}$$

As an approximation, an additional condition for central through crack and double-edge cracks under symmetrical loading was used (Shen and Glinka 1991) as given below:

$$\frac{\partial u(x,a)}{\partial x}\Big|_{x=0} = 0 \tag{4.5}$$

From Eqs. (4.2) and (4.5), it can be shown that

$$\frac{\partial m(x,a)}{\partial x}\Big|_{x=0} = 0 \tag{4.6}$$

For the single-edge cracks, the following condition is found to be more appropriate (Fett et al. 1987, Shen and Glinka 1991):

$$\frac{\partial^2 u(x,a)}{\partial x^2}\Big|_{x=0} = 0 \tag{4.7}$$

The following relation may be obtained using Eqs. (4.2) and (4.7):

$$\frac{\partial^2 m(x,a)}{\partial x^2}\Big|_{x=0} = 0 \tag{4.8}$$

After satisfying Eqs. (4.6) and (4.8), one may get an additional condition which reduces the number of known reference SIFs.

4.3 Determination of Universal Weight Function for Edge Cracks in Finite Width Plate

In the present work, first of all an attempt is made to derive the universal weight function using the reference SIFs for the edge crack in a finite width of plate. The two reference SIFs for two different loadings as shown in Figs. 4.2 and 4.3 along with Eq. (4.8) are used to obtain the desired weight function. For the first reference case (Fig. 4.2), an edge crack in a finite width of plate subjected to uniform stress σ_o is taken. The standard equation of SIF for this case is written as

$$K_I = F_1 \sigma_o \sqrt{\pi a} \tag{4.9}$$

in which

$$F_1 = 1.122 - 0.231a/D + 10.55(a/D)^2 - 21.71(a/D)^3 + 30.382(a/D)^4 \tag{4.10}$$

$$\frac{\pi F_1}{\sqrt{2}} = 2 + M_1 + 2M_2/3 + M_3/2 \tag{4.15}$$

Using Eqs. (4.4), (4.13), and (4.14), following equation can be obtained as

$$\sqrt{2}F_2 = 1 + M_1 + M_2 + M_3 \tag{4.16}$$

Equations (4.4) and (4.8) yield

$$M_2 = 3 \tag{4.17}$$

After solving Eqs. (4.15), (4.16), and (4.17), the following expressions for parameters of weight function are obtained:

$$\begin{aligned} M_2 &= 3 \\ M_3 &= 2\sqrt{2}F_2 - 2\pi F_1 \\ M_1 &= \sqrt{2}F_2 - 1 - M_2 - M_3 \end{aligned} \tag{4.18}$$

Thus, the three parameters of weight function for the edge cracks in a finite width of plate are expressed by expression (4.18) and then the four-term universal weight function can be characterized by the following expression.

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\begin{bmatrix} 1 + M_1(1-x/a)^{1/2} + M_2(1-x/a) \\ +M_3(1-x/a)^{3/2} \end{bmatrix} \tag{4.19}$$

The nondimensional forms of Tada Green's function and the derived equivalent weight function having coefficients M_1, M_2, and M_3 determined using Eq. (4.19) are compared. It is found that the maximum absolute error up to $a/D = 0.2$ is less than 10% and this error increases for deeper cracks.

From the above comparison it is noticed that the standard procedures to find out the parameters of the universal weight function for the case of edge cracks in finite width of plate is not suitable. One of the obvious reasons may be attributed to the geometrical factor expressed in Eq. (4.10) which is suitable for $a/D \leq 0.6$. Therefore, an alternative procedure is to be sought to get the correct values of the parameters of the universal weight function for the assumed cracked geometry. As a result, it is proposed to fit the Tada Green's function using least square technique for determining the different parameters of the universal weight function.

4.3.1 Four-Term Universal Weight Function

The form of four-term universal weight function is expressed in Eq. (4.19). In this method, the values of the Tada Green's function at different values of *a/D* (ranging $0 \leq a/D < 1$) and *x/D* (ranging $0 \leq x/D < 1$) are computed. Then the values of three parameters M_1, M_2, and M_3 are determined using least square technique. From the least square fitting it was found that at every point of *a/D* ratio (for values

Tada H, Paris PC, Irwin G (1985) The Stress Analysis of Cracks Handbook. Hellertown, PA, Del Research Corporation.
Wittmann FH, Rokugo K, Bruhwiller E, Mihashi H, Simopnin P (1988) Fracture energy and strain softening of concrete as determined by compact tension specimens. Mater Struct 21(1): 21–32.
Xu S (1999) Determination of parameters in the bilinear, Reinhardt's non-linear and exponentially non-linear softening curves and their physical meanings. Werkstoffe und Werkstoffprüfung im Bauwesen, Hamburg, Libri BOD: 410–424.
Xu S, Reinhardt HW (1998) Crack extension resistance and fracture properties of quasi-brittle materials like concrete based on the complete process of fracture. Int J Fract 92: 71–99.
Xu S, Reinhardt HW (1999a) Determination of double-K criterion for crack propagation in quasi-brittle materials, Part I: Experimental investigation of crack propagation. Int J Fract 98: 111–149.
Xu S, Reinhardt HW (1999b) Determination of double-K criterion for crack propagation in quasi-brittle materials, Part II: Analytical evaluating and practical measuring methods for three-point bending notched beams. Int J Fract 98: 151–177.
Xu S, Reinhardt HW (1999c) Determination of double-K criterion for crack propagation in quasi-brittle materials, Part III: Compact tension specimens and wedge splitting specimens. Int J Fract 98: 179–193.
Xu S, Reinhardt HW (2000) A simplified method for determining double-K fracture meter parameters for three-point bending tests. Int J Fract 104: 181–209.
Xu S, Zhang X (2008) Determination of fracture parameters for crack propagation in concrete using an energy approach. Eng Fract Mech 75: 4292–4308.
Xu S, Zhao Y, Wu Z (2006) Study on the average fracture energy for crack propagation in concrete. J Mater Civil Eng ASCE 18(6): 817–824.
Zhao G, Jiao H, Xu S (1991) Study on fracture behavior with wedge splitting test method. Fracture Processes in Concrete, Rock and Ceramics (Edited by van Mier et al.), London, E & F.N. Spon: 789–798.
Zhu X (1997) Transport organischer Flüssigkeiten in Betonbauteilen mit Mikro- und Biegerissen. Berlin, Deutscher Ausschuss für Stahlbeton, Heft 475, Beuth Verlag GmbH: S.1–104.
Zi G, Bažant ZP (2003) Eignvalue method for computing size effect of cohesive cracks with residual stress, with application to kink-bands in composites. Int J Eng Sci 41: 1519–1534.
Zi G, Belytschko T (2003) New crack-tip elements for X-FEM and applications to cohesive cracks. Int J Numer Methods Eng 57: 2221–2240.

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