Preface

In vector optimization one investigates optimal elements such as minimal, strongly minimal, properly minimal or weakly minimal elements of a nonempty subset of a partially ordered linear space. The problem of determining at least one of these optimal elements, if they exist at all, is also called a vector optimization problem. Problems of this type can be found not only in mathematics but also in engineering and economics. Vector optimization problems arise, for example, in functional analysis (the Hahn-Banach theorem, the Bishop-Phelps lemma, Ekeland's variational principle), multiobjective programming, multi-criteria decision making, statistics (Bayes solutions, theory of tests, minimal covariance matrices), approximation theory (location theory, simultaneous approximation, solution of boundary value problems) and cooperative game theory (cooperative n player differential games and, as a special case, optimal control problems). In the last two decades vector optimization has been extended to problems with set-valued maps. This new field of research, called set optimization, seems to have important applications to variational inequalities and optimization problems with multivalued data.

The roots of vector optimization go back to F.Y. Edgeworth (1881) and V. Pareto (1906) who have already given the definition of the standard optimality concept in multiobjective optimization. But in mathematics this branch of optimization has started with the legendary paper of H.W. Kuhn and A.W. Tucker (1951). Since about the end of the 1960's research is intensively made in vector optimization.

It is the aim of this book to present various basic and important results of vector optimization in a general mathematical setting and to demonstrate its usefulness in mathematics and engineering. An extension to set optimization is also given. The first three parts are a revised edition of the former book [160] of the author. The forth part on engineering applications and the fifth part entitled extensions to set optimization have been added.

The theoretical vector optimization results are contained in the second part of this book. For a better understanding of the proofs several theorems of convex analysis are recalled in the first part. This part concisely summarizes the necessary background material and may be viewed as an appendix.

The main part of this book begins on page 102 with a discussion of several optimality notions together with some simple relations. Necessary and sufficient conditions for optimal elements are obtained by scalarization, i.e. the original vector optimization problem is replaced by an optimization problem with a real-valued objective map. The scalarizing functionals being used are certain linear functionals and norms. Existence theorems for optimal elements are proved using Zorn's lemma and the scalarization theory. For vector optimization problems with inequality and equality constraints a generalized Lagrange multiplier rule is given. Moreover, a duality theory is developed for convex maps. These results are also specialized to abstract linear optimization problems. The third part of this book is devoted to the application of the preceding general theory. For vector approximation problems the connections to simultaneous approximation problems are shown and a generalized Kolmogorov condition is formulated. Furthermore, nonlinear and linear Chebyshev problems are considered in detail. The last section is entitled cooperative n player differential games. These include optimal control problems. For these games a maximum principle is proved.

In the part on engineering applications the developed theoretical results are applied to multiobjective optimization problems arising in engineering. After a presentation of the theoretical basics of multiobjective optimization numerical methods are discussed. Some of these methods are applied to concrete nonlinear multiobjective optimization problems from electrical engineering, computer science, chemical engineering and medical engineering. The last part extends the second part of this book to set optimization. After an introduction to this field of research including basic concepts the notion of the contingent epiderivative is discussed in detail. Subdifferentials are the topic together with a comprehensive chapter on optimality conditions in set optimization.

This book should be readable for students in mathematics whose background includes a basic knowledge in optimization and linear functional analysis. Mathematically oriented engineers may be interested in the forth part on engineering applications.

The bibliography contains only a selection of references. A reader who is interested in the first papers of vector optimization is requested to consult the extensive older bibliographies of Achilles-Elster-Nehse [1], Nehse [258] and Stadler [312].

This second edition is a revised version containing two new sections, additional remarks on the contribution of Edgeworth and Pareto and an updated bibliography.

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