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Perspectives on projective geometry. A guided tour through real and complex geometry. (English)

Berlin: Springer. xxii, 571 p. EUR 64.95/net; £ 58.99; SFR 93.50; $ 84.95 (2011).
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The author of this very well written and detailed book is an expert in projective geometry, especially in computational projective geometry, as the books [Geometries. (Geometriekalküle.) Springer-Lehrbuch. Berlin: Springer. (2009; Zbl 1181.00003), Die interaktive Geometrie-Software Cinderella. Version 1. 2. Einzelplatzversion. Berlin: Springer. CD-ROM, Handbuch x, (1999; Zbl 0969.51001)], and numerous scientific articles show. The reader of the present book on classical projective geometry is expected to be familiar with elementary linear algebra, apart from students this book is accessible to mathematicians as well as computer scientists and physicists. The author presents the rich interplay of geometric structures and their algebraic counterparts, thus following the philosophy of J. Plücker (1801-1868) and F. Klein (1849-1925). Definitions and theorems never stand alone, they are patiently explained and always discussed from the view of computer implementation. The emphasis is on structures in order to express the fundamental objects and operations in a most elegant way. The book points out the advantages of projective geometry with respect to Euclidean, hyperbolic, and elliptic geometry.

Each chapter of the book starts with a motto, for instance chapter 17 on the complex projective line is introduced as follows: *The shortest route between two truths in the real domain passes through the complex domain* (J. S. Hadamard (1865-1963)).

Many of the approximately 230 very aesthetical, computer generated, partially colored illustrations show that beside rigorous abstractions also visualizations can play a helpful role in the study of geometric objects.

Except for chapter 1, the book is divided into three parts.

Chapter 1. Pappos's theorem: nine proofs and three variations: among the nine are those via the Pascal theorem, the Cayley-Bacharach-Chasles theorem, and the Miquel theorem.

Part I Projective Geometry

Chapter 2. Projective planes: drawings and perspectives, the axioms, the smallest projective plane.

Chapter 3. Homogeneous coordinates: the real projective plane, joins and meets, parallelism, duality, projective transformations, basic facts on finite projective planes.

Chapter 4. Lines and cross-ratio: the real projective line, elementary properties of the cross-ratio.

Chapter 5. Calculating with points on lines: harmonic points, projective scales, from geometry to real numbers, the fundamental theorem, a note on other fields, von Staudt's original constructions, Pappos's theorem.

Chapter 6. Determinants: (this chapter demonstrates the importance of determinants and multihomogeneous bracket polynomials in expressing projectively invariant properties) the "determinantal" point of view, Plücker's μ, invariant properties, the Grassmann-Plücker relations.
Chapter 7. More on bracket algebra: from points to determinants and back, a glimpse of invariant theory, projectively invariant functions, the bracket algebra.

Part II Working and playing with Geometry

Chapter 8. Quadrilateral sets and liftings: symmetry and generalizations of quadrilateral sets, involutions and quadrilateral sets.
Chapter 9. Conics and their duals: equation of a conic, polars and tangents, dual quadratic forms, transformation of conics, degenerate conics, primal-dual pairs.
Chapter 10. Conics and perspectivity: conic through five points, conics and cross-ratio, perspective generation of a conic, transformations and conics, Hesse's "Übertragungsprinzip", the theorems of Pascal and Brianchon, harmonic points on a conic.
Chapter 11. Calculating with conics: splitting a degenerate conic, the necessity of "If" Operations, intersecting a conic and a line, intersecting two conics, the role of complex numbers, one tangent and four points.
Chapter 12. Projective d-space: elements at infinity, homogeneous coordinates and transformations, points, planes, and lines in 3-space, joins and meets (a universal system and how to use it).
Chapter 13. Diagram techniques: (shows that diagrams formed by geometric objects and ε-tensors form invariants under projective transformations) from points, lines, and matrices to tensors, tensor diagrams, how transformations work, the δ- and the ε-tensors, the ε-δ rule, transforming ε-tensors, invariants of line and point configurations.
Chapter 14. Working with diagrams: (gives more advanced applications of tensors and diagrams) a trace condition, Pascal's theorem, closed ε-cycles, conics quadratic forms and tangents, diagrams in the real projective 3-space, the ε-δ rule in rank 4, co- and contravariant lines in rank 4, tensors versus Plücker coordinates.
Chapter 15. Configurations, theorems, and bracket expressions: Desargues's theorem, binomial proofs, chains and cycles of cross-ratios, Ceva and Menelaus, gluing Ceva and Menelaus configurations,

Part III Measurements

Chapter 16. Complex numbers: a primer: historical background, fundamental theorem, geometry of complex numbers, Euler's formula, complex conjugation.
Chapter 17. The complex projective line: geometric properties, projective transformations, inversions and Möbius reflections, Grassmann-Plücker relations, intersection angles, stereographic projection.
Chapter 18. Euclidean geometry: the circular points of plane Euclidean geometry (Reviewer's remark: The mentioned name is avoided by the author probably in view of the later chapters about Cayley-Klein geometries.), cocircularity, the robustness of the cross-ratio, projective, affine, similarity, Euclidean transformations, perpendicularity, Laguerre's formula, distances.
Chapter 19. Euclidean structures from a projective perspective: (Euclidean geometry is projective geometry together with the pair of circular points) mirror images, angle bisectors, center of a circle, constructing the foci of a conic, constructing a conic by foci, triangle theorems, hybrid thinking (demonstrated with the nine-point circle).

Chapter 20. Cayley-Klein geometries: (concentrates on the planar case) interpretation of the circular points of plane Euclidean geometry as degenerate dual conic, distance and angle measurement, hyperbolic, elliptic, and parabolic measurements along a line, an investigation of distances and angles in the hyperbolic plane with 7 helpful and well described figures, the seven types of planar Cayley-Klein geometries, coarser and finer classifications.
Chapter 21. Measurements and transformations: (focusses on transformations, their projective invariants, and the behavior of measurements under these transformations) measurements versus oriented measurements, comparing measurements, reflections and pole/polar pairs, rotations.
Chapter 22. Cayley-Klein geometries at work: orthogonality, constructive versus implicit representations, commonalities and differences, midpoints and angle bisector, trigonometry.
Chapter 23. Circles and cycles: circles via distances, relation to the fundamental conic, centers at infinity, organizing principle (cycles as limiting cases, duality of circles, curves of constant curvature), cycles in Galilean geometry (=plane isotropic geometry).
Chapter 24. Non-Euclidean geometry: A historical interlude: the inner geometry of a space, Euclid's postulates, Gauss, Bolyai, and Lobachevsky, Beltrami and Klein, the Beltrami-Klein model, Poincaré.
Chapter 25. Hyperbolic geometry: hyperbolic transformations, angles and boundaries, the Poincaré disk, transformations of the complex projective line and the Poincaré disk, angles and distances in the Poincaré disk.
Chapter 26. Selected topics in hyperbolic geometry: circles and cycles in the Poincaré disk, area and angle defect, (hyperbolic) Thales and (hyperbolic) Pythagoras, constructing regular n-gons, symmetry groups (accompanied among others by a figure showing a pentagonal hyperbolic checkerboard).
Chapter 27. What we did not touch: (gives a brief overview of a loose selection of topics) algebraic projective geometry (cubics, Cayley-Bacharach-Chasles theorem, Bézout's theorem, special points, duality), projective geometry and discrete mathematics (arrangement of pseudolines), projective geometry and quantum theory, a dynamic projective geometry.

Rolf Riesinger (Wien)
Keywords: conic; bracket algebra; Grassmann-Plücker relations; Hesse's Übertragungsprinzip; diagram techniques; Cayley-Klein geometry
Classification :
∗51A05 General theory of linear incidence geometry
51A25 Algebraization (linear incidence geometry)
51M05 Euclidean geometries (general) and generalizations
51M10 Hyperbolic and elliptic geometries (general) and generalizations

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