Preface

This book deals with occurrences of patterns in permutations and, more generally, in words with repeated letters. An occurrence of a pattern in a word is a subsequence of the word whose letters appear in the same order of size as the letters in the pattern. As a simple example, an occurrence of the pattern 321 is simply a decreasing subsequence of length 3, such as the one formed by the letters 632 in the permutation 4631725.

The patterns we deal with have been studied sporadically, often implicitly, for over a century, but in the last 20 years or so, the area has grown dramatically, resulting in many hundreds of published papers and in the annual conference "Permutation Patterns" organized for the first time at the University of Otago in Dunedin, New Zealand, in 2003. The introduction of the area of (permutation) patterns is traditionally attributed to Donald Knuth and in particular to exercises on pages 242–243 in his first volume of "The Art of Computer Programming" [540] in 1968, while the first systematic study of pattern avoidance was done by Rodica Simion and Frank W. Schmidt [721] in 1985.

There are several survey papers on this subject [136, 183, 515, 512, 352, 749, 751, 805] and the entire Volume 9 (2) of the *Electronic Journal of Combinatorics* is devoted to it. Moreover, there are proceedings of the Permutation Patterns 2007 conference, edited by Steve Linton, Nik Ruškuc and Vincent Vatter, and published in the London Mathematical Society Lecture Note Series, Cambridge University Press (vol. 376). The book "A walk through combinatorics" [141] by Miklós Bóna contains material on permutation patterns, while the book "Combinatorics of permutations" [137] by the same author provides a comprehensive and accessible introduction to so-called *classical permutation patterns*. Finally, there is the book "Combinatorics of compositions and words" [461] by Silvia Heubach and Toufik Mansour directly related to the subject.

However, the area has grown far beyond the content of the books mentioned above. The notion of a "pattern" has been extended in many different ways, often bringing new connections to other disciplines. Even such an important and relatively well-studied class of patterns as "generalized patterns" (called "vincular patterns"

in this book) introduced by Eric Babson and Einar Steingrímsson in 2000, received almost no attention in the books by Bóna and was not considered for permutations in the book by Heubach and Mansour. One of the goals of this book is to introduce a new notation for vincular patterns, because it has been somewhat confusing so far.

The two main objectives of this book are the following:

1. **To provide a motivation** to study patterns by demonstrating as many links to other areas of research as possible — Chapters 2 and 3 are entirely dedicated to this, and many other parts of the book contain such motivating material. These links provide connections to different combinatorial structures appearing in the literature, but also references to computer science (*sorting*, *generating*, and *complexity issues*), statistical mechanics (*Partially Asymmetric Simple Exclusion Process*), and computational biology (*whole genome duplication-random loss model*).

2. **To be comprehensive** in mentioning existing publications, and in sketching new research directions and trends in the field. In particular, we hope to have gathered, in our bibliography, almost all published papers related to the area (the book contains more than 800 references). Of course, there is a price to pay for being comprehensive, while keeping the size of the book within reasonable limits — we give very few proofs and there are no exercises. However, references are given to all results mentioned, so that the interested reader should have no problems finding the details.

While the book mentions several original results from papers in preparation, a couple of important topics were not covered the way they deserve to be. These topics include, but are not limited to, the intensively studied theory of *pattern classes* (see Remark 6.1.65 for a collection of references on it) and *enumeration schemes* (see [662, 663, 781, 816]). Moreover, we do not discuss at all a couple of research directions, for example, the results on the Möbius function on posets defined by different notions of pattern containment (see [107, 122, 202, 513, 708, 752]). Except for that, the book is a comprehensive collection of up-to-date results on patterns, most of which will be accessible to a broad audience, from undergraduate students to active researchers in the area of patterns in words and permutations, or adjacent fields.

The book is organized as follows.

- In Chapter 1 we introduce the main classes of patterns of interest in this book (classical patterns, barred patterns, vincular patterns, bivincular patterns, and

partially ordered patterns) and also provide examples of typical problems on these patterns. Moreover, we list a bibliography related to each of the pattern classes.

- In Chapters 2 and 3 we provide motivation points to study patterns in words and permutations. They include links to theoretical computer science through several sorting devices, planar maps and relevant objects, Schubert varieties and Kazhdan-Lusztig polynomials, computational biology through the tandem duplication-random loss model, statistical mechanics through the Partially Asymmetric Simple Exclusion Process, the theory of partially ordered sets, classification of Mahonian statistics, bijective combinatorics through encoding combinatorial objects by pattern-restricted permutations, and more.

- In Chapter 4 we present the more than thirty year history of bijections between permutations avoiding any classical pattern trivially equivalent to 321 and any pattern trivially equivalent to 132. We discuss a recent classification of these bijections and a philosophical question on what is a "good" bijection from the point of view of bijective combinatorics. Additionally, this chapter provides a collection of approaches to deal with classical patterns, along with general theorems about Wilf-equivalence of certain classical patterns.

- Chapter 5 contains an overview of results on consecutive patterns, and various approaches to studying them. For example, in this chapter we discuss the symbolic method, the symmetric functions approach, and the cluster and chain methods.

- In Chapters 6 and 7 we collect known (mostly enumerative) results on various patterns other than consecutive ones.

- In Chapter 8 we discuss several topics without a common thread. These topics include simple permutations, pattern matching problems, Gray codes, packing patterns, a link to combinatorics on words, universal cycles, simsun permutations, and games on patterns in permutations.

- In Chapter 9 we present several extensions and generalizations of the study of patterns discussed in the previous chapters.

- Appendix A serves as an easy access to the definitions for most of the permutation/word statistics and number sequences appearing in this book, while in Appendix B we provide a couple of basic algebraic facts, including the notion of Chebyshev polynomials of the second kind.

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