# Preface

## Preface to the First Edition

Combinatorial mathematics has been pursued since time immemorial, and at a reasonable scientific level at least since Leonhard Euler (1707–1783). It rendered many services to both pure and applied mathematics. Then along came the prince of computer science with its many mathematical problems and needs – and it was combinatorics that best fitted the glass slipper held out. Moreover, it has been gradually more and more realized that combinatorics has all sorts of deep connections with "mainstream areas" of mathematics, such as algebra, geometry and probability. This is why combinatorics is now a part of the standard mathematics and computer science curriculum.

This book is as an introduction to *extremal combinatorics* – a field of combinatorial mathematics which has undergone a period of spectacular growth in recent decades. The word "extremal" comes from the nature of problems this field deals with: if a collection of finite objects (numbers, graphs, vectors, sets, etc.) satisfies certain restrictions, how large or how small can it be?

For example, how many people can we invite to a party where among each three people there are two who know each other and two who don't know each other? An easy Ramsey-type argument shows that at most five persons can attend such a party. Or, suppose we are given a finite set of nonzero integers, and are asked to mark an as large as possible subset of them under the restriction that the sum of any two marked integers cannot be marked. It turns out that (independent of what the given integers actually are!) we can always mark at least one-third of them.

Besides classical tools, like the pigeonhole principle, the inclusion-exclusion principle, the double counting argument, induction, Ramsey argument, etc., some recent weapons – the probabilistic method and the linear algebra method – have shown their surprising power in solving such problems. With a mere knowledge of the concepts of linear independence and discrete probability, completely unexpected connections can be made between algebra, probability, and combinatorics. These techniques have also found striking applications in other areas of discrete mathematics and, in particular, in the theory of computing.

Nowadays we have comprehensive monographs covering different parts of extremal combinatorics. These books provide an invaluable source for students and researchers in combinatorics. Still, I feel that, despite its great potential and surprising applications, this fascinating field is not so well known for students and researchers in computer science. One reason could be that, being comprehensive and in-depth, these monographs are somewhat too difficult to start with for the beginner. I have therefore tried to write a "guide tour" to this field – an introductory text which should

- be self-contained,
- be more or less up-to-date,
- present a wide spectrum of basic ideas of extremal combinatorics,
- show how these ideas work in the theory of computing, and
- be accessible to graduate and motivated undergraduate students in mathematics and computer science.

Even if not all of these goals were achieved, I hope that the book will at least give a first impression about the power of extremal combinatorics, the type of problems this field deals with, and what its methods could be good for. This should help students in computer science to become more familiar with combinatorial reasoning and so be encouraged to open one of these monographs for more advanced study.

Intended for use as an introductory course, the text is, therefore, far from being all-inclusive. Emphasis has been given to theorems with elegant and beautiful proofs: those which may be called the gems of the theory and may be relatively easy to grasp by non-specialists. Some of the selected arguments are possible candidates for *The Book*, in which, according to Paul Erdős, God collects the perfect mathematical proofs.* I hope that the reader will enjoy them despite the imperfections of the presentation.

A possible feature and main departure from traditional books in combinatorics is the choice of topics and results, influenced by the author's twenty years of research experience in the theory of computing. Another departure is the inclusion of combinatorial results that originally appeared in computer science literature. To some extent, this feature may also be interesting for students and researchers in combinatorics. In particular, some impressive applications of combinatorial methods in the theory of computing are discussed.

**Teaching.** The text is *self-contained.* It assumes a certain mathematical maturity but *no* special knowledge in combinatorics, linear algebra, prob-

* "You don't have to believe in God but, as a mathematician, you should believe in The Book." (Paul Erdős)

For the first approximation see M. Aigner and G.M. Ziegler, *Proofs from THE BOOK*. Second Edition, Springer, 2000.

ability theory, or in the theory of computing — a standard mathematical background at undergraduate level should be enough to enjoy the proofs. All necessary concepts are introduced and, with very few exceptions, all results are proved before they are used, even if they are indeed "well-known." Fortunately, the problems and results of combinatorics are usually quite easy to state and explain, even for the layman. Its accessibility is one of its many appealing aspects.

The book contains much more material than is necessary for getting acquainted with the field. I have split it into relatively short chapters, each devoted to a particular proof technique. I have tried to make the chapters almost *independent*, so that the reader can choose his/her own order to follow the book. The (linear) order, in which the chapters appear, is just an extension of a (partial) order, "core facts first, applications and recent developments later." Combinatorics is broad rather than deep, it appears in different (often unrelated) corners of mathematics and computer science, and it is about techniques rather than results – this is where the independence of chapters comes from.

Each chapter starts with results demonstrating the particular technique in the simplest (or most illustrative) way. The relative importance of the topics discussed in separate chapters is not reflected in their length – only the topics which appear for the first time in the book are dealt with in greater detail. To facilitate the understanding of the material, over 300 exercises of varying difficulty, together with hints to their solution, are included. This is a vital part of the book – many of the examples were chosen to complement the main narrative of the text. Some of the hints are quite detailed so that they actually sketch the entire solution; in these cases the reader should try to fill out all missing details.

**Acknowledgments.** I would like to thank everybody who was directly or indirectly involved in the process of writing this book. First of all, I am grateful to Alessandra Capretti, Anna Gál, Thomas Hofmeister, Daniel Kral, G. Murali Krishnan, Martin Mundhenk, Gurumurthi V. Ramanan, Martin Sauerhoff and P.R. Subramania for comments and corrections.

Although not always directly reflected in the text, numerous earlier discussions with Anna Gál, Pavel Pudlák, and Sasha Razborov on various combinatorial problems in computational complexity, as well as short communications with Noga Alon, Aart Blokhuis, Armin Haken, Johan Håstad, Zoltan Füredi, Hanno Lefmann, Ran Raz, Mike Sipser, Mario Szegedy, and Avi Wigderson, have broadened my understanding of things. I especially benefited from the comments of Aleksandar Pekec and Jaikumar Radhakrishnan after they tested parts of the draft version in their courses in the BRICS International Ph.D. school (University of Aarhus, Denmark) and Tata Institute (Bombay, India), and from valuable comments of László Babai on the part devoted to the linear algebra method.

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My deepest thanks to my wife, Daiva, and my daughter, Indrė, for being there.

Frankfurt/Vilnius March 2001 *Stasys Jukna*

## Preface to the Second Edition

This second edition has been extended with substantial new material, and has been revised and updated throughout. In particular, it offers three new chapters about expander graphs and eigenvalues, the polynomial method and error-correcting codes. Most of the remaining chapters also include new material such as the Kruskal–Katona theorem about shadows, the Lovász–Stein theorem about coverings, large cliques in dense graphs without induced 4-cycles, a new lower bounds argument for monotone formulas, Dvir's solution of finite field Kakeya's conjecture, Moser's algorithmic version of the Lovász Local Lemma, Schöning's algorithm for 3-SAT, the Szemerédi–Trotter theorem about the number of point-line incidences, applications of expander graphs in extremal number theory, and some other results. Also, some proofs are made shorter and new exercises are added. And, of course, all errors and typos observed by the readers in the first edition are corrected.

I received a lot of letters from many readers pointing to omissions, errors or typos as well as suggestions for alternative proofs – such an enthusiastic reception of the first edition came as a great surprise. The second edition gives me an opportunity to incorporate all the suggestions and corrections in a new version. I am therefore thankful to all who wrote me, and in particular to: S. Akbari, S. Bova, E. Dekel, T. van Erven, D. Gavinsky, Qi Ge, D. Gunderson, S. Hada, H. Hennings, T. Hofmeister, Chien-Chung Huang, J. Hünten, H. Klauck, W. Koolen-Wijkstra, D. Krämer, U. Leck, Ben Pak Ching Li, D. McLaury, T. Mielikäinen, G. Mota, G. Nyul, V. Petrovic, H. Prothmann, P. Rastas, A. Razen, C. J. Renteria, M. Scheel, N. Schmitt, D. Sieling, T. Tassa, A. Utturwar, J. Volec, F. Voloch, E. Weinreb, A. Windsor, R. de Wolf, Qiqi Yan, A. Zilberstein, and P. Zumstein.

I thank everyone whose input has made a difference for this new edition. I am especially thankful to Thomas Hofmeister, Detlef Sieling and Ronald

de Wolf who supplied me with the reaction of their students. The "error-probability" in the 2nd edition was reduced by Ronald de Wolf and Philipp Zumstein who gave me a lot of corrections for the new stuff included in this edition. I am especially thankful to Ronald for many discussions—his help was extremely useful during the whole preparation of this edition. All remaining errors are entirely my fault.

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