

Contents

Notation	xiii
----------------	------

Part I The Classics

1 Counting	3
1.1 The binomial theorem	3
1.2 Selection with repetitions	6
1.3 Partitions	7
1.4 Double counting	8
1.5 The averaging principle	11
1.6 The inclusion-exclusion principle	13
Exercises	16
2 Advanced Counting	23
2.1 Bounds on intersection size	23
2.2 Graphs with no 4-cycles	24
2.3 Graphs with no induced 4-cycles	26
2.4 Zarankiewicz's problem	29
2.5 Density of 0-1 matrices	33
2.6 The Lovász–Stein theorem	34
2.6.1 Covering designs	36
Exercises	37
3 Probabilistic Counting	41
3.1 Probabilistic preliminaries	41
3.2 Tournaments	44
3.3 Universal sets	45
3.4 Covering by bipartite cliques	46
3.5 2-colorable families	47
3.6 The choice number of graphs	49
Exercises	50

4	The Pigeonhole Principle	53
4.1	Some quickies	53
4.2	The Erdős–Szekeres theorem	55
4.3	Mantel’s theorem	56
4.4	Turán’s theorem	58
4.5	Dirichlet’s theorem	59
4.6	Swell-colored graphs	60
4.7	The weight shifting argument	61
4.8	Schur’s theorem	63
4.9	Ramseyan theorems for graphs	65
4.10	Ramsey’s theorem for sets	68
	Exercises	70
5	Systems of Distinct Representatives	77
5.1	The marriage theorem	77
5.2	Two applications	79
5.2.1	Latin rectangles	79
5.2.2	Decomposition of doubly stochastic matrices	80
5.3	Min–max theorems	81
5.4	Matchings in bipartite graphs	82
	Exercises	85
<hr/>		
Part II Extremal Set Theory		
<hr/>		
6	Sunflowers	89
6.1	The sunflower lemma	89
6.2	Modifications	91
6.2.1	Relaxed core	91
6.2.2	Relaxed disjointness	92
6.3	Applications	93
6.3.1	The number of minterms	93
6.3.2	Small depth formulas	94
	Exercises	96
7	Intersecting Families	99
7.1	Ultrafilters and Helly property	99
7.2	The Erdős–Ko–Rado theorem	100
7.3	Fisher’s inequality	101
7.4	Maximal intersecting families	102
7.5	Cross-intersecting families	104
	Exercises	105

8	Chains and Antichains	107
8.1	Decomposition in chains and antichains	108
8.2	Application: the memory allocation problem	110
8.3	Sperner's theorem	111
8.4	The Bollobás theorem	112
8.5	Strong systems of distinct representatives	115
8.6	Union-free families	116
	Exercises	117
9	Blocking Sets and the Duality	119
9.1	Duality	119
9.2	The blocking number	121
9.3	Helly-type theorems	122
9.4	Blocking sets and decision trees	123
9.5	Blocking sets and monotone circuits	126
	Exercises	132
10	Density and Universality	135
10.1	Dense sets	135
10.2	Hereditary sets	136
10.3	Matroids and approximation	139
10.4	The Kruskal–Katona theorem	143
10.5	Universal sets	148
10.6	Paley graphs	149
10.7	Full graphs	151
	Exercises	153
11	Witness Sets and Isolation	155
11.1	Bondy's theorem	155
11.2	Average witnesses	156
11.3	The isolation lemma	159
11.4	Isolation in politics: the dictator paradox	160
	Exercises	162
12	Designs	165
12.1	Regularity	166
12.2	Finite linear spaces	167
12.3	Difference sets	168
12.4	Projective planes	169
	12.4.1 The construction	171
	12.4.2 Bruen's theorem	172
12.5	Resolvable designs	173
	12.5.1 Affine planes	174
	Exercises	175

Part III The Linear Algebra Method

13 The Basic Method	179
13.1 The linear algebra background	179
13.2 Graph decompositions	185
13.3 Inclusion matrices	186
13.4 Disjointness matrices	187
13.5 Two-distance sets	189
13.6 Sets with few intersection sizes	190
13.7 Constructive Ramsey graphs	191
13.8 Zero-patterns of polynomials	192
Exercises	193
14 Orthogonality and Rank Arguments	197
14.1 Orthogonal coding	197
14.2 Balanced pairs	198
14.3 Hadamard matrices	200
14.4 Matrix rank and Ramsey graphs	203
14.5 Lower bounds for boolean formulas	205
14.5.1 Reduction to set-covering	205
14.5.2 The rank lower bound	207
Exercises	210
15 Eigenvalues and Graph Expansion	213
15.1 Expander graphs	213
15.2 Spectral gap and the expansion	214
15.2.1 Ramanujan graphs	218
15.3 Expanders and derandomization	220
Exercises	221
16 The Polynomial Method	223
16.1 DeMillo–Lipton–Schwartz–Zippel lemma	223
16.2 Solution of Kakeya’s problem in finite fields	226
16.3 Combinatorial Nullstellensatz	228
16.3.1 The permanent lemma	230
16.3.2 Covering cube by affine hyperplanes	231
16.3.3 Regular subgraphs	231
16.3.4 Sum-sets	232
16.3.5 Zero-sum sets	233
Exercises	235

17 Combinatorics of Codes	237
17.1 Error-correcting codes	237
17.2 Bounds on code size	239
17.3 Linear codes	243
17.4 Universal sets from linear codes	245
17.5 Spanning diameter	245
17.6 Expander codes	247
17.7 Expansion of random graphs	250
Exercises	251

Part IV The Probabilistic Method

18 Linearity of Expectation	255
18.1 Hamilton paths in tournaments	255
18.2 Sum-free sets	256
18.3 Dominating sets	257
18.4 The independence number	258
18.5 Crossings and incidences	259
18.5.1 Crossing number	259
18.5.2 The Szemerédi–Trotter theorem	261
18.6 Far away strings	262
18.7 Low degree polynomials	264
18.8 Maximum satisfiability	265
18.9 Hash functions	267
18.10 Discrepancy	268
18.11 Large deviation inequalities	273
Exercises	276
19 The Lovász Sieve	279
19.1 The Lovász Local Lemma	279
19.2 Disjoint cycles	283
19.3 Colorings	284
19.4 The k -SAT problem	287
Exercises	290
20 The Deletion Method	293
20.1 Edge clique covering	293
20.2 Independent sets	294
20.3 Coloring large-girth graphs	295
20.4 Point sets without obtuse triangles	296
20.5 Affine cubes of integers	298
Exercises	301

21	The Second Moment Method	303
21.1	The method	303
21.2	Distinct sums	304
21.3	Prime factors	305
21.4	Separators	307
21.5	Threshold for cliques	309
	Exercises	311
22	The Entropy Function	313
22.1	Quantifying information	313
22.2	Limits to data compression	314
22.3	Shannon entropy	318
22.4	Subadditivity	320
22.5	Combinatorial applications	322
	Exercises	325
23	Random Walks	327
23.1	The satisfiability problem	327
23.1.1	Papadimitriou's algorithm for 2-SAT	328
23.1.2	Schöning's algorithm for 3-SAT	329
23.2	Random walks in linear spaces	331
23.2.1	Small formulas for complicated functions	333
23.3	Random walks and derandomization	336
	Exercises	339
24	Derandomization	341
24.1	The method of conditional probabilities	341
24.1.1	A general frame	342
24.1.2	Splitting graphs	343
24.1.3	Maximum satisfiability: the algorithmic aspect	344
24.2	The method of small sample spaces	345
24.2.1	Reducing the number of random bits	346
24.2.2	k -wise independence	347
24.3	Sum-free sets: the algorithmic aspect	350
	Exercises	352

Part V Fragments of Ramsey Theory

25	Ramseyan Theorems for Numbers	357
25.1	Arithmetic progressions	357
25.2	Szemerédi's cube lemma	360
25.3	Sum-free sets	362
25.3.1	Kneser's theorem	363
25.4	Sum-product sets	365
	Exercises	368

26 The Hales–Jewett Theorem 371

26.1 The theorem and its consequences 371

26.1.1 Van der Waerden’s theorem 373

26.1.2 Gallai–Witt’s Theorem 373

26.2 Shelah’s proof of HJT 374

Exercises 377

27 Applications in Communication Complexity 379

27.1 Multi-party communication 379

27.2 The hyperplane problem 381

27.3 The partition problem 383

27.4 Lower bounds via discrepancy 385

27.5 Making non-disjoint coverings disjoint 388

Exercises 389

References 393

Index 407



<http://www.springer.com/978-3-642-17363-9>

Extremal Combinatorics

With Applications in Computer Science

Jukna, S.

2011, XXIV, 412 p., Hardcover

ISBN: 978-3-642-17363-9