Chapter 1
Survey of the Classical Theory

1.1 Why is the Classical Theory Needed?

Notwithstanding that quantum physics is necessary to obtain a good grasp of the theory of near-field electrodynamics classical approaches are still indispensable for several reasons.

Quantum theory is a rational generalization of the classical theory, and the correspondence principle of Niels Bohr [1–4] links the two together. In a certain limit the result obtained by a quantum physical calculation thus agrees with that of a classical calculation. One may illustrate this by an example of importance in mesoscopic optics, viz., the calculation of the linear polarizability of a mesoscopic spherical particle [5]. The quantum calculation aims at a determination of the microscopic nonlocal conductivity of the particle, and may conveniently be carried out on the basis of the equation of motion for the density matrix operator. A knowledge of the conductivity leads directly to a quantum expression for the polarizability. This expression depends on the energies and wave functions of the various stationary states of the mesoscopic particle, as well as on the probability that a given state is occupied. In the Fourier domain of the electromagnetic field, the polarizability depends on the frequency and wave vector of the externally impressed field. As a function of the particle radius (R) the polarizability increases on the average proportional to R^3, the classical Rayleigh result [6]. Superimposed on this average behavior the polarizability exhibits quantum fluctuations. These fluctuations become less prominent as the radius increases, and the spectrum of the energy eigenstates begins to form a quasi-continuum. Asymptotically, the classical Lorenz–Mie theory for the polarizability of a spherical object is reached [7–9].

For many applications of near-field electrodynamics in the fields of near-field optics and nano-optics it appears sufficient to have an average (effective medium) theory for the physics. In such cases a classical approach is of particular value. In almost all situations a classical calculation is much easier to carry out than a quantum calculation. This, in itself makes the classical approach useful as a starting point for a deeper analysis.

We study the theory of near-field electrodynamics to gain insight in unifying principles, to predict new phenomena, and not least to account for experimental

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observations. For the last purpose, it is useful to remember the words of Bohr [10]: "The experimental conditions can be varied in many ways, but the point is that in each case we must be able to communicate to others what we have done and what we have learned, and therefore the functioning of the measuring instruments must be described within the framework of classical physical ideas." A theoretical account of observations related even to quantum processes (here in near-field electrodynamics) always involve a certain amount of classical theory.

The physics of near-field electrodynamics brings into focus the old question concerning the epistemological difference between classical theory and quantum theory in a fresh and inspiring manner. Alone for this reason it is important to have a clear classical picture of near-field interactions. The central point of quantum theory (and Bohr's point) can be formulated as follows: No (elementary) phenomenon is a phenomenon until it is a registered (observed) phenomenon [11]. In contrast to this, classical theory tells us that phenomena exists (have a "physical reality" in the words of Einstein, Podolsky, and Rosen [12]) independent of all acts of observation. In both classical and quantum physics isolated systems develop in a deterministic manner in time. All attempts to observe physical properties of an isolated system (object) break its isolation, at least temporarily. The interaction between the system and the measuring apparatus inevitably results in correlations which are conceptually different in classical and quantum physics [13]. In far-field electrodynamic studies these correlations are often (but not always!) so weak that the conceptual difference between classical and quantum measurements can be neglected. In near-field electrodynamics the correlations are in most situations strong, and the joint state of the object and the apparatus appears quite complicated, even in a classical framework. On top of the classical complications, the quantum mechanical measurement process "only" establishes statistical correlations between the states of the object and measuring apparatus [14, 15].

1.2 Classical Electrodynamics: Macroscopic vs. Microscopic Theory

In classical studies of the electrodynamics of macroscopic media, one is often not interested in the detailed behavior of the electromagnetic field over atomic distances. What matters is the average of the field, and the matter properties, over a volume large compared to the volume occupied by a single atom or molecule. Roughly speaking, what is of relevance is an averaging over macroscopically small, but microscopically large, regions. Let us denote such an averaging by $\langle \cdots \rangle$. As it is well-known, macroscopic electrodynamics is governed by the following so-called macroscopic Maxwell equations [16–19]:

$$\nabla \times \langle \boldsymbol{E} \rangle = -\frac{\partial}{\partial t} \langle \boldsymbol{B} \rangle, \tag{1.1}$$

$$\nabla \cdot \langle \boldsymbol{D} \rangle = \langle \rho \rangle, \tag{1.2}$$

$$\nabla \times \langle \boldsymbol{H} \rangle = \langle \boldsymbol{J} \rangle + \frac{\partial}{\partial t} \langle \boldsymbol{D} \rangle, \tag{1.3}$$

$$\nabla \cdot \langle \boldsymbol{B} \rangle = 0. \tag{1.4}$$

As they stand, the macroscopic Maxwell equations are a set of eight equations involving the four fields $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{D}$, and $\boldsymbol{H}$. It is a consequence of (1.2) and (1.3) that the "free" charge ($\langle \rho \rangle$) and current ($\langle \boldsymbol{J} \rangle$) densities satisfy the equation of continuity, $\nabla \cdot \langle \boldsymbol{J} \rangle + \partial \langle \rho \rangle / \partial t = 0$. To close the set of equations in (1.1)–(1.4), these are supplemented by so-called constitutive relations connecting the "derived" electric ($\langle \boldsymbol{D} \rangle$) and magnetic ($\langle \boldsymbol{H} \rangle$) fields to the "primary" electric ($\langle \boldsymbol{E} \rangle$) and magnetic ($\langle \boldsymbol{B} \rangle$) fields [19–22]:

$$\langle \boldsymbol{D} \rangle = \langle \boldsymbol{D} \rangle \left(\langle \boldsymbol{E} \rangle, \langle \boldsymbol{B} \rangle \right), \tag{1.5}$$

$$\langle \boldsymbol{H} \rangle = \langle \boldsymbol{H} \rangle \left(\langle \boldsymbol{E} \rangle, \langle \boldsymbol{B} \rangle \right). \tag{1.6}$$

The constitutive relations describe the electric and magnetic response of the "bound" charges to the primary electromagnetic field, in an average sense. In addition to (1.5) and (1.6), a "generalized Ohm's law,"

$$\langle \boldsymbol{J} \rangle = \langle \boldsymbol{J} \rangle \left(\langle \boldsymbol{E} \rangle, \langle \boldsymbol{B} \rangle \right), \tag{1.7}$$

is needed to account for the average response of the "free" charges to the field. The constitutive relations in (1.5)–(1.7) in general have a complicated form. Basically, they originate in equations of motions for the charged particles in the selfconsistent (here macroscopic) electromagnetic field.

To obtain the actual (physical) solution of (1.1)–(1.7) also a study of the relevant initial-value problem is needed. In many situations this problem is difficult to solve in a satisfactory manner.

The macroscopic scheme outlined above often is employed even in near-field electrodynamics, where physical phenomena on a subwavelength scale are in focus [23–25]. Great caution must be exercised if the scheme is used in the optical regime, and the results obtained (predicted) in general should be taken with a grain of salt. In nano-optics, where one aims at an understanding of optical phenomena on the nanometer scale, the spatial averaging procedure of macroscopic electrodynamics certainly may be doubtful.

Classical electrodynamics can be based on a quite different scheme in which in a way no spatial averaging procedure is needed. The scheme is based on the so-called microscopic Maxwell–Lorentz equations, which are presented in the following chapter ((2.1)–(2.4)). These equations dates back to Lorentz, who introduced atomic concepts into Maxwell's theory, in his attempts to "separate ether and matter [26, 27]." In the Lorentz programme only the microscopic $\boldsymbol{E}$- and $\boldsymbol{B}$-fields appear, and in general these fluctuate rapidly in space over atomic distances. The material particles are considered as charged point-particles, and in the classical Maxwell–Lorentz equations only the current ($\boldsymbol{J}$) and charge (ρ) densities of these particles appear on the matter side [16–18]. In order to obtain a closed set of equations

for the microscopic classical approach, the Maxwell–Lorentz equations are supplemented by the Newton–Lorentz equations of motion for the various point-particles; see (2.30).

What has been gained in going from macroscopic to microscopic classical electrodynamics? From a quantum mechanical point of view, the correspondence principle leads to the microscopic classical Maxwell–Lorentz theory, an important fact for the quantum theory of near-field electrodynamics. The microscopic electrodynamic field ($\boldsymbol{E}$, $\boldsymbol{B}$) concept remains its validity on the atomic length scale (and below). This does not mean that the Maxwell–Lorentz theory can be trusted on the atomic length scale, but if one calculates the microscopic current and charge densities of the particles via the nonrelativistic Schrödinger equation or, if needed, via the Dirac equation, and *not* by means of the Newton–Lorentz equation, a theory valid on the atomic length scale emerges. For most purposes in near-field electrodynamics this so-called microscopic semiclassical theory is a good starting point. The word semiclassical here refers to the fact that the electromagnetic field is treated as a classical (unquantized) quantity. When it comes to classical studies of the near-field interaction between just a few atomic particles the microscopic Maxwell–Lorentz approach is indispensable.

1.3 Maxwell–Lorentz Electrodynamics

Most of the material presented in Part I is based on the framework offered us by the microscopic classical theory of electrodynamics, and the purpose of Chap. 2 is to review and discuss basic aspects of the Maxwell–Lorentz approach with a view to subsequent developments in the quantum theory of near-field electrodynamics.

The potential formulation of the Maxwell–Lorentz theory is emphasized because it is of central importance in near-field electrodynamics. The redundancy in the potentials only affects the rotational-free part of the vector potential and the scalar potential, and the relevant combination of these quantities relates exclusively to the matter-attached part of the electromagnetic field. This part is nonvanishing only in the near-field zone of matter. The gauge freedom allows one a certain flexibility in the description of near-field interactions, a circumstance of great importance in both the semiclassical and field-quantized theory, as we shall realize later on. In the Lorenz gauge, simple and form-identical inhomogeneous wave equations appear for the scalar and vector potentials. The most general and physically acceptable, integral solutions to these equations have the retarded scalar (Huygens) propagator as kernel. With a knowledge of the Lorenz-gauge potentials it is easy to determine the electric and magnetic fields, but one cannot from the standard solutions conclude that these fields are retarded (with the vacuum speed of light) in the near-field zone of matter. From a fundamental point of view the electromagnetic field of a single moving point-charge is of particular interest. From the potentials in the Lorenz gauge, the so-called Liénard–Wiechert potentials, we obtain the electric and magnetic fields of the point-particle. Each of these fields divides naturally into two parts, named

the velocity and acceleration field. Only the velocity field is present if the particle moves with uniform velocity (in the given inertial frame), and this field, also called the attached field, plays an important role in near-field interactions.

Since the dynamical Maxwell–Lorentz and Newton–Lorentz equations exhibit translational invariance in time and space, and rotational invariance in space, the total energy, the total momentum, and the total angular momentum of the global field-particle system are time independent quantities, as we show by a direct calculation. Besides these global conservation laws, one has related local conservation laws of great importance in near-field electrodynamics, not least for conceptual reasons. Perhaps, these laws take the most physical transparent forms when matter, as here, is treated as consisting of an assembly of point-particles [28]. The local energy conservation leads to microscopic expressions for the energy density and energy flow (Poynting vector) of the electromagnetic field. The local momentum conservation allows us to identify the momentum density of the field, and to introduce a stress tensor for the particle-field system. This tensor consists of additive parts from the electromagnetic field (the Maxwell stress tensor (multiplied by -1)) and the point-particles. The local momentum conservation law is important for the studies of, e.g., mechanical forces in near-field optics [29–31] and on atoms [32–34] and mesoscopic (and nano-sized) objects [35]. A classical account for the forces acting on the atoms and molecules in confined fields, and for the radiation pressure (light drag) phenomenon [36], can be made on the basis of the local momentum conservation law. The angular momentum light drag effect [37–39] can be studied starting from the local conservation law for the angular momentum, possibly extended to the semiclassical level in the case of mesoscopic objects [40–42].

1.4 The Standard Green Functions (Not Propagators)

In the Lorenz gauge the four-vector potential is related to the four-current density by means of an integral relation where the kernel is the Huygens propagator. This propagator describes how light spreads out from a source point in space-time. Since the Huygens propagator is nonvanishing (and in fact singular) only on the light cone the interpretation at first sight perhaps appears satisfactory from the point of view of special relativity. However, one must not forget that the four-potential is not an observable quantity in physics. With this in mind, it seems of interest to establish a propagator formalism which relates the electric and magnetic fields to the three-current density. Within the so-called scalar theory of scattering (diffraction) the Huygens propagator continues to be the relevant kernel for the description, but the scalar approach is an approximation [43, 44].

In near-field electrodynamics it has for many years been popular to base theoretical studies on what I here call the *standard Green function* formalism [18,24,45–48]. In Chap. 3, the standard Green functions (now dyadic quantities) relating respectively the electric field and magnetic field to the current density of the Maxwell–Lorentz theory are determined and discussed. As everyone else, we carry out the

calculations in the space-frequency domain, *but*, and this is an important point, *we do not say that the result can be taken back to the space-time domain by Fourier transformation, if wished*. From the perspective of near-field electrodynamics we face what might seem to be a conceptual problem: The near-field part of the standard Green function which relates the electric field to the current density does not exist in the space-time domain. This fact is the reason that I use the term standard Green function and not the name standard propagator, as many people do. The root of the above-mentioned problem is the fact that it is the photon field which propagates (with the vacuum speed of light), and this field is associated only to the divergence-free (transverse) part of the electromagnetic field. The related so-called transverse propagator has been studied in [5, 49–51]. The total electric field has a rotational-free part in the near-zone of matter, and if this part is subtracted we arrive at a genuine propagator relation between the transverse electric field and the current density, as we shall realize in Chap. 14. In a quantum context, where the matter field is a continuum field in the probabilistic sense, a related problem arises for the standard Green function for the electric field if the "point of observation" is inside matter: The spatial integral of the Green function is only conditionally convergent, a physically unacceptable feature. The solution of this aspect of the problem is also given in Chap. 14 (see also [45]).

1.5 Evanescent Electromagnetic Fields

The information contained in the standard Green functions $\boldsymbol{G}(\boldsymbol{R};\omega)$ and $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{R};\omega)$, which relate the electric and magnetic fields to the particle current density, can be represented in a variety of different forms, which basically are physically equivalent. All the forms have in common that they belong to the microscopic Maxwell–Lorentz equations in the frequency (ω) domain. They differ in the manner in which the spatial ($\boldsymbol{R}$) information is represented. For investigations of a given problem one form of representation can be more useful than the others. In studies of the electrodynamics of atomic, molecular, and mesoscopic objects which linear extensions are (much) less than the relevant wavelengths of the electromagnetic field the standard Green functions are most often given in spherical coordinates. In these coordinates the local unit vectors and the magnitude $R = |\boldsymbol{R}|$ appear in the Green functions. The R-dependence contains terms proportional to R^{-1}, R^{-2}, and R^{-3}. A term proportional to R^{-3} only appears in the Green function relating to the electric field, and it is this part we referred to as the near-field part of $\boldsymbol{G}(\boldsymbol{R};\omega)$ in the previous section.

If one, at least as an attempt, seeks to make a spatial Fourier transform of the standard Green functions one arrives at a plane-wave (wave vector $\boldsymbol{q}$) representation of these ($\boldsymbol{G}(\boldsymbol{q},\omega)$, $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{q},\omega)$). The assumption of the existence of Fourier integral transformations turns out to be incorrect, since $\boldsymbol{G}(\boldsymbol{q},\omega)$ and $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{q},\omega)$ have poles. Nevertheless, closely related integral representations do exist for $\boldsymbol{G}(\boldsymbol{R};\omega)$ (see also [50]) and $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{R};\omega)$, and as we shall see later on in this book, these representations are extremely useful. In Chap. 4, we use the plane-wave representation

as a springboard for the introduction of the so-called mixed representation of the standard Green functions. In this representation, also called the angular spectrum representation [44, 52, 53], a Fourier expansion is made in only two coordinates, say X and Y, leading to $\boldsymbol{G}(Z;\boldsymbol{q}_\parallel,\omega)$ and $\boldsymbol{G}_{\mathrm{M}}(Z;\boldsymbol{q}_\parallel,\omega)$, where $\boldsymbol{q}_\parallel$ is the wave vector in the XY-plane. The two-dimensional plane-wave expansion is useful in (near-field) studies of the electrodynamics of plane structures (surfaces, interfaces, sheets and layered media), and to a certain extent for the description of the electrodynamic interaction between mesoscopic (or microscopic) particles and plane structures.

The theoretical studies of the electromagnetic coupling between a small particle and a planar surface have a 100-year-old history, starting around 1909 with Sommerfeld's theory for the radiation from an electric dipole oriented vertically above a conducting medium (ground) [54]. In Sommerfeld's asymptotic solution electromagnetic surface waves, investigated earlier by Zenneck [55], appear. A few years later, in 1911, Hörschelmann analyzed the radiation from a horizontally oriented dipole [56]; see also [57]. In Sommerfeld's and Hörschelmann's calculations an expansion of the field in cylindrical coordinates was used. In an important paper, dealing with the propagation of electromagnetic waves over a conducting sphere, Weyl derived a new contour integral representation of a diverging spherical scalar wave [58]. In this representation the mode functions are so-called inhomogeneous waves (more on these below). For a particular contour the Weyl representation provides us with a decomposition of the spherical wave into homogeneous and evanescent field modes, the angular spectrum representation. The early developments in the field is summarized in [59]. Almost two decades ago the Weyl expansion was employed by Agarwal in a theoretical study of the optics of an atom placed in front of a phaseconjugating mirror [60]. Further analyses were made by Hendriks and Nienhuis [61], Milonni et al. [62], Arnoldus and George [63], Agarwal and Gupta [64]. The work of Agarwal inspired me to suggest that phaseconjugation of the field from the tip of a near-field microscope would allow one to focus light beyond the classical diffraction limit [65], an idea which was confirmed experimentally shortly afterwards [66]. An account of the microscopic theory for phaseconjugation from a mesoscopic particle is given in [67]; see also [40], Part B. The question of phaseconjugation of evanescent waves plays an important role for the studies in [60–67], and for further analyses of this the reader is referred to [68, 69], and references therein. Because the field from a point-charge has a singularity at the position of the charge, a rigorous derivation of the Weyl representation of a spherical wave requires some care [70]. The dipole–surface interaction gives rise to a mutual attraction, also found between a pair of parallel, uncharged, conducting plates in vacuum. The attraction, referred to as the Van der Waals attraction, was calculated originally by Casimir [71] in 1949; see also Casimir and Polder [72]. From a quantum electrodynamic point of view, the attraction can be accounted for by assuming that the related force is a consequence of the separation-dependent energy in the vacuum field trapped between the plates [73, 74]. The Weyl expansion has been used by Agarwal in a study of basic aspects of quantum electrodynamics in the presence of dielectrics and conductors [75]. Theoretical studies of the electrodynamic interaction between an electric dipole and a planar (metallic) medium

exhibiting a so-called nonlocal (spatially dispersive) response have also been carried out in the nonretarded regime [76], and with retardation effects included [77, 78], and the results obtained used in calculations of the surface-dressed dipole polarizability. The renormalized polarizability agrees with that obtained in a local approach [79, 80]. The dipole emission near planar interfaces is also discussed in the book by Novotny and Hecht [24], and here further references of interest in nano-optics are listed.

Under certain conditions an evanescent field may be generated by a current sheet, as we shall see. In a sense sheet electrodynamics shows the evanescent field concept in its simplest form, and starting from the result for a single sheet the evanescent fields belonging to more complicated layered structures can be determined by superposition. Evanescent fields play a key role in studies of surface electromagnetic waves; see [24,25,81–88], and references therein. In the condensed matter physics literature these waves are commonly referred to as surface polaritons. Roughly speaking a surface polariton is an admixture of the electromagnetic field and a collective particle excitation that propagates along the surface of a medium, or along the interface between two media. The strength of the electromagnetic field associated with a surface polariton decays exponentially as one moves away from the interface into either medium. Surface electromagnetic waves are of particular importance when carried by (bound to) a metal surface, and in the literature they are called surface plasmaritons, surface plasmon polaritons, or just surface plasmons. In the context of near-field optics and nano-optics the study of optical phenomena related to surface waves on metals recently has been termed plasmonics or nanoplasmonics [24]. The overwhelming majority of the theoretical (numerical) calculations describes the metal response on the basis of macroscopic (local) electrodynamics. The diversity of electromagnetic surface phenomena, which one catches a glimpse of when microscopic (nonlocal) response theory is used [84, 89–95] has not been very well explored theoretically nor experimentally up to now. To understand the basic physical properties of surface polaritons bound to the surface of a (BCS) superconductor [96–98] microscopic electrodynamics is necessary [99–102]. Surface electromagnetic waves can be generated (excited) by various methods. Thus, the substantial increase in the interest of surface polaritons which appeared around 1970, mainly was due to excitation schemes suggested by Otto [103] and Kretschmann [104]. In the wake of the birth of near-field optics the use of local (dipole-like) excitation (and detection) methods for surface modes gained renewed interest in the scientific community, and the tremendous recent interest in studies of surface polaritons is bound up with the scanning near-field optical microscopy technique [24, 25]. Evanescent fields also show up when a plane monochromatic electromagnetic wave is incident on a planar dielectric (glass)-vacuum interface at an angle larger than the critical angle. On the vacuum side the electromagnetic field decays exponentially with the distance from the interface. Although the energy density of the evanescent field is nonzero, no energy is transported away from the interface. Frustrated total internal reflection (FTIR) results if a second planar dielectric is brought within an optical wavelength from the first. From a fundamental point of view FTIR is the paradigm of optical tunneling [105–107], and it has been

maintained that photon tunneling is a near-field phenomenon which originates in our inability to localize a photon completely in space [108, 109]. There are certain indications that information on the photon tunneling phenomenon may be obtained from studies of the Goos–Hänchen shift [110–113], a phenomenon which shows up in FTIR experiments. A mixed representation also can be used to describe the current density of a moving point-charge. If the particle trajectory is confined to lie in a plane, a classical expression for the related sheet current density can be established. In near-field electrodynamics it is of importance to investigate also the (cycle-averaged) field momentum density outside the current sheet. We do this paying particular attention to the case of evanescent modes.

Starting from the two-dimensional plane-wave expansion other useful representations of the Huygens propagator and the standard Green functions can be obtained. Thus, for studies of the electrodynamic interaction between a small particle (possibly atom or molecule) and a planar structure it may be advantageous to describe the interaction over the $\boldsymbol{q}_\parallel$-plane in polar coordinates [50, 114]. The angular integrations associated with the magnetic part of the standard Green function thus can be expressed in terms of Bessel functions of the first kind and zero and first order, for instance. For the electric part of the Green function also the Bessel function of the first kind and second order appears. Altogether, one ends up with a representation of the various Green functions in which only a single integral over the magnitude of $\boldsymbol{q}_\parallel$ ($0 \leq q_\parallel < \infty$) enters. The region $0 \leq q_\parallel \leq \omega/c_0$ (c_0 is the vacuum speed of light) is associated with homogeneous waves, and the region $\omega/c_0 < q_\parallel < \infty$ is connected to evanescent modes. If a different set of variables is used for the integration over the $\boldsymbol{q}_\parallel$-plane, one may express the Green functions as a contour integral in a complex α-plane. A segment along the $\Re\alpha$-axis now gives the homogeneous wave contribution, and a segment parallel to the $\Im\alpha$-axis associates to the evanescent modes. By modifying the contour above one can obtain an expansion of the Green functions in terms of so-called generalized inhomogeneous waves [53, 58, 70]. For these waves the surfaces of constant amplitude and of constant phase are distinct, except for special values of the variables.

1.6 Multipole Electrodynamics: A Richly Faceted Subject

The theoretical analysis of field–matter interactions is simplified in cases where the linear extension of the particle system (object) is much smaller than the "relevant" wavelengths of the electromagnetic field. What we mean by the word relevant depends on the context. In order to exemplify this let us consider the interaction of a monochromatic external (incoming) field with an object of extension much less than the wavelength of the external field. During the interaction local fields of different wavelengths are generated inside the object and these fields do not have wavelengths which are large in comparison with the linear size of the object. Despite this, the relevant wavelength is that of the externally impressed field in this case. If the monochromatic external field is replaced by a quasi-monochromatic field of the

same mean frequency, the relevant wavelengths are those of this field. From the point of view of electrodynamics the object may be characterized as mesoscopic. An object which appears mesoscopic when subjected to an external field may also be characterized as mesoscopic in a field emission process if the dominating wavelengths of the radiated field all are much larger than the electronic size of the object. The field–matter interaction in the case of mesoscopic objects often is characterized as mesoscopic electrodynamics. Nano-optics, which deals with the electromagnetic interaction between visible (or infrared or ultraviolet) light and nano-sized objects, is a form of mesoscopic electrodynamics. The optical near-field interaction with material structures may also be characterized as a sort of mesoscopic electrodynamics.

The prevailing microscopic classical current density, $\boldsymbol{J}(\boldsymbol{r},t)$, in a mesoscopic object can often with advantage be described in terms of a so-called moment expansion, in which only terms of the lowest orders are retained. The spatial integral (zeroth-order moment) of $\boldsymbol{J}(\boldsymbol{r},t)$ over the mesoscopic object gives the so-called electric dipole current density, $\boldsymbol{J}^{\mathrm{ED}}$, of the object (at a given time). The spatial first-order moment of $\boldsymbol{J}(\boldsymbol{r},t)$, a tensorial quantity, gives the sum of the electric quadrupole ($\boldsymbol{J}^{\mathrm{EQ}}$) and the magnetic dipole ($\boldsymbol{J}^{\mathrm{MD}}$) moments of the current density distribution. The symmetric part of the tensor is the EQ-moment, and the antisymmetric part is the MD-moment. The moments $\boldsymbol{J}^{\mathrm{ED}}$ and $\boldsymbol{J}^{\mathrm{EQ}}$ are just the time derivatives of the well-known electric dipole ($\boldsymbol{p}(t)$) and quadrupole ($\boldsymbol{Q}(t)$) moments of a point-charge distribution. The quantity $\boldsymbol{J}^{\mathrm{MD}}$ relates in a simple fashion to the magnetic dipole moment ($\boldsymbol{m}(t)$) of a point-charge distribution, a moment which in turn can be expressed in terms of the (relativistic) orbital angular momenta of the individual point-particles.

In the space-frequency domain the electric ($\boldsymbol{E}^{\mathrm{ED}}(\boldsymbol{r};\omega)$) and magnetic ($\boldsymbol{B}^{\mathrm{ED}}(\boldsymbol{r};\omega)$) fields generated by $\boldsymbol{p}(\omega)$ can be expressed in compact form with the help of the standard Green functions $\boldsymbol{G}(\boldsymbol{r};\omega)$ and $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{r};\omega)$, respectively. The fields $\boldsymbol{E}^{\mathrm{MD}}$ and $\boldsymbol{B}^{\mathrm{MD}}$ generated by $\boldsymbol{m}(\omega)$ can also be expressed in terms of the standard Green functions. In this case $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{r};\omega)$ relates to $\boldsymbol{E}^{\mathrm{MD}}(\boldsymbol{r};\omega)$, and $\boldsymbol{G}(\boldsymbol{r};\omega)$ to $\boldsymbol{B}^{\mathrm{MD}}(\boldsymbol{r};\omega)$. As the reader may have anticipated, the fields $\boldsymbol{E}^{\mathrm{EQ}}(\boldsymbol{r};\omega)$ and $\boldsymbol{B}^{\mathrm{EQ}}(\boldsymbol{r};\omega)$ which are generated by $\boldsymbol{Q}(\omega)$ relate to the third rank tensors $\nabla\boldsymbol{G}(\boldsymbol{r};\omega)$ and $\nabla\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{r};\omega)$.

In near-field electrodynamics one is faced with a complicated interplay between the divergence-free (transverse) and rotational-free (longitudinal) parts of the electromagnetic field. This interplay is important for our understanding of the space-time generation and destruction of photons in their interaction with matter, whether it be in the context of the first or second-quantized theory, and for fundamental studies of our ability to localize photons in space-time. In mesoscopic electrodynamics it is therefore fruitful to seek to establish a multipole formalism which respects the division of the electromagnetic field into its transverse and longitudinal parts [19, 115–117]. In Sects. 5.4 and 5.5, a systematic approach for higher-order multipoles is developed for the transverse part of the field. The calculations are carried out in the space-frequency domain, and we start by considering the electromagnetic field in a source-free region of space. In such a region the electric ($\boldsymbol{E}(\boldsymbol{r};\omega)$) and magnetic ($\boldsymbol{B}(\boldsymbol{r};\omega)$) fields satisfy the Helmholtz equation, and so

do the scalar quantities $\boldsymbol{r}\cdot\boldsymbol{E}(\boldsymbol{r};\omega)$ and $\boldsymbol{r}\cdot\boldsymbol{B}(\boldsymbol{r};\omega)$ because the $\boldsymbol{E}$- and $\boldsymbol{B}$-fields are divergence-free in vacuum. In preparation for the introduction of the so-called transverse electromagnetic multipole waves one seeks the general solution to the Helmholtz equation for the scalars $\boldsymbol{r}\cdot\boldsymbol{E}$ and $\boldsymbol{r}\cdot\boldsymbol{B}$ in spherical coordinates. The solution can be written as a double sum with parameters J and M. The parameter J can take on the values $J = 0, 1, 2, \cdots$, and for a given value of J the possible M's are $M = -J, -J+1, \cdots, 0, \cdots, J-1, J$. Not surprisingly, these parameters are identical to those entering the quantum theory for the orbital angular momentum. Each (J, M)-term consists of a product of a spherical harmonic (Y_{JM}) and a radial function. With an eye to studies of radiation problems, the radial function is given in the form of a superposition of spherical Hankel functions of the first ($h_J^{(1)}$) and second ($h_J^{(2)}$) kind and order J. The electromagnetic multipole waves are divided into two groups: (1) Electric multipole waves (superscript (E) for which $\boldsymbol{r}\cdot\boldsymbol{B}_{JM}^{(E)} = 0$, and (2) magnetic multipole waves (superscript (M)) for which $\boldsymbol{r}\cdot\boldsymbol{E}_{JM}^{(M)} = 0$. The subscript JM indicates that there belongs a multipole wave to each combination of J and M. In the transverse-electromagnetic-multipole-wave decomposition of the general solution to the free-space Maxwell equations still appears linear combinations of the Hankel functions $h_J^{(1)}$ and $h_J^{(2)}$, but instead of the spherical harmonics Y_{JM} we now find the so-called vector spherical harmonics $\boldsymbol{X}_{JM} \propto i^{-1}\boldsymbol{r}\times\boldsymbol{\nabla}Y_{JM}$.

In the multipole wave expansion of the electromagnetic field in source-free regions of space appears yet arbitrary so-called electric and magnetic multipole coefficients, which basically give the strengths of the various multipole fields. In order to determine these coefficients one needs to know the current density distribution ($\boldsymbol{J}(\boldsymbol{r};\omega)$) in the source which generates the field.

The examination of the connection between the multipole field strengths and the source current density distribution gives us a glimpse of the important question of how to choose appropriate field variables in the source domain. A key point here is to work with a new "electric-field" variable which is divergence-free everywhere in space (the magnetic field *is* transverse everywhere according to the Maxwell theory). The new electric-field variable ($\boldsymbol{\mathcal{E}}$) is given by $\boldsymbol{\mathcal{E}}(\boldsymbol{r};\omega) = \boldsymbol{E}(\boldsymbol{r};\omega) + i\,\boldsymbol{J}(\boldsymbol{r};\omega)/(\epsilon_0\omega)$. In Sect. 10.4 we shall introduce the generalized electric displacement field $\boldsymbol{D}$ and the generalized magnetization $\boldsymbol{M}$ [118, 119]. In terms of these, one has

$$\epsilon_0\boldsymbol{\mathcal{E}}(\boldsymbol{r};\omega) = \boldsymbol{D}(\boldsymbol{r};\omega) + \frac{i}{\omega}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r};\omega), \tag{1.8}$$

as the reader may show after having studied Sect. 10.4. The $\boldsymbol{D}$-field plays an important role in linear nonlocal response theory, and for studies of near-field aspects of the photon emission from micro- and mesoscopic sources. It is possible to make the choice $\boldsymbol{M} = \boldsymbol{0}$ without changing any physical properties of the field-particle system as we shall learn in Sect. 10.4.1. With such a choice the new transverse field variable $\boldsymbol{\mathcal{E}}$ is just the generalized displacement field (divided by ϵ_0). Via the Helmholtz equations for $\boldsymbol{r}\cdot\boldsymbol{\mathcal{E}}$ and $\boldsymbol{r}\cdot\boldsymbol{B}$, one is able to express the actual electric and magnetic multipole coefficients in terms of space-integrals involving $\boldsymbol{\nabla}\times\boldsymbol{J}$ and $\boldsymbol{J}$, respectively.

As we shall see, the multipole expansion of the source current density distribution is useful in studies of the rate of energy, momentum, and angular momentum transfer to a mesoscopic particle from a prescribed external field.

1.7 Local Electromagnetic Fields and Resonances

In near-field electrodynamics the local field concept is of central importance. Thus, if one has a knowledge of this field *all* electrodynamic properties of the medium under study can be predicted. Although quantum physics is needed for a rigorous calculation of local fields, because these vary on a length scale (much) shorter than that of the externally impressed field in most cases, it is of some interest to study local fields in the framework of the microscopic Maxwell–Lorentz equations. For an assembly of point-particles the local electric field at a given point ($\boldsymbol{r}$) in space is the sum of the external field at this point and the fields generated at $\boldsymbol{r}$ by the various particles of the system [5,24,25,114,120–129]. The field generated by a given point-particle is not known a priori, however, because the current density distribution of this particle, which is the source of the emitted field, is determined by the local field acting on the particle. To cut the Gordian knot, one must in a selfconsistent manner determine the local fields acting on all the particles in the system. This task is a formidable difficult one in general. If we treat the particles as electric dipoles, and limit ourselves to linear response theory, the task can be solved. A supermatrix $(\boldsymbol{\mathcal{U}} - \boldsymbol{\mathcal{T}})^{-1}$ relates the local fields at the sites of the, say N, dipoles to the external fields at these sites. The quantity $\boldsymbol{\mathcal{T}}$ contains the electric-dipole polarizabilities of the various particles, and the Green functions $\boldsymbol{G}(\boldsymbol{r}_i - \boldsymbol{r}_j; \omega)$ $[(i, j) = 1, 2, \cdots, N]$ describing the field communication between all pairs, (i, j), of dipoles. In some cases the particles are so close to each other that only the near-field part of the $\boldsymbol{G}$'s needs to be retained. Once the selfconsistent solution for the electric field on the dipole sites has been obtained the local field easily can be determined everywhere else in space. The procedure described above incorporates all kinds of multipole scattering between the electric dipoles, and from the general result the Born series approach to the N-particle scattering problem can be established by making a power series expansion of $(\boldsymbol{\mathcal{U}} - \boldsymbol{\mathcal{T}})^{-1}$ in $\boldsymbol{\mathcal{T}}$.

In a closed system no external fields act, and possible selfconsistent solutions for the local field therefore are selfsustaining. For an assembly of electric dipoles self-sustaining solutions must satisfy the condition $\mathrm{Det}\{\boldsymbol{\mathcal{U}} - \boldsymbol{\mathcal{T}}\} = 0$, where Det stands for determinant. The inevitable presence of irreversible electronic damping mechanisms and spontaneous emission makes all systems open to some extend, perhaps with the exception that the whole universe may be considered closed. Selfsustaining solutions are also called local-field resonances, and this name is also used to cover the resonances one may obtain in weakly open systems. Roughly speaking, a local-field resonance occurs whenever a component of the supermatrix $(\boldsymbol{\mathcal{U}} - \boldsymbol{\mathcal{T}})^{-1}$ has a deep minimum. Local-field resonances can occur at certain frequencies $\omega = \omega_{\mathrm{Res}}$, or at a given frequency for certain spatial arrangements of the dipole positions.

In the last case, which most often occurs in few-particle systems, one speaks of a configurational resonance.

The classical multipole scattering among magnetic dipole, and between electric quadrupoles, can of course be studied by analogy with the treatment for electric dipoles.

1.8 Radiation Reaction in a Classical Perspective

In the procedure for classical local-field calculations, which we briefly described in the foregoing section, the radiative reaction which the field emitted from a point-particle exerts on the particle itself was not taken into account. The radiation reaction problem raises deep questions in electrodynamics, and the problem cannot be dealt with in a satisfactory manner in the framework of classical electrodynamics. Nevertheless, it certainly is worthwhile to study the phenomena in a purely classical context because this gives us some insight in the problems we are facing [19, 130–133]. Nonrelativistic and fully relativistic quantum mechanical treatments have been given by Moniz and Sharp [134], and by Low [135], respectively.

An electrically charged point-particle emits energy when accelerated, and the radiative energy loss affects the motion of the particle. In classical electrodynamics a point-charge placed in a prescribed (external) electromagnetic field moves according to the Newton–Lorentz equation. To account for the radiative energy loss some force, called the radiation reaction force, must act on the particle besides the force exerted by the external field. In the nonrelativistic Abraham–Lorentz approach the radiation reaction force is determined in some approximate and time-averaged sense via the instantaneous power radiated by the particle's acceleration field [19, 130, 136–138]. By demanding that the negative of the work carried out on the particle by the yet unknown radiative reaction force in a *certain* time interval equals the power radiated in this interval one finds that the radiative reaction force is proportional to the time derivative of the particle's instantaneous (!) acceleration. To reach this conclusion the particle motion must have such a form that the scalar product of the particle velocity and acceleration has the same value at the two endpoints of this "certain" time interval.

From our nonrelativistic classical standpoint the radiation reaction effect is of importance if the relative change in the particle acceleration is appreciable on a time scale $\tau_0 = Q^2/(6\pi\epsilon_0 m c_0^3)$, where Q and m are the charge and mass of the particle. For the electron $\tau_0 \approx 6.3 \cdot 10^{-24}$ s. Among other doubtful consequences, the Abraham–Lorentz model predicts a preacceleration effect on a time scale of the order of τ_0 [116, 130]. To observe such an effect the external force must be switched on over a time interval $\sim \tau_0$, an impossibility for classical interactions. The classical theory for the radiation reaction is useful in physical optics, where it gives an important contribution to the electric dipole polarizability, for instance [18]. As a part of the particle's "inner" dynamics the so-called (radiation reaction) dressed polarizability is the observable quantity in (near-field) electrodynamics. The electric radiation

reaction field $\boldsymbol{E}_{\mathrm{RR}}(\omega)$ of an electric dipole (moment $\boldsymbol{p}(\omega)$) also can be obtained via the standard Green function $\boldsymbol{G}(\boldsymbol{R};\omega)$ if one near $\boldsymbol{R} = \boldsymbol{0}$ keeps the small imaginary part of $\boldsymbol{G}$. Because this part varies only little on a length scale comparable to the linear extension of a mesoscopic object, the radiative reaction on such an object does not depend (significantly) on the electronic structure of the object. Via the Green function $\boldsymbol{G}_{\mathrm{M}}(\boldsymbol{R};\omega)$ the magnetic radiation reaction field $\boldsymbol{B}_{\mathrm{RR}}(\omega)$ on a magnetic dipole (moment $\boldsymbol{m}(\omega)$) is easily calculated. The duality between the electric and magnetic fields shows that $|\boldsymbol{E}_{\mathrm{RR}}(\omega)|/|\boldsymbol{p}(\omega)| = c_0^2|\boldsymbol{B}_{\mathrm{RR}}(\omega)|/|\boldsymbol{m}(\omega)|$.

In the classical relativistic domain the modification of a charged point-particle's motion by radiation reaction is described by the Lorentz–Dirac equation [139–144]. Starting from the derivation of a manifestly covariant expression for the energy-momentum radiation rate, and the conclusion following from the fact an elementary particle keeps its rest mass intact during the radiation reaction process, we end up with the Lorentz–Dirac equation. In order to cancel the rest-mass changing radiation, one suspects that the attached self-field of the particle will be temporarily distorted during the acceleration process. An unaccelerated charged object carries along its velocity field, and in the particle's rest frame this is just the electrostatic Coulomb field. The related field energy is equivalent to some so-called electromagnetic mass in a relativistic context. Observed in an inertial frame moving with respect to the rest frame, the velocity field contributes a self-momentum to the uniformly moving particle. A distortion of the self-momentum appears during accelerated particle motion. In an external (ext) electromagnetic field the transverse vector potential $\boldsymbol{A}_{\mathrm{T}}^{\mathrm{ext}}$ adds a momentum $Q\boldsymbol{A}_{\mathrm{T}}^{\mathrm{ext}}$ to the particle, and this momentum spreads over the Coulomb self-field region. As soon as the external magnetic field overlaps this region the attached field momentum starts to be distorted. For an elementary discussion of this aspect the reader is referred to [116].

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