# Preface

Riemann surfaces appear in many branches of mathematics and physics, e.g. in differential and algebraic geometry and the theory of moduli spaces, in topological field theories, quantum chaos and integrable systems. The practical use of Riemann surface theory has been limited for a long time by the absence of efficient computational approaches. In recent years considerable progress has been achieved in the numerical treatment of Riemann surfaces which stimulated further research in the subject and led to new applications. The existing computational approaches follow from the various definitions of Riemann surfaces: via non-singular algebraic curves, as quotients under the action of Fuchsian or Schottky groups, or via polyhedral surfaces.

It is the purpose of the present volume to give a coherent presentation of the existing or currently being developed computational approaches to Riemann surfaces. The authors of the contributions are representants from the groups providing publically available numerical codes in this field. Thus this volume illustrates which software tools are available and how they can be used in practice. In addition examples for solutions to partial differential equations and in surface theory are presented.

In the introduction, A.I. Bobenko presents a comprehensive summary of the theory of compact Riemann surfaces, Abelian differentials, periods on Riemann surfaces, theta functions and uniformization theory. Riemann originally introduced Riemann surfaces as plane algebraic curves. B. Deconinck and M. Patterson have followed this approach together with M. v. Hoeij for a number of years. They have devised several algorithms facilitating different aspects of the effective computation with Riemann surfaces represented by plane algebraic curves. Their algorithms have led to the *algcurves* Maple package: a collection of Maple programs for computations with algebraic curves. B. Deconinck and M. Patterson describe their algorithms, the Maple implementation and give instructive examples. The numerical approach via algebraic curves involves the computation of contour integrals on Riemann surfaces. To study the moduli spaces associated to Riemann surfaces numerically, an efficient computation of these integrals is necessary. J. Frauendiener and C. Klein present a fully numerical approach based on Gauss integration which provides high accuracy. Explicit solutions of integrable partial differential equations are discussed as applications.

A complementary approach to compact Riemann surfaces is based on the uniformization theory. M. Schmies discusses numerics of the Schottky uniformization of Riemann surfaces and in particular the convergence of Poincaré theta series and their use in the numerical treatment of Riemann surfaces. It is incorporated in the Java project *jtem*. The use of this package is demonstrated for concrete examples from surface theory. R. Hidalgo and M. Seppälä discuss the uniformization of hyperelliptic algebraic curves. Using a method originally due to Myrberg, they construct an algorithm that approximates the generators of a Schottky group uniformizing a given hyperelliptic algebraic curve. D. Crowdy and J. Marshall study conformal mappings for multiply connected domains both analytically and numerically. They discuss a formulae for these mappings in terms of the Schottky–Klein prime function. The latter function is numerically evaluated by using Schottky uniformization.

The relation of Riemann surfaces to polyhedral surfaces offers yet another computational approach. A.I. Bobenko, C. Mercat and M. Schmies discuss the computation of period matrices of Euclidean surfaces by methods of discrete differential geometry. The latter are based on the notions of discrete holomorphicity and discrete Riemann surfaces. As an application period matrices of Lawson surfaces are computed. An interesting object associated to the modular space of Riemann surfaces are determinants of Laplacians. These determinants appear in particular in topological field theories. A. Kokotov presents a review of determinants of Laplacians for surfaces with polyhedral metrics. These determinants are given in terms of explicit functions, and provide a way to study global aspects of the geometry of the associated modular space numerically, an investigation which is currently being performed.

The intended audience of this book is twofold. It can be used as a textbook for a graduate course in numerics of Riemann surfaces. The standard undergraduate background, i.e., calculus and linear algebra, is required. In particular, no knowledge of the theory of Riemann surfaces is expected, the necessary background in this theory is contained in the Introduction chapter.

On the other hand, this book is also written for specialists in geometry and mathematical physics applying the theory of Riemann surfaces in their research. It is the first book on numerics of Riemann surfaces which reflects the progress in this field during the last decade, and it contains original results. There is a growing number of applications where one is interested in the evaluation of concrete characteristics of models analytically described in terms of Riemann surfaces. Many problem settings and computations in this volume are motivated by such concrete applications in geometry and mathematical physics.

This work was supported in part by the MISGAM program of the European Science Foundation. A.I. Bobenko is partially supported by the DFG Research Unit ‘Polyhedral Surfaces’. C. Klein thanks for support by the project

FroM-PDE funded by the European Research Council through the Advanced Investigator Grant Scheme, the Conseil Régional de Bourgogne via a FABER grant and the ANR via the program ANR-09-BLAN-0117-01.

Berlin, Dijon
June, 2010

*Alexander I. Bobenko*
*Christian Klein*

http://www.springer.com/978-3-642-17412-4

Computational Approach to Riemann Surfaces
Bobenko, A.I.; Klein, C. (Eds.)
2011, XII, 264 p. 58 illus., 14 illus. in color., Softcover
ISBN: 978-3-642-17412-4