

Chapter 2

Fractal Analysis in CNC End Milling

Abstract This chapter deals with the fractal dimension modeling in CNC end milling operation. Milling operations are carried out for three different materials viz. mild steel, brass and aluminium work-pieces for different combinations of spindle speed, feed rate and depth of cut. The generated surfaces are measured with Talysurf instrument and analyzed to get fractal dimension. The experimental results are further processed to model fractal dimension using response surface methodology (RSM). It is seen that spindle speed and depth of cut are the significant factors affecting fractal dimension for mild steel. For brass material, the significant factors are spindle speed and feed rate but for aluminium the significant factor is depth of cut. In general, for mild steel and brass, with increase in spindle speed, D increases. Comparing the developed response surface models, it is concluded that the models are material specific and the tool-work-piece material combination plays a vital role in fractal dimension of the generated surface profile.

Keywords Fractal dimension (D) • CNC End Milling • RSM • Mild steel • Brass • Aluminium

2.1 Introduction

CNC milling is a popular machining process in the modern industry because of its ability to remove materials with a multi-point cutting tool at a faster rate with a reasonably good surface quality. In order to get specified surface roughness, selection of controlling parameters is necessary. There has been a great many research developments in modeling surface roughness and optimization of the controlling parameters to obtain a surface finish of desired level since only proper selection of cutting parameters can produce a better surface finish. But such studies

Table 2.1 Variable levels used in the experimentation

Levels	Aluminium			Brass			Mild steel		
	d	N	f	d	N	f	d	N	f
−1	0.10	4,500	900	0.10	1,500	550	0.150	2,500	300
−0.5	0.15	4,750	950	0.15	1,800	600	0.175	2,750	350
0	0.20	5,000	1,000	0.20	2,100	650	0.200	3,000	400
0.5	0.25	5,250	1,050	0.25	2,400	700	0.225	3,250	450
1	0.30	5,500	1,100	0.30	2,700	750	0.250	3,500	500

are far from complete since it is very difficult to consider all the parameters that control the surface roughness for a particular manufacturing process. In CNC milling there are several parameters which control the surface quality. The analysis of surface roughness on CNC end milling process is a big challenge for research development. Several factors involved in machining process have to be optimized to obtain a desired surface quality. In this study, three machining parameters are considered viz. spindle speed, feed rate and depth of cut. Also the study is conducted on three different materials, viz. mild steel, brass and aluminium to consider the effect of work-piece material variation on fractal dimension of machined surfaces. The experimental results are analyzed using RSM.

2.2 Experimental Details

2.2.1 Design of Experiments

A full factorial design is used with five levels of each of the three design factors viz. depth of cut (d , mm), spindle speed (N , rpm) and feed rate (f , mm/min). Thus the design chosen was five level-three factor (5^3) full factorial design consisting of 125 sets of coded combinations for each work-piece material. Three cutting parameters are selected as design factors while other parameters have been assumed to be constant over the experimental domain. The upper and lower limits of a factor were coded as +1 and −1 respectively using Eq. 1.8. The process variables/design factors with their values on different levels are listed in Table 2.1 for three different work-piece materials.

2.2.2 Machine Used

The machine used for the milling tests is a ‘DYNA V4.5’ CNC end milling machine having the control system SINUMERIK 802 D with a vertical milling head. The specification of CNC end milling machine has been shown in Table 2.2. For generating the milled surfaces, CNC part programs for tool paths were created with specific commands. The compressed coolant servo-cut was used as cutting environment.

Table 2.2 Specification of CNC end milling machine

Table size	450 × 250 mm
Table load capacity	200 Kgs
X Travel	250 mm
Y Travel	175 mm
Z Travel	175 mm
Spindle nose to table	300 mm
Spindle centre to column	280 mm
Taper of spindle nose	BT 30
Spindle speed	9,000 rpm
Rapid on X and Y axis	15 m/min
Rapid on Z axis	10 m/min
Spindle motor	3.7 kW
X axis motor	3 Nm
Y axis motor	3 Nm
Z axis motor	6 Nm
Contro system	802 D SINUMERIK
Power requirement	7.5 kW/10 H.P.
Lubricating oil	Tellus 33 or EN KLO 68

2.2.3 Cutting Tool Used

Coated carbide tools are known to perform better than uncoated carbide tools. Thus commercially available CVD coated carbide tools were used in this investigation. The tools used were flat end mill cutters produced by WIDIA (EM-TiAlN). The tools were coated with TiAlN coating. For each material a new cutter of same specification was used. The details of the end milling cutters are given below:

Cutter diameter = 8 mm

Overall length = 108 mm

Fluted length = 38 mm

Helix angle = 30°

Hardness = 1,570 HV

Density = 14.5 g/cc

Transverse rupture strength = 3,800 N/mm²

2.2.4 Work-Piece Materials

The present study was carried out with three different materials, viz., 6061-T4 Aluminium, AISI 1040 steel and Medium leaded Brass UNS C34000. The chemical composition and mechanical properties of the work-piece materials are shown in Table 2.3. All the specimens were in the form of 100 × 75 × 25 mm blocks.

Table 2.3 Composition and mechanical properties of work-piece materials

Work material	Chemical composition (W%)	Mechanical property
Aluminium (6061-T4)	0.2%Cr, 0.3%Cu, 0.85%Mg, 0.04%Mn, 0.5%Si, 0.04%Ti, 0.25%Zn, 0.5%Fe and balance Al	Hardness—65 BHN, Density—2.7 g/cc, Tensile Strength—241 MPa
Brass (UNS C34000)	0.095%Fe, 0.9%Pb, 34%Zn and balance Cu	Hardness—68 HRF, Density—8.47 g/cc, Tensile strength—340 MPa
Mild Steel (AISI 1040)	0.42%C, 0.48%Mn, 0.17%Si, 0.02%P, 0.018%S, 0.1%Cu, 0.09%Ni, 0.07%Cr and balance Fe	Hardness—201 BHN, Density—7.85 g/cc, Tensile strength—620 MPa

2.3 Results and Discussion

CNC milling operations are carried out on mild steel, brass and aluminium work-pieces to get machined surfaces for different combinations of spindle speed, feed rate and depth of cut. The generated surfaces are measured using Talysurf instrument and further processed to get fractal dimension (D). Full factorial design of experiments is considered in the study and the experimental results are presented in Table 2.4.

The influences of the cutting parameters (d , N and f) on the profile fractal dimension D have been assessed for three different materials. The second order model was postulated in obtaining the relationship between the fractal dimension and the machining variables using response surface methodology (RSM). The analysis of variance (ANOVA) was used to check the adequacy of the second order model. The results for the three different materials are presented one by one.

2.3.1 RSM for Mild Steel

The second order response surface equation for the fractal dimension in mild steel milling is obtained in terms of coded values of design factors as:

$$D = 1.3836 + 0.0136d + 0.0115N + 0.0069f - 0.0063dN + 0.0003df - 0.0106Nf - 0.0283d^2 + 0.0169N^2 + 0.0032f^2 \quad (2.1)$$

Table 2.4 Experimental results for CNC milling considering full factorial design

Sl No	Depth of cut(d)	Spindle speed(N)	Feed rate(f)	<i>D</i> for mild steel	<i>D</i> for brass	<i>D</i> for aluminium
1	-1	-1	-1	1.31	1.28	1.34
2	-1	-1	-0.5	1.33	1.31	1.34
3	-1	-1	0	1.29	1.22	1.37
4	-1	-1	0.5	1.30	1.28	1.29
5	-1	-1	1	1.32	1.27	1.36
6	-1	-0.5	-1	1.29	1.30	1.38
7	-1	-0.5	-0.5	1.33	1.29	1.32
8	-1	-0.5	0	1.37	1.30	1.35
9	-1	-0.5	0.5	1.37	1.32	1.35
10	-1	-0.5	1	1.34	1.27	1.36
11	-1	0	-1	1.32	1.38	1.35
12	-1	0	-0.5	1.35	1.33	1.34
13	-1	0	0	1.38	1.31	1.33
14	-1	0	0.5	1.34	1.30	1.34
15	-1	0	1	1.39	1.31	1.34
16	-1	0.5	-1	1.38	1.36	1.35
17	-1	0.5	-0.5	1.36	1.33	1.36
18	-1	0.5	0	1.36	1.30	1.35
19	-1	0.5	0.5	1.40	1.31	1.34
20	-1	0.5	1	1.34	1.32	1.34
21	-1	1	-1	1.40	1.37	1.34
22	-1	1	-0.5	1.38	1.35	1.35
23	-1	1	0	1.41	1.34	1.35
24	-1	1	0.5	1.36	1.35	1.38
25	-1	1	1	1.37	1.30	1.38
26	-0.5	-1	-1	1.41	1.30	1.37
27	-0.5	-1	-0.5	1.39	1.27	1.36
28	-0.5	-1	0	1.35	1.26	1.35
29	-0.5	-1	0.5	1.39	1.25	1.31
30	-0.5	-1	1	1.38	1.28	1.37
31	-0.5	-0.5	-1	1.31	1.31	1.36
32	-0.5	-0.5	-0.5	1.37	1.29	1.39
33	-0.5	-0.5	0	1.40	1.31	1.31
34	-0.5	-0.5	0.5	1.41	1.29	1.35
35	-0.5	-0.5	1	1.40	1.29	1.32
36	-0.5	0	-1	1.38	1.38	1.35
37	-0.5	0	-0.5	1.32	1.34	1.34
38	-0.5	0	0	1.37	1.31	1.34
39	-0.5	0	0.5	1.39	1.28	1.34
40	-0.5	0	1	1.39	1.29	1.38
41	-0.5	0.5	-1	1.41	1.35	1.33
42	-0.5	0.5	-0.5	1.40	1.33	1.31
43	-0.5	0.5	0	1.36	1.32	1.37
44	-0.5	0.5	0.5	1.41	1.32	1.36

(continued)

Table 2.4 (continued)

Sl No	Depth of cut(d)	Spindle speed(N)	Feed rate(f)	<i>D</i> for mild steel	<i>D</i> for brass	<i>D</i> for aluminium
45	-0.5	0.5	1	1.36	1.29	1.35
46	-0.5	1	-1	1.38	1.37	1.36
47	-0.5	1	-0.5	1.38	1.37	1.35
48	-0.5	1	0	1.32	1.36	1.34
49	-0.5	1	0.5	1.39	1.32	1.34
50	-0.5	1	1	1.41	1.31	1.38
51	0	-1	-1	1.38	1.26	1.41
52	0	-1	-0.5	1.42	1.25	1.35
53	0	-1	0	1.43	1.26	1.37
54	0	-1	0.5	1.43	1.27	1.34
55	0	-1	1	1.41	1.29	1.35
56	0	-0.5	-1	1.38	1.36	1.36
57	0	-0.5	-0.5	1.41	1.27	1.36
58	0	-0.5	0	1.38	1.32	1.35
59	0	-0.5	0.5	1.38	1.27	1.39
60	0	-0.5	1	1.40	1.29	1.31
61	0	0	-1	1.38	1.35	1.34
62	0	0	-0.5	1.37	1.35	1.34
63	0	0	0	1.34	1.32	1.37
64	0	0	0.5	1.41	1.31	1.35
65	0	0	1	1.38	1.31	1.31
66	0	0.5	-1	1.40	1.36	1.28
67	0	0.5	-0.5	1.39	1.34	1.34
68	0	0.5	0	1.36	1.32	1.34
69	0	0.5	0.5	1.40	1.32	1.37
70	0	0.5	1	1.38	1.36	1.35
71	0	1	-1	1.43	1.37	1.36
72	0	1	-0.5	1.41	1.33	1.35
73	0	1	0	1.40	1.35	1.37
74	0	1	0.5	1.39	1.34	1.36
75	0	1	1	1.43	1.36	1.36
76	0.5	-1	-1	1.40	1.24	1.38
77	0.5	-1	-0.5	1.39	1.27	1.32
78	0.5	-1	0	1.38	1.23	1.29
79	0.5	-1	0.5	1.43	1.26	1.33
80	0.5	-1	1	1.38	1.28	1.32
81	0.5	-0.5	-1	1.39	1.27	1.38
82	0.5	-0.5	-0.5	1.35	1.27	1.38
83	0.5	-0.5	0	1.37	1.33	1.33
84	0.5	-0.5	0.5	1.40	1.25	1.33
85	0.5	-0.5	1	1.41	1.28	1.34
86	0.5	0	-1	1.35	1.38	1.33
87	0.5	0	-0.5	1.32	1.33	1.36
88	0.5	0	0	1.37	1.32	1.36

(continued)

Table 2.4 (continued)

Sl No	Depth of cut(d)	Spindle speed(N)	Feed rate(f)	<i>D</i> for mild steel	<i>D</i> for brass	<i>D</i> for aluminium
89	0.5	0	0.5	1.39	1.29	1.31
90	0.5	0	1	1.41	1.31	1.28
91	0.5	0.5	-1	1.36	1.39	1.33
92	0.5	0.5	-0.5	1.38	1.33	1.36
93	0.5	0.5	0	1.37	1.32	1.33
94	0.5	0.5	0.5	1.39	1.31	1.37
95	0.5	0.5	1	1.38	1.36	1.34
96	0.5	1	-1	1.44	1.36	1.34
97	0.5	1	-0.5	1.43	1.37	1.34
98	0.5	1	0	1.44	1.37	1.3
99	0.5	1	0.5	1.43	1.34	1.3
100	0.5	1	1	1.42	1.34	1.36
101	1	-1	-1	1.30	1.29	1.34
102	1	-1	-0.5	1.42	1.28	1.32
103	1	-1	0	1.38	1.26	1.32
104	1	-1	0.5	1.38	1.24	1.29
105	1	-1	1	1.39	1.26	1.36
106	1	-0.5	-1	1.35	1.31	1.37
107	1	-0.5	-0.5	1.38	1.28	1.24
108	1	-0.5	0	1.33	1.31	1.33
109	1	-0.5	0.5	1.36	1.27	1.33
110	1	-0.5	1	1.40	1.30	1.22
111	1	0	-1	1.39	1.37	1.36
112	1	0	-0.5	1.37	1.33	1.34
113	1	0	0	1.35	1.34	1.34
114	1	0	0.5	1.39	1.27	1.32
115	1	0	1	1.41	1.33	1.31
116	1	0.5	-1	1.40	1.39	1.32
117	1	0.5	-0.5	1.38	1.37	1.34
118	1	0.5	0	1.38	1.31	1.32
119	1	0.5	0.5	1.36	1.29	1.35
120	1	0.5	1	1.39	1.35	1.33
121	1	1	-1	1.41	1.37	1.35
122	1	1	-0.5	1.41	1.37	1.33
123	1	1	0	1.40	1.36	1.31
124	1	1	0.5	1.40	1.33	1.3
125	1	1	1	1.36	1.31	1.32

The developed model is checked for adequacy by ANOVA and F-test. Table 2.5 presents the ANOVA table for the second order model proposed for fractal dimension, *D* given in Eq. 2.1. It can be seen that the *P*-value is less than 0.05 which means that the model is significant at 95% confidence level. Also the

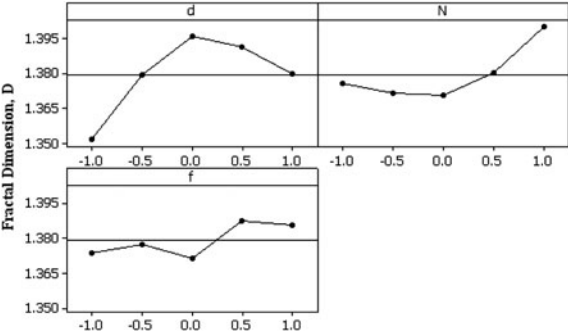
Table 2.5 ANOVA for second order model for *D* in CNC milling of mild steel

Source	Degrees of freedom	Sum of squares	Mean squares	$F_{\text{calculated}}$	$F_{0.05}$	P
Regression	9	0.051657	0.005740	8.25	1.96	0
Residual error	115	0.080004	0.000696			
Total	124	0.131661				

Table 2.6 ANOVA for model coefficients for *D* in CNC milling of mild steel

Source	Degrees of freedom	Sum of squares	Mean squares	$F_{\text{calculated}}$	$F_{0.05}$	P
d	4	0.0293648	0.0073412	13.15	2.52	0.000
N	4	0.0146848	0.0036712	6.58	2.52	0.000
f	4	0.0052688	0.0013172	2.36	2.52	0.063
d*N	16	0.0232112	0.0014507	2.60	1.82	0.004
d*f	16	0.0075072	0.0004692	0.84	1.82	0.636
N*f	16	0.0159072	0.0009942	1.78	1.82	0.054
Error	64	0.0357168	0.0005581			
Total	124	0.1316608				

Fig. 2.1 Main effect plot for mild steel



calculated value of the *F*-ratio is more than the standard value of the *F*-ratio for *D*. It means the model is adequate at 95% confidence level to represent the relationship between the machining response and the considered machining parameters of the CNC end milling process on mild steel. Table 2.6 represents the ANOVA table for individual model coefficients where it can be seen that there are three effects with a *P*-value less than 0.05 which means that they are significant at 95% confidence level. These significant effects are: depth of cut, spindle speed and the interaction between spindle speed and depth of cut. Figure 2.1 depicts the main effects plot for the fractal dimension and the design factors considered in the present study. From this figure also, it is seen that spindle speed and depth of cut have the significant effect on fractal dimension. To see the effects of process parameters on fractal dimension in the experimental regime, three dimensional surface as well as contour plots are presented at high level and low level of the parameters (Figs. 2.2, 2.3, 2.4).

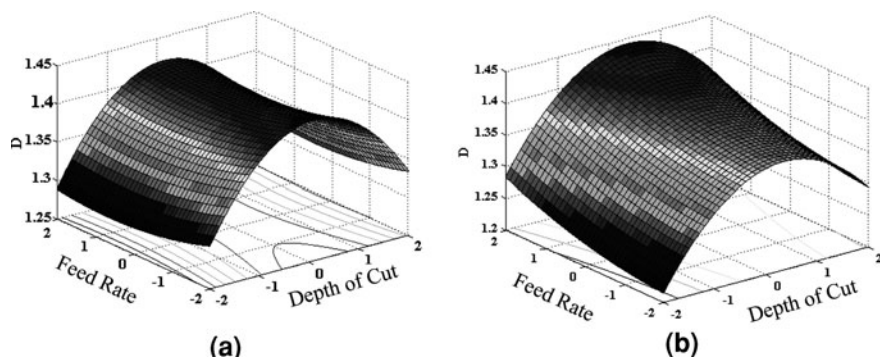


Fig. 2.2 Surface and contour plot of fractal dimension for mild steel: **a** at high level of spindle speed, **b** at low level of spindle speed

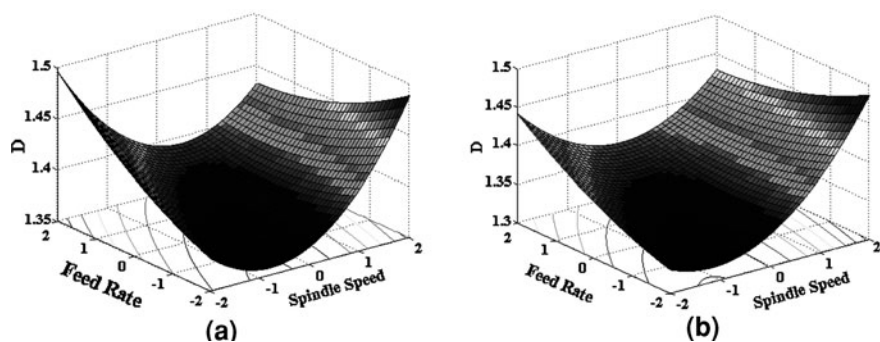


Fig. 2.3 Surface and contour plot of fractal dimension for mild steel: **a** at high level of depth of cut, **b** at low level of depth of cut

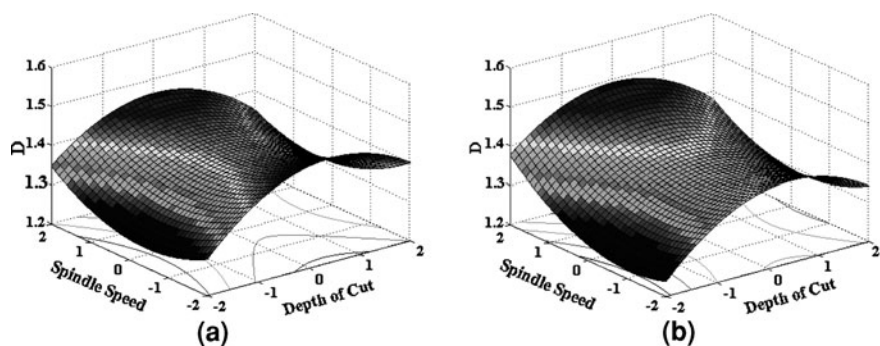


Fig. 2.4 Surface and contour plot of fractal dimension for mild steel: **a** at high level of feed rate, **b** at low level of feed rate

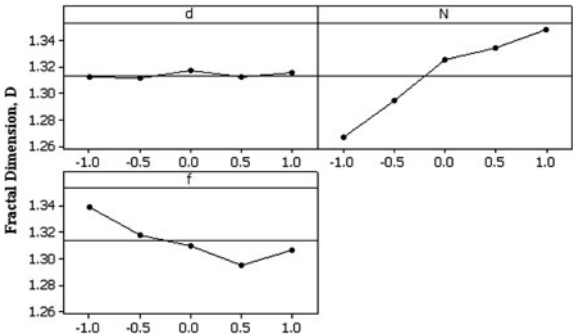
Table 2.7 ANOVA for second order model for *D* in CNC milling of brass

Source	Degrees of freedom	Sum of squares	Mean squares	$F_{\text{calculated}}$	$F_{0.05}$	P
Regression	9	0.138293	0.015366	36.35	1.96	0
Residual Error	115	0.048614	0.000423			
Total	124	0.186907				

Table 2.8 ANOVA for model coefficients for *D* in CNC milling of brass

Source	Degrees of freedom	Sum of squares	Mean squares	$F_{\text{calculated}}$	$F_{0.05}$	P
d	4	0.0006512	0.0001628	0.61	2.52	0.654
N	4	0.1095792	0.0273948	103.26	2.52	0.000
f	4	0.0264432	0.0066108	24.92	2.52	0.000
d*N	16	0.0043968	0.0002748	1.04	1.82	0.433
d*f	16	0.0092528	0.0005783	2.18	1.82	0.015
N*f	16	0.0196048	0.0012253	4.62	1.82	0.000
Error	64	0.0169792	0.0002653			
Total	124	0.1869072				

Fig. 2.5 Main effect plot for brass



2.3.2 RSM for Brass

The second order response surface equation for fractal dimension in brass milling is obtained in terms of coded values of design factors as:

$$D = 1.3130 + 0.0015d + 0.0408N - 0.0175f + 0.0071dN + 0.0014df - 0.0098Nf - 0.0008d^2 - 0.0142N^2 + 0.0163f^2 \tag{2.2}$$

The developed model is checked for adequacy by ANOVA and F-test. Table 2.7 presents the ANOVA table for the second order model proposed for *D* given in Eq. 2.2. It can be seen that the *P*-value is less than 0.05 which means that the model is significant at 95% confidence level. Also the calculated value of the *F*-ratio is more than the standard value of the *F*-ratio for *D*. It means the model is adequate at 95% confidence level to represent the relationship between the

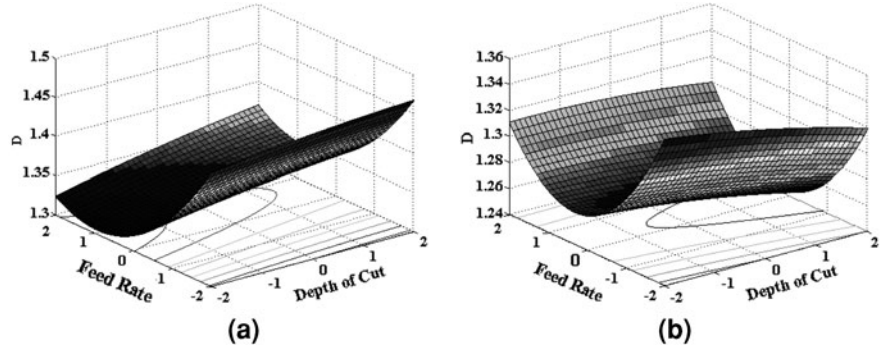


Fig. 2.6 Surface and contour plot of fractal dimension for brass: **a** at high level of spindle speed, **b** at low level of spindle speed

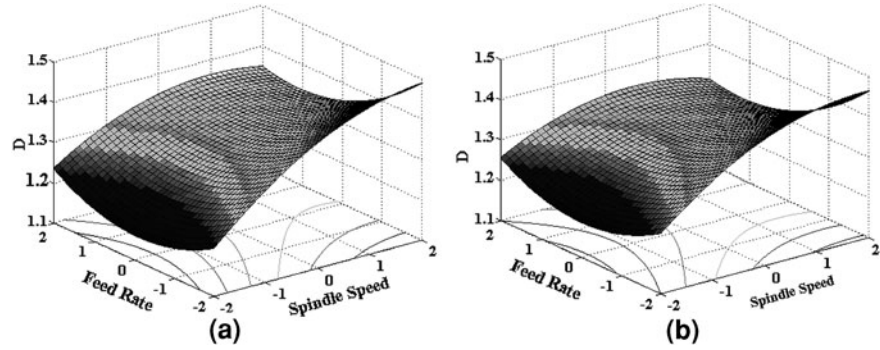


Fig. 2.7 Surface and contour plot of fractal dimension for brass: **a** at high level of depth of cut, **b** at low level of depth of cut

machining response and the considered machining parameters of the CNC end milling process on brass. Table 2.8 represents the ANOVA table for individual model coefficients where it can be seen that spindle speed, feed rate, the interaction between spindle speed and feed rate and the interaction of depth of cut and feed rate are significant factors at 95% confidence level. Figure 2.5 depicts the main effects plot for the fractal dimension and the design factors considered in the present study. From this figure also, it is seen that spindle speed and feed rate have the significant effect on fractal dimension. Figures 2.6, 2.7, 2.8 show the estimated three-dimensional surface as well as contour plots for fractal dimension as functions of the independent machining parameters. All these figures clearly depict the variation of fractal dimension with controlling variables within the experimental regime.

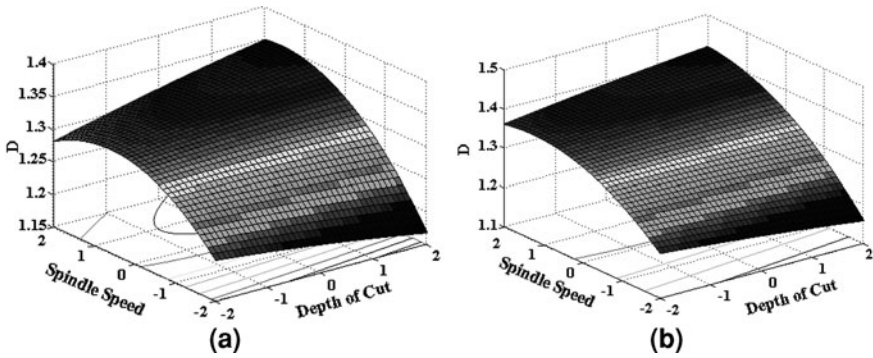


Fig. 2.8 Surface and contour plot of fractal dimension for brass: **a** at high level of feed rate, **b** at low level of feed rate

Table 2.9 ANOVA for second order model for D in CNC milling of aluminium

Source	Degrees of freedom	Sum of squares	Mean squares	$F_{\text{calculated}}$	$F_{0.05}$	P
Regression	9	0.025241	0.002805	4.5	1.96	0
Residual error	115	0.0717	0.000624			
Total	124	0.096941				

2.3.3 RSM for Aluminium

The second order response surface equation has been fitted using Minitab software for the response variable D . The equation can be given in terms of the coded values of the independent variables as:

$$D = 1.3433 - 0.0128d + 0.0013N - 0.0062f - 0.0011dN - 0.0095df + 0.0122Nf - 0.0135d^2 + 0.0041N^2 + 0.0056f^2 \quad (2.3)$$

Table 2.9 presents the ANOVA table for the second order model proposed for D given in Eq. 2.3. It can be appreciated that the P -value is less than 0.05 which means that the model is significant at 95% confidence level. Also the calculated value of the F -ratio is more than the standard value of the F -ratio for D . It means the model is adequate at 95% confidence level to represent the relationship between the machining response and the considered machining parameters of the CNC end milling process. Table 2.10 represents the ANOVA table for individual model coefficients where it can be seen that depth of cut and the interaction between spindle speed and feed rate are significant at 95% confidence level. Figure 2.9 depicts the main effects plot for the fractal dimension and the design factors considered in the present study. From this figure also, it is seen that depth of cut has the significant effect on fractal dimension. Figures 2.10, 2.11, 2.12 show the estimated three-dimensional surface as well as contour plots for fractal

Table 2.10 ANOVA for model coefficients for *D* in CNC milling of aluminium

Source	Degrees of freedom	Sum of squares	Mean squares	$F_{\text{calculated}}$	$F_{0.05}$	P
d	4	0.0146608	0.0036652	6.76	2.52	0.000
N	4	0.0004928	0.0001232	0.23	2.52	0.922
f	4	0.0032048	0.0008012	1.48	2.52	0.219
d*N	16	0.0110272	0.0006892	1.27	1.82	0.243
d*f	16	0.0102352	0.0006397	1.18	1.82	0.307
N*f	16	0.0226432	0.0014152	2.61	1.82	0.003
Error	64	0.0346768	0.0005418			
Total	124	0.0969408				

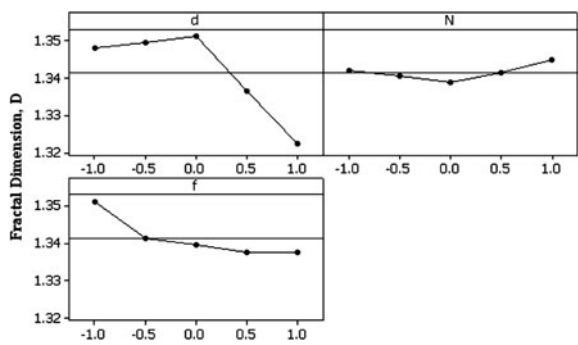


Fig. 2.9 Main effect plot for aluminium

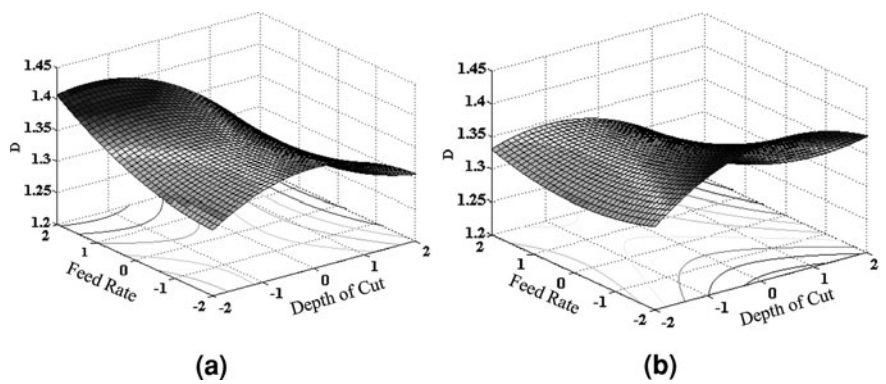


Fig. 2.10 Surface and contour plot of fractal dimension for aluminium: **a** at high level of spindle speed, **b** at low level of spindle speed

dimension as functions of the independent machining parameters. All these figures clearly depict the variation of fractal dimension with controlling variables within the experimental regime.

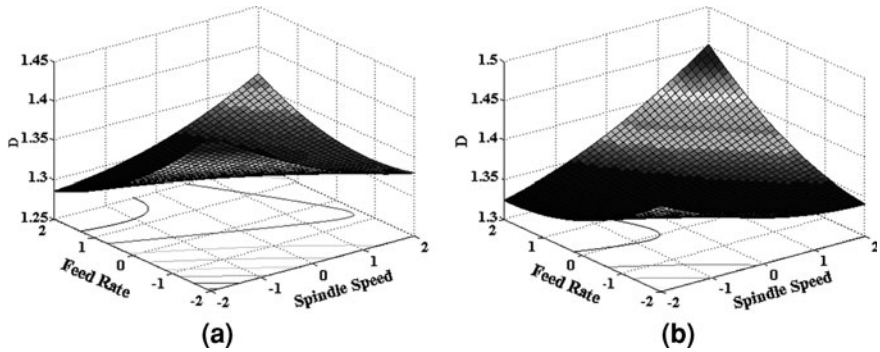


Fig. 2.11 Surface and contour plot of fractal dimension for aluminium: **a** at high level of depth of cut, **b** at low level of depth of cut

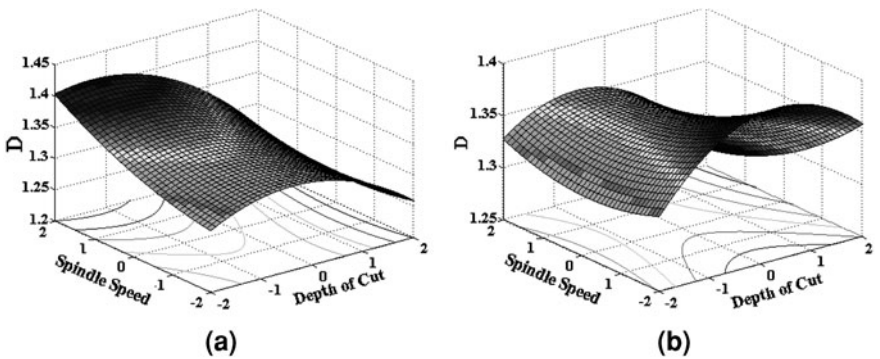


Fig. 2.12 Surface and contour plot of fractal dimension for aluminium: **a** at high level of feed rate, **b** at low level of feed rate

2.4 Closure

For three different work-piece materials, fractal dimension models are developed in CNC end milling using response surface method. The second order response models have been validated with analysis of variance. A comparison of the response surface models for fractal dimension in different materials reveals the fact that these models are material specific or in other words, the tool-work-piece material combination plays a vital role in fractal dimension of the generated surface profile. Also the effect of the cutting parameters on fractal dimension is different for different materials as evidenced from Tables 2.6, 2.8 and 2.10. Accordingly, optimum machining parameter combinations for fractal dimension depend greatly on the work-piece material within the experimental domain.

However, it can be concluded that it is possible to select a combination of spindle speed, depth of cut and feed rate for achieving the surface topography with desired fractal dimension within the constraints of the available machine. Thus with the known boundaries of desired fractal dimension and machining parameters, machining can be performed with a relatively high rate of success.



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