Preface

Models in mathematical finance, for example stock price processes, are often defined in continuous-time. Hence optimization problems like consumption-investment problems lead to stochastic control problems in continuous-time. However, only a few of these problems can be solved explicitly. When numerical methods have to be applied, it is sometimes wise to start with a process in discrete-time, as done for example in the *approximating Markov chain approach*. The resulting optimization problem is then a *Markov Decision Problem* and there is a rich toolbox available for solving these kind of problems theoretically and numerically.

The purpose of this book is to present selected parts of the theory of *Markov Decision Processes* and show how they can be applied in particular to problems in finance and insurance. We start by explaining the theory for problems with finite time horizon. Since we have financial applications in mind and since we do not want to restrict to binomial models we have to work with Borel state and action spaces. This framework is also needed for studying *Partially Observable Markov Decision Processes* and *Piecewise Determinsitic Markov Decision Processes*. In contrast to the case of a discrete (finite or countable) state space the theory is more demanding since non-trivial measurability problems have to be solved. However, we have decided to circumvent these kind of problems by introducing a so-called *structure assumption* for the model. The advantage is that in applications this structure assumption is often easily verified and avoids some of the technicalities. This makes the book accessible to readers who are not familiar with general probability and measure theory. Moreover, we present numerous different applications and show how this structure assumption can be verified. Applications range from consumption-investment problems, mean-variance problems, dividend problems in risk theory to indifference pricing and pricing of American options, just to name a few. The book is unique in the presentation and collection of these financial applications. Some of them appear for the first time in a book.

We also consider the theory of infinite horizon *Markov Decision Processes* where we treat so-called *contracting* and *negative* Markov Decision Problems in a unified framework. *Positive* Markov Decision Problems are also presented as well as *stopping problems*. A particular focus is on problems with *partial observation*. These kind of problems cover situations where the decision maker is not able to observe all driving factors of the model. Special cases are Hidden Markov Models and Bayesian Decision Problems. They include statistical aspects, in particular *filtering theory* and can be solved by so-called *filtered Markov Decision Processes*. Moreover *Piecewise Deterministic Markov Decision Processes* are discussed and we give recent applications to finance.

It is our aim to present the material in a mathematically rigorous framework. This is not always easy. For example, the last-mentioned problems with partial observation need a lot of definitions and notation. However each chapter on theory is followed by a chapter with applications and we give examples throughout the text which we hope shed some light on the theory. Also at the end of each chapter on theory we provide a list of exercises where the reader can test her knowledge.
Having said all this, not much general probability and optimization theory is necessary to enjoy this book. In particular we do not need the theory of stochastic calculus which is a necessary tool for continuous-time problems. The reader should however be familiar with concepts like *conditional expectation* and *transition kernels*. The only exception is Section 2.4 which is a little bit more demanding. Special knowledge in finance and insurance is not necessary. Some fundamentals are covered in the appendix. As outlined above we provide an example-driven approach. The book is intended for graduate students, researchers and practitioners in mathematics, finance, economics and operations research. Some of the chapters have been tried out in courses for masters students and in seminars.
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