

Chapter 1

Intense-Field Dirac Theory of Ionization of Hydrogen and Hydrogenic Ions in Infrared and Free-Electron Laser Fields

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Abstract We investigate ionization of Dirac hydrogen atom and hydrogenic ions of charge states Z using the relativistic intense-field theory of spin-resolved ionization process. We discuss the energy dependence, emission angle dependence, spin dependence and charge state Z dependence of ionization rates for free-electron laser (FEL) and infrared frequencies. The results of relativistic intense-field model calculations are compared with the corresponding nonrelativistic calculations, and in specific cases also with the results of relativistic single-photon first Born approximation. Furthermore, we discuss two counterintuitive predictions made earlier: (a) an asymmetry of the up and down spin electron currents from *unpolarized* target H-atoms, and (b) an increase of the ionization probability with the increase of the ionization potential in certain domains of the charge states Z for a given laser intensity and frequency.

1.1 Introduction

Laser fields with intensities as high as $10^{21} \text{ W cm}^{-2}$ or more have become available recently (e.g., [1, 2]). At such intensities, even nonrelativistic bound electrons are accelerated to velocities comparable to the light velocity c , and the dynamics of electrons bound in atoms, molecules, clusters or solids could be strongly modified. Unlike in the traditional quantum electrodynamical problems, which are concerned with weak electromagnetic fields and spontaneous emission/absorption processes, where the electron motion can be treated successfully by relativistic perturbation theory [3], for highly intense laser fields the usual perturbation treatment

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breaks down. The breakdown intensity, e.g., for optical/near-infrared wavelengths is approximately about $10^{13} \text{ W cm}^{-2}$ [4].

One usually defines a super-intense laser field as one for which the mean quiver energy of a free electron in the field, known as the ponderomotive potential, U_p , becomes comparable to or greater than the rest mass energy of the electron, i.e., $U_p \equiv \frac{e^2 F^2}{4m\omega^2} \geq mc^2$, where F is the peak value of the electric field strength. In such fields, in general a huge number of photons can be exchanged significantly in a transition process, and the usual non-perturbative approach of direct time-dependent simulation, e.g., of the corresponding Dirac equation of the system, or the Floquet expansion technique in the case of periodic fields, can be extremely arduous if not impossible. In the former case the space-time grid, and in the latter case the size of the Floquet matrix, would tend to be too large to be practicable. One alternative possibility of investigating relativistic dynamics in super-intense laser fields is to use the relativistic generalization of the so-called intense-field KFR (Keldysh–Faisal–Reiss)-like approximation (e.g., review [5]). In this approximation, *all* order interactions of the field with a relativistic free electron are first accounted for by using the Volkov solution of the Dirac equation of a free electron in a plane wave electromagnetic field [6]. This solution satisfies the final state plane wave boundary condition when the interaction with the field is over. In contrast, for an electron in a Coulomb potential plus the laser field, no such exact solution is known. A systematic approximation of the wavefunction of the *interacting system* satisfying such a typical bound state initial condition, nevertheless, can be developed within the so-called KFR-ansatz (e.g., [5]).

One aspect of the intense-field laser–atom interaction in the relativistic domain is the coupling of the photon field with the spin degrees of freedom. The latter gives rise to phenomena-like stimulated Mott-scattering [7–9] and other spin-dependent bound state effects (e.g., [10]) and the ubiquitous phenomenon of ionization in intense fields. In the latter context, most intense-field calculations of ionization probability in relativistic domains do not provide specific spin information since the rates are mostly obtained using the convenient tracing rules. The electrons emitted from target atoms in the initial bound states and/or in the final free states of course could be prepared and/or detected with or without spin selection. Thus for the ionization process, in strong fields the rates of spin-flips and the characters of up and down spin currents can be of much experimental interest. Intense-field 4-component Dirac analysis of such spin-dependence of the ionization rates has been reported in [11] that revealed among other things a remarkable spin *asymmetry* in the ionization currents even from spin *unpolarized* ground state H-atom. The asymmetry had been predicted to survive also for ionization by weak and long wavelength photons that possess a negligible photon momentum. Another relativistic aspect in ionization process in super-intense fields is the effect of retardation due to the finite velocity of light or the effect due to a finite photon momentum (that goes beyond the usual dipole approximation of the field). This can lead to deviations from the well-known return of the electron to the core along the polarization direction responsible for the various re-scattering phenomena in intense fields (e.g., [5]), due to the transverse effect of the Lorentz force exerted by the nonnegligible magnetic field

of light. An interesting prediction of an earlier Klein–Gordon theoretical relativistic model calculation [12] has been *anomalous effect* in which the probability of ionization increases with increase of the ionization potential of the target atom and ion in certain domain of Z , keeping the intensity and the frequency of the laser field the same.

Here, we shall discuss relativistic ionization processes in Hydrogen atom and hydrogenic ions theoretically, keeping the above and related aspects of the process in mind. To begin with, in the next section, we give a systematic expansion of the strong-field Dirac relativistic wavefunction of the interacting system satisfying the initial *bound* state condition. Next, we apply the leading term of the wavefunction to obtain an analytic relativistic expression of the ionization amplitudes for a general elliptically polarized laser field of arbitrary frequency and intensity. Then the ionization rates for all combinations of the spin-specific (up or down spin) ground state of the atom or ion and the (up or down spin) free state of the ejected electron are obtained. Results of specific numerical calculations for the interesting case of a circularly polarized field with a given helicity are presented and discussed, and a summary of the results is given in the end.

1.2 Relativistic Theory of Intense-Field Processes

The Dirac equation of the system of atom (or ion) + laser field is

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = H_D(t) \Psi(t), \quad (1.1)$$

where the total Dirac Hamiltonian of the system is

$$H_D(\mathbf{x}, t) = c\boldsymbol{\alpha} \cdot \left(-i\hbar \frac{\partial}{\partial \mathbf{x}} - \frac{e_0}{c} \mathbf{A}(\mathbf{x}, t) \right) + V(\mathbf{x}) + \beta m_0 c^2. \quad (1.2)$$

We have denoted the external electromagnetic vector potential of an elliptically polarized intense laser field (ellipticity parameter $\xi [-\pi/2, \pi/2]$) by

$$\mathbf{A}(\mathbf{x}, t) = A_0(x) (\boldsymbol{\epsilon}_1 \cos(\xi/2) \cos(\omega t - \boldsymbol{\kappa} \cdot \mathbf{x}) - \boldsymbol{\epsilon}_2 \sin(\xi/2) \sin(\omega t - \boldsymbol{\kappa} \cdot \mathbf{x})), \quad (1.3)$$

where $A_0(x)$ is the envelope and $\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2$ are the orthogonal unit polarization vectors along the semi-major and the semi-minor axes, respectively, of the laser field. The corresponding scalar potential $\mathcal{A}_0(\mathbf{x})$ is defined by the potential energy for the hydrogenic atom (or ion) of charge Z : $V(\mathbf{x}) \equiv e_0 \mathcal{A}_0(\mathbf{x}) = -\frac{Ze_0^2}{|\mathbf{x}|}$. On introducing the Dirac–Feynman four-vector notations: $x \equiv (x_0, \mathbf{x})$, $x_0 = ct$, $|\mathbf{x}| = r$; $p \equiv (p_0, \mathbf{p})$, $a \cdot b = a_\mu b_\mu = a_0 b_0 - \mathbf{a} \cdot \mathbf{b}$, $\not{a} = \gamma_\mu a_\mu$, $\bar{u} = u^\dagger \gamma_0$, γ_μ are Dirac γ -matrices, and defining the constants $e \equiv \frac{e_0}{\hbar c}$ and $m \equiv \frac{m_0 c}{\hbar}$, the Dirac equation above can be rewritten in the covariant form:

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - e\gamma_\mu \mathcal{A}_\mu(x) \right) \Psi(t) = m\Psi(t). \quad (1.4)$$

Note that in the present notation only two constants e and m appear in the Dirac equation of interest, instead of the original four constants. Moreover, at the end of any calculation if desired the natural constants could be restored by simply substituting them in place of e and m . In Feynman slash notation, the Dirac equation becomes

$$(i \not{\partial} - e \not{\mathcal{A}}(x) - m)\Psi(t) = 0, \quad (1.5)$$

where $\not{\partial} \equiv \gamma_\mu \frac{\partial}{\partial x_\mu}$ and $\not{\mathcal{A}} \equiv \gamma_\mu \mathcal{A}_\mu$. For a later reference, we formally also define here the total Green's function associated with the same total Dirac Hamiltonian by the inhomogeneous equation

$$(i \not{\partial} - e \not{\mathcal{A}}(x) - m)G(x, x') = \delta^4(x - x'). \quad (1.6)$$

1.2.1 The Initial and Final Rest-Interaction Hamiltonians

The unperturbed equation satisfied by the initial bound state $\phi^{(0)}$ in the Coulomb potential of the nucleus is

$$(i \not{\partial} - \gamma_0 U(x) - m)\phi^{(0)}(x) = 0. \quad (1.7)$$

Therefore, the initial state rest-interaction Hamiltonian is given by $V_i(x) = e \not{\mathcal{A}}(x)$, where we have introduced the four-vector notation: $A(x) = (0, \mathbf{A}(x))$, for the external vector potential only. We take the final-state as the plane wave solution of the Dirac-Volkov equation

$$(i \not{\partial} - e \not{\mathcal{A}}(x) - m)\phi(t) = 0, \quad (1.8)$$

and, hence, the final state rest-interaction Hamiltonian is $V_f(x) = \gamma_0 U(x)$, with $U(x) \equiv -\frac{Z\alpha}{|x|}$.

1.2.2 Relativistic Volkov Solutions

The relativistic plane wave Volkov solutions [6] of (1.8) for an electron in the four-momentum state $p = (p_0, \mathbf{p})$ can be conveniently expressed with the help of the quantity

$$\phi_p(x) = \sqrt{\frac{m}{E}} e^{-ip \cdot x - i f_p(x)} \left(1 + \frac{e \not{\epsilon} \not{\mathcal{A}}(x)}{2\kappa \cdot p} \right) \hat{u}_p, \quad (1.9)$$

where $E \equiv p_0 = +\sqrt{(\mathbf{p}^2 + m^2)}$, and

$$f_p(x) = \frac{1}{2\kappa \cdot p} \int^{\kappa \cdot x} [(2eA(\xi) \cdot p - e^2 A^2(\xi))] d\xi, \quad (1.10)$$

$$\hat{u}_p = \frac{\not{p} + m}{\sqrt{2m(m + E)}}, \quad (1.11)$$

$$u_p^{(s)} = \hat{u}_p w^{(s)}, \quad (1.12)$$

where the 4-spinors $w^{(s)}$ ($s = 1, 2, 3, 4$) are defined by

$$w^{(1=\uparrow)} = (1000)^\dagger \quad (1.13)$$

$$w^{(2=\downarrow)} = (0100)^\dagger \quad (1.14)$$

$$w^{(3=\uparrow)} = (0010)^\dagger \quad (1.15)$$

$$w^{(4=\downarrow)} = (0001)^\dagger. \quad (1.16)$$

The two positive energy Volkov solutions are given by

$$\phi_p^{(s)}(x) = \phi_p(x) w^{(s)} \quad (s = 1, 2), \quad (1.17)$$

and the two negative energy Volkov solutions are given by

$$\phi_p^{(s)}(x) = \phi_{-p}(x) w^{(s)} \quad (s = 3, 4). \quad (1.18)$$

1.2.3 Volkov–Feynman Propagator

The associated relativistic Green's function of an electron interacting with the external electromagnetic field is defined by the inhomogeneous equation

$$(i \not{\partial} - e \not{A}(x) - m) G_F(x, x') = \delta^4(x - x'), \quad A(x) = (0, \mathbf{A}(x)). \quad (1.19)$$

Its solution satisfying the Feynman–Stückelberg boundary condition (that automatically incorporates the interpretation of the positron states as the negative energy electron states evolving backward in time) is given by the Volkov–Feynman propagator:

$$\begin{aligned} G_F(x, x') = & -i \frac{m}{E} \theta(x_0 - x'_0) \sum_{s=1,2} \left[\int \frac{d^3 p}{(2\pi)^3} \phi_p^{(s)}(x) \bar{\phi}_p^{(s)}(x') \right] \\ & + i \frac{m}{E} \theta(x'_0 - x_0) \sum_{s=3,4} \left[\int \frac{d^3 p}{(2\pi)^3} \phi_p^{(s)}(x) \bar{\phi}_p^{(s)}(x') \right]. \end{aligned} \quad (1.20)$$

1.2.4 Floquet Representation of Relativistic Volkov Solutions

We note that for a vector potential of a constant envelope the phase function $f_p(x)$ reduces to

$$f_p(x) = -a_p \sin(u + \chi_p) + b_p \sin(2u) + \lambda_p u, \quad (1.21)$$

where

$$\begin{aligned} u &= \kappa \cdot x \\ &= \omega t - \kappa \cdot \mathbf{x} \end{aligned} \quad (1.22)$$

$$a_p = \frac{\Lambda(\mathbf{p}, \xi) e A_0}{\kappa \cdot p} \quad (1.23)$$

$$\Lambda(\mathbf{p}, \xi) = \mathbf{p} \cdot \boldsymbol{\epsilon}_1 \cos(\xi/2) \sqrt{1 + (\tan \phi_p \tan(\xi/2))^2} \quad (1.24)$$

$$\tan \phi_p = \frac{\mathbf{p} \cdot \boldsymbol{\epsilon}_2}{\mathbf{p} \cdot \boldsymbol{\epsilon}_1} \quad (1.25)$$

$$\chi_p = \tan^{-1}[\tan \phi_p \tan(\xi/2)] \quad (1.26)$$

$$b_p = \frac{(e A_0)^2}{8 \kappa \cdot p} \cos \xi \quad (1.27)$$

$$\lambda_p = \frac{(e A_0)^2}{4 \kappa \cdot p} \quad (1.28)$$

$$\kappa \cdot p = \kappa_0 p_0 - \kappa \cdot \mathbf{p}, \quad (1.29)$$

and the light four-wavevector $\kappa = (\kappa_0, \boldsymbol{\kappa})$, $\kappa_0 = \omega/c$. Using the Jacobi–Anger relation

$$e^{iz \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\theta}, \quad (1.30)$$

in $e^{-i f_p(x)}$ appearing above, combining it with the factors in (1.9) and shifting the index n as required we obtain the Floquet representation of the relativistic Volkov solution (cf. [13]),

$$\phi_p^{(s)}(x) = \sum_{n=-\infty}^{\infty} \sqrt{\frac{m}{E}} e^{-i(p + \lambda_p \kappa - n \kappa) \cdot x} Q_n(p) u_p^{(s)}, \quad (1.31)$$

where

$$Q_n(p) = \left(J_n + \frac{e A_0}{4 \kappa \cdot p} (\not{\epsilon}(\xi) J_{n-1} + \not{\epsilon}^*(\xi) J_{n+1}) \right), \quad (1.32)$$

and J_n stands for the generalized Bessel function of three arguments

$$J_n \equiv J_n(a_p, b_p, \chi_p) = \sum_{m=-\infty}^{\infty} J_{n+2m}(a_p) J_m(b_p) e^{(n+2m)\chi_p}, \quad (1.33)$$

and, $u_p^{(s)} = \hat{u}_p w^{(s)}$ for $(s = 1, 2)$ are the positive energy Dirac spinors. The corresponding negative energy Volkov solutions, $\phi_p^{(s)}(x)$ for $(s = 3, 4)$, are defined as above but by replacing $p \rightarrow -p$ and $w^{(s=1,2)} \rightarrow w^{(s=3,4)}$.

1.2.5 Floquet Representation of Volkov–Feynman Propagator

A useful Floquet representation of the Volkov–Feynman Green’s function, for a constant amplitude vector potential

$$A(x) = A_0(\epsilon_1 \cos \xi \cos(\kappa \cdot x + \delta) - \epsilon_2 \sin \xi \cos(\kappa \cdot x + \delta)), \quad (1.34)$$

where $\delta[0, 2\pi]$ is an arbitrary phase, can be derived [14] by first expanding the Floquet solution of (1.19) as

$$G_F(x, x') = \sum_{n=-\infty}^{\infty} e^{in\kappa \cdot x} G_{n,0}(x, x'). \quad (1.35)$$

Thus, writing a factor of unity on the right-hand side of (1.19) as $1 = \sum_{n=-\infty}^{\infty} e^{in\delta} \delta_{n,0}$, and substituting it and (1.35) in (1.19) and projecting with respect to δ , or equating the coefficients of $e^{in\delta}$ on both sides, we get the Floquet–Volkov propagator equation

$$\left(i \not{\partial} - n \not{\epsilon} - \frac{eA_0}{2} (\not{\epsilon}(\xi) a_n^- + \not{\epsilon}^*(\xi) a_n^+) - m \right) G_{n,0}(x, x') = \delta_{n,0} \delta^4(x - x') \quad (1.36)$$

with

$$\epsilon(\xi) = \epsilon_1 \cos(\xi/2) + i\epsilon_2 \sin(\xi/2), \quad (1.37)$$

where $a_n^\pm$ simply increases (+), or decreases (−), the index n of $G_{n,0}$ (or any other quantity with index n) on its right. It is interesting to note that $a_n^\pm$ are in fact the semiclassical analogs of the quantum creation and annihilation operators acting on the quantum number state $|n\rangle$, and replace the corresponding quantum operators very well for large initial occupation number of the field, i.e., in the so-called “laser approximation” ([4] Sect. 6.4).

The plane wave solutions of the homogeneous Floquet–Volkov equation (1.36) (with the right hand = 0) are (cf. [14]),

$$\psi_n^{(s)}(p; x) = e^{-ip \cdot x} \sqrt{\frac{m}{E}} Q_n(p) u_p^{(s)}, \quad (1.38)$$

which can be verified by substitution and using the properties of the generalized Bessel functions (cf. Table I, [15]). They satisfy the useful bi-orthonormal relations:

$$\sum_{n=-\infty}^{\infty} \sum_{s=1}^4 \int d^4x \bar{\psi}_{n-N'}^{(s)}(p'; x) \psi_{n-N}^{(s)}(p; x) = (2\pi)^4 \delta(p - p') \delta_{N, N'} \quad (1.39)$$

$$\sum_{N=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \psi_{n-N}^{(s)}(p; x) \bar{\psi}_{n'-N}^{(s')}(p; x') = \delta(x - x') \delta_{s, s'} \delta_{n, n'}, \quad (1.40)$$

which may be readily proved on using the orthonormality and completeness properties (cf. Table I, [15]) of the generalized Bessel functions J_{n-N} of the plane wave states and of the Dirac spinors. Therefore, following [14] the solution of the propagator equation (1.36), satisfying the Feynman–Stückelberg boundary condition, can be written as

$$G_{n, n'}(x, x') = \sum_{N=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - x')} Q_{n-N}(p) \frac{\not{p}_N + m}{p_N^2 - m^2 + i0} Q_{n'-N}^\dagger(p), \quad (1.41)$$

where $Q_n(p)$ are given by (1.32), and the four-vector $p_N \equiv p + \lambda_p \kappa - N\kappa$. Note that in this form the relativistic Floquet–Volkov propagator strongly resembles the usual Feynman propagator [3] to which it reduces in the absence of the field (for $A_0 = 0$). It permits one to extend the usual Feynman covariant perturbative technique in a rather analogous way to processes in the presence of an intense external field. Finally, the Floquet representation of the Green's function $G_F(x, x')$ for a constant envelope time periodic field is given by the simple substitution of $G_{n,0}(x, x')$ from the above (1.41) into (1.35):

$$\begin{aligned} G_F(x, x') &= \sum_{n=-\infty}^{\infty} e^{in\kappa \cdot x} \sum_{N=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - x')} Q_{n-N}(p) \frac{\not{p}_N + m}{p_N^2 - m^2 + i0} Q_{-N}^\dagger(p). \end{aligned} \quad (1.42)$$

1.2.6 Dirac Wavefunction of the Interacting System

To derive a systematic approximation of the wavefunction of the interacting system, we rewrite the Dirac equation (1.4) in the integral equation form as

$$\Psi(x) = \phi^{(0)}(x) + \int d^4x' G(x, x') V_i(x') \phi^{(0)}(x'), \quad (1.43)$$

where, $G(x, x')$ is the total Green's function of the system, $\phi^{(0)}(x)$ is the unperturbed initial state in the absence of the laser field, and $V_i(x')$ is the laser–matter interaction Hamiltonian in the initial state. The total Green's function $G(x, x')$ can

be expanded in terms of the final state Volkov–Feynman propagator $G_F(x, x')$ as

$$G(x, x') = G_F(x, x') + \int d^4x'' G_F(x, x'') V_f(x'') G_F(x'', x') + \dots \quad (1.44)$$

Substituting the above expansion in the equation for the wavefunction (1.43), we get

$$\begin{aligned} \Psi(x) = & \phi^{(0)}(x) + \int d^4x' G_F(x, x') V_i(x') \phi^{(0)}(x') \\ & + \int \int d^4x'' d^4x' G_F(x, x'') V_f(x'') G_F(x'', x') V_i(x') \phi^{(0)}(x') + \dots \end{aligned} \quad (1.45)$$

Thus, we get the laser-modified Dirac wavefunction for the interacting system in the form:

$$\Psi(x) = \phi^{(0)}(x) + \Psi^{(1)}(x) + \Psi^{(2)}(x) + \dots, \quad (1.46)$$

where the first-order (KFR1) wavefunction is given by

$$\Psi^{(1)}(x) = \int d^4x' G_F(x, x') V_i(x') \phi_i^{(0)}(x'), \quad (1.47)$$

and the second-order (KFR2) correction is given by

$$\Psi^{(2)}(x) = \int d^4x'' d^4x' G_F(x, x'') V_f(x'') G_F(x'', x') V_i(x') \phi_i^{(0)}(x'), \quad (1.48)$$

and so on.

1.3 Relativistic Ionization Amplitudes

To obtain the transition amplitude (or the S-matrix element) for ionization, we project the final Volkov state of a given momentum $\mathbf{p}$ on to the change of the wavefunction from the initial state ($\Psi(x) - \phi^{(0)}(x)$) due to the laser interaction. Thus, the amplitude for ionization is

$$\begin{aligned} \mathcal{A}_{\text{fi}} = & \int d^4x \bar{\phi}_p(x) (\Psi(x) - \phi^{(0)}(x)) \\ = & \int d^4x \bar{\phi}_p(x) (\Psi^{(1)}(x) + \Psi^{(2)}(x) + \dots) \\ = & -i \int d^4x \bar{\phi}_p(x) e \mathcal{A}(x) \phi^{(0)}(x) - i \int \int d^4x d^4x' \bar{\phi}_p(x) \gamma_0 U(x) G_F(x, x') \\ & \times e \mathcal{A}(x') \phi^{(0)}(x') + \dots, \end{aligned} \quad (1.49)$$

where we have used the orthogonality of the Volkov wavefunctions and the relation

$$\bar{\phi}_p(x) = \phi_p^\dagger(x)\gamma_0, \quad (1.50)$$

which is the Dirac-adjoint of the Volkov wavefunction with four-momentum p . The lowest order relativistic intense-field amplitude (denoted as KFR1) is therefore given by the leading term

$$\mathcal{A}_{\text{if}}^{(1)} = -i \int d^4x \bar{\phi}_p(x) e \mathcal{A}(x) \phi_i^{(0)}(x), \quad (1.51)$$

and the second-order amplitude (denoted as KFR2) is given by

$$\mathcal{A}_{\text{if}}^{(2)} = -i \int \int d^4x d^4x' \bar{\phi}_p(x) \gamma_0 U(x) G_F(x, x') e \mathcal{A}(x') \phi_i^{(0)}(x'), \quad (1.52)$$

and so on.

In the sequel, we shall restrict ourselves to the lowest order (KFR1) amplitude for bound-free transitions, (1.51). The evaluation of the four-dimensional integrations can be carried out analytically [11]. The results are given for the ionization amplitudes in terms of simple algebraic formulas involving explicit functions. They permit us to readily calculate not only the energy and angle differential and integrated rates but also the individual *spin-resolved* ionization probabilities in intense fields [11]. The analytical formulas are given below for the *general* case of an *elliptically* polarized radiation field, which automatically reduces to the important special cases of linear and circular polarizations for the ellipticity parameter $\xi = 0$ and $\xi = \pm\pi/2$ (right, left) helicity. We use the expressions to investigate the energy spectra, angular distributions, the spin currents, and the total rates of ionization at both infrared and XUV frequencies. They are applied to study the dependence of the ionization probability of H and a sequence of hydrogenic ions. Results of specific relativistic calculations are compared with the corresponding nonrelativistic calculations.

1.4 Analytic Evaluations

In this section, we evaluate the lowest order (KFR1) ionization amplitude in greater detail and give the analytical formulas for the corresponding spin-specific ionization transition amplitudes and rates.

1.4.1 Covariant Expressions for Ground-State Dirac Wavefunction and Its Fourier Transform

To evaluate the relativistic ionization amplitudes, it is found very useful first to express the Dirac hydrogenic bound state wavefunction of charge state Z in a covariant form as ([16] p. 391):

$$\phi_{1s}^{(s)}(\mathbf{r}) = N_{1s} r^{\gamma'-1} e^{-p_B r} \not{r}_\mu(\hat{\mathbf{r}}) w^{\uparrow, \downarrow} \quad (s = 1, 2), \quad (1.53)$$

$$N_{1s} = (2p_B)^{\gamma'+\frac{1}{2}} \left(\frac{1 + \gamma'}{8\pi\Gamma(1 + 2\gamma')} \right)^{\frac{1}{2}}$$

$$p_B = mZ\alpha$$

$$\gamma' = \sqrt{1 - (Z\alpha)^2}, \quad (1.54)$$

$$\beta' = (1 - \gamma')/(Z\alpha), \quad (1.55)$$

$$\not{r}_\mu = \gamma_\mu n_\mu$$

$$n_\mu = (n_0, \mathbf{n}) \equiv (1, i\beta'\hat{\mathbf{r}}). \quad (1.56)$$

Next, it is useful to derive the Fourier transform (FT) of the bound state also in the covariant form. To this end, we obtain the following two integrals:

$$c_0(p) = \int d^3r e^{-i\mathbf{p}\cdot\mathbf{r}} r^{\gamma'-1} e^{-p_B r}$$

$$= \frac{4\pi}{p} \frac{\Gamma(\gamma' + 1)}{(p_B^2 + p^2)^{\frac{\gamma'+1}{2}}} s_0(p), \quad (1.57)$$

$$s_0(p) = \sin \left((\gamma' + 1) \tan^{-1} \frac{p}{p_B} \right), \quad (1.58)$$

and

$$g_1(\mathbf{p}) = \int d^3r e^{-i\mathbf{p}\cdot\mathbf{r}} r^{\gamma'-1} e^{-p_B r} \boldsymbol{\gamma} \cdot \hat{\mathbf{r}}$$

$$= i c_1(p) \boldsymbol{\gamma} \cdot \hat{\mathbf{p}}, \quad (1.59)$$

$$c_1(p) = \frac{4\pi}{p} \frac{\Gamma(1 + \gamma')}{(p^2 + p_B^2)^{\frac{\gamma'+1}{2}}} \left((p_B/p) s_0(p) - \frac{\gamma' + 1}{\gamma'} (1 + (p_B/p)^2)^{\frac{1}{2}} s_1(p) \right)$$

$$s_1(p) = \sin \left(\gamma' \tan^{-1} \frac{p}{p_B} \right). \quad (1.60)$$

Thus, the FT of Dirac H atom ground state:

$$\tilde{\phi}_{1s}^{(s)}(\mathbf{p}) = \int d^3r e^{-i\mathbf{p}\cdot\mathbf{r}} \phi_{1s}^{(s)}(\mathbf{r})$$

$$= N_{1s} (c_0(p) + \beta' c_1(p) \boldsymbol{\gamma} \cdot \hat{\mathbf{p}}) w^{\uparrow, \downarrow}$$

$$= N_{1s} c_0(p) \not{b}_\mu(\mathbf{p}) w^{\uparrow, \downarrow} \quad (s = 1, 2), \quad (1.61)$$

where

$$\begin{aligned} b_\mu(\mathbf{p}) &= (1, \mathbf{b}(\mathbf{p})) \\ \not{b} &= \gamma_\mu b_\mu \\ \mathbf{b}(\mathbf{p}) &= -g(p) \hat{\mathbf{p}}, \\ g(p) &= \beta' \left(\frac{p_B}{p} - \frac{\gamma' + 1}{\gamma'} (1 + (p_B/p)^2)^{\frac{1}{2}} \frac{\sin(\gamma' \tan^{-1}(p/p_B))}{\sin((\gamma' + 1) \tan^{-1}(p/p_B))} \right) \\ w^\uparrow &= w^{(s=1)} \\ w^\downarrow &= w^{(s=2)}. \end{aligned} \quad (1.62)$$

1.4.2 Explicit Formulas: Spin-Specific Ionization Amplitudes and Rates

We may now evaluate the transition amplitude for ionization (1.51) and the probability per unit time (or rates) of ionization.

1.4.2.1 Explicit Ionization Amplitudes

The amplitudes are obtained by substituting the ground-state wavefunction (1.53) and the Volkov solution (1.31) in (1.51), performing the dx_0 integration in terms of the delta-function and the spatial integration using the FT of the ground state, (1.61). Thus, for the transition between the spin states $s \rightarrow s'$, one gets:

$$\mathcal{A}_{s \rightarrow s'}^{(1)} = -2\pi i \sum_n \delta(p_0 + p_0^B + \lambda_p \kappa_0 - n \kappa_0) \times T_{s \rightarrow s'}^{(n)}(\mathbf{q}), \quad (1.63)$$

where $p_0^B \equiv \sqrt{m^2 - p_B^2}$, $p_B = mZ\alpha$, and

$$T_{s \rightarrow s'}^{(n)}(\mathbf{q}) = \frac{eA_0}{2} N(E) \bar{u}_p^{(s')} \not{B}_n^* \tilde{\phi}_{1s}^{(s)}(\mathbf{q}), \quad s, s' = (u, d) \quad (1.64)$$

$$= \frac{eA_0}{2} N(E) N_{1s} c_0(\mathbf{q}) t_{s \rightarrow s'}^{(n)}(\mathbf{q}), \quad (1.65)$$

$$t_{s \rightarrow s'}^{(n)}(\mathbf{q}) = \bar{u}_p^{(s')} \not{B}_n^* \not{b} w^{(s)}. \quad (1.66)$$

The four vectors $B_n = (B_n^0, \mathbf{B}_n)$ are defined in terms of the generalized Bessel functions J_n , (1.33),

$$B_n^0 = \frac{eA_0\kappa_0}{4\kappa \cdot p} (2J_n + \cos \xi (J_{n+2} + J_{n-2})) , \quad (1.67)$$

$$\mathbf{B}_n = \boldsymbol{\epsilon}(\xi)J_{n-1} + \boldsymbol{\epsilon}^*(\xi)J_{n+1} + \hat{\kappa} B_n^0 \quad (1.68)$$

$$\boldsymbol{\epsilon}(\xi) = \boldsymbol{\epsilon}_1 \cos(\xi/2) + i\boldsymbol{\epsilon}_2 \sin(\xi/2) \quad (1.69)$$

$$\kappa \cdot p = \kappa_0 p_0 - \boldsymbol{\kappa} \cdot \mathbf{p} . \quad (1.70)$$

We list the other symbols together:

$$\mathbf{q} \equiv q\hat{\mathbf{q}} = \mathbf{p} + (\lambda_p - n) \boldsymbol{\kappa} \quad (1.71)$$

$$\gamma' = \sqrt{1 - (Z\alpha)^2} , \quad (1.72)$$

$$\beta' = (1 - \gamma')/(Z\alpha) , \quad (1.73)$$

$$N_{1s} = (2p_B)^{\gamma' + \frac{1}{2}} \left(\frac{1 + \gamma'}{8\pi\Gamma(1 + 2\gamma')} \right)^{\frac{1}{2}} , \quad (1.74)$$

$$m_1 = \sqrt{(E + m)/(2m)} , \quad (1.75)$$

$$m_2 = -\sqrt{(E - m)/(2m)} , \quad (1.76)$$

$$N_E = \sqrt{\frac{m}{E}} , \quad (1.77)$$

$$E = +\sqrt{\mathbf{p}^2 + m^2} ; \quad (1.78)$$

$$\kappa_0 = \frac{\omega}{c} \quad (1.79)$$

$$\boldsymbol{\kappa} = \kappa_0 \hat{\boldsymbol{\kappa}} \quad (1.79)$$

$$\kappa = (\kappa_0, \boldsymbol{\kappa}) . \quad (1.80)$$

Finally, evaluating the matrix elements explicitly (with respect to $w^{(s)}, w^{(s')}$) for $s, s' = (u, d)$, we get the algebraic expressions for the four reduced matrix elements $t_{s \rightarrow s'}^{(n)}$ [11]:

$$\begin{aligned} t_{u \rightarrow u}^{(n)} &= B_n^{0*} (m_1 + m_2 g(q) (\hat{\mathbf{p}} \cdot \hat{\mathbf{q}} + i (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_z)) \\ &\quad + \mathbf{B}_n^* \cdot (m_2 \hat{\mathbf{p}} + m_1 g(q) \hat{\mathbf{q}}) \\ &\quad - i (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_z \end{aligned} \quad (1.81)$$

$$\begin{aligned} t_{u \rightarrow d}^{(n)} &= m_2 g(q) B_n^{0*} (i (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_x - (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_y) \\ &\quad - i (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_x \\ &\quad + (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_y \end{aligned} \quad (1.82)$$

$$\begin{aligned} t_{d \rightarrow u}^{(n)} &= m_2 g(q) B_n^{0*} (i (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_x + (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_y) \\ &\quad - i (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_x \\ &\quad - (\mathbf{B}_n^* \times (m_2 \hat{\mathbf{p}} - m_1 g(q) \hat{\mathbf{q}}))_y , \end{aligned} \quad (1.83)$$

and

$$\begin{aligned}
 t_{d \rightarrow d}^{(n)} = & B_n^{0*} (m_1 + m_2 g(q) (\hat{\mathbf{p}} \cdot \hat{\mathbf{q}} - i (\hat{\mathbf{p}} \times \hat{\mathbf{q}})_z)) \\
 & + \mathbf{B}_n^* \cdot (m_2 \hat{\mathbf{p}} + m_1 g(q) \hat{\mathbf{q}}) \\
 & + i \left(\mathbf{B}_n^* \times (m_2 \hat{\mathbf{k}} - m_1 g(q) \hat{\mathbf{q}}) \right)_z.
 \end{aligned} \tag{1.84}$$

1.4.3 Spin-Specific Ionization Rates

The differential rates of ionization by absorption of n laser photons from the bound spin states $s = (u = \uparrow, d = \downarrow)$ to the continuum spin states $s' = (u, d)$ can be easily expressed in terms of the absolute square of the transition amplitude (1.63), divided by the effectively long interaction time τ , and summed over the final wavenumber states. To this end, we use a convenient representation of the square of the 1-dimensional delta-function

$$\begin{aligned}
 \delta^2(k - k') &= \delta(k - k') \times \frac{1}{2\pi} \lim_{\tau \rightarrow \infty} \int_{-c\tau/2}^{c\tau/2} e^{i(k-k')x_0} dx_0 \\
 &= \delta(k - k') \times \lim_{\tau \rightarrow \infty} \frac{1}{2\pi} (c\tau).
 \end{aligned} \tag{1.85}$$

Thus, we get the differential rate $dW^{(n)}$ of ionization:

$$\begin{aligned}
 dW_{s \rightarrow s'}^{(n)} &= \left(\frac{eA_0}{2} N(E) N_{1s} c_0(q) \right)^2 \left| t_{s \rightarrow s'}^{(n)}(\mathbf{q}) \right|^2 \\
 &\times (2\pi) c \delta(p_0 + p_0^B + \lambda_p \kappa_0 - n \kappa_0) \frac{1}{(2\pi)^3} d^3 p.
 \end{aligned} \tag{1.86}$$

Finally, noting $d^3 p = p^2 d|p| d\Omega_p$, and performing the $d|p|$ integration $[0, \infty]$, the rate of ionization for the final electron momentum $\mathbf{p}$ in the solid angle element $d\Omega_p$ takes the simple form [11]:

$$\frac{dW_{s \rightarrow s'}}{d\Omega_p}(\mathbf{p}) = \sum_{n \geq n_0} \left(\frac{eA_0}{2} N(E) N_{1s} c_0(q) \right)^2 \left| t_{s \rightarrow s'}^{(n)}(\mathbf{q}) \right|^2 c \frac{p_0 |p|}{(2\pi)^2}. \tag{1.87}$$

In the above, the minimum integer value of $n = n_0$ is determined by the energy-momentum conservation relation,

$$n\omega = \epsilon_B + \epsilon_{\text{kin}} + \lambda_p \omega, \tag{1.88}$$

where $\epsilon_B = m_0 c^2 \left(1 - \sqrt{1 - \left(\frac{p_B}{m}\right)^2}\right)$ is the binding energy (or ionization potential), and $\epsilon_{\text{Kin}} = m_0 c^2 \left(\sqrt{1 + \left(\frac{p}{m}\right)^2} - 1\right)$ is the kinetic energy. We note here parenthetically that because of the implicit p -dependence of λ_p , (1.88) is an implicit equation for n that requires to be solved for its root before the contributions of all allowed $n \geq n_0$, where n_0 is the minimum allowed value of n , can be calculated. The other parameters are

$$p_0 \equiv \sqrt{m^2 + p^2} = (n - \lambda_p) \kappa_0 + \sqrt{m^2 - p_B^2} \quad (1.89)$$

$$\mathbf{q} = |\mathbf{q}| \hat{\mathbf{q}} \equiv \mathbf{p} + (\lambda_p - n) \boldsymbol{\kappa}. \quad (1.90)$$

1.4.3.1 Special Polarizations

The above results hold for the general case of elliptical polarization. The results for the two most common cases of linear and circular polarization of the field are readily obtained from above by simply setting the ellipticity parameter $\xi = 0$ and $\xi = \frac{\pi}{2}$ (right helicity), $-\frac{\pi}{2}$ (left helicity), respectively.

Linear Polarization

The same rate formulas for the reduced amplitudes (1.81)–(1.84) hold, except that, since in this case $\xi = 0$, therefore

$$a_p(\xi = 0) = \frac{\mathbf{p} \cdot \boldsymbol{\epsilon}_1 e A_0}{\kappa \cdot \mathbf{p}} \quad (1.91)$$

$$\chi_p(\xi = 0) = 0 \quad (1.92)$$

$$b_p = \frac{e A_0^2}{8 \kappa \cdot \mathbf{p}}. \quad (1.93)$$

Thus, the generalized Bessel function of three arguments (1.33) now simplifies to the generalized Bessel function of two arguments, i.e.,

$$J_n = J_n \left(\frac{\mathbf{p} \cdot \boldsymbol{\epsilon}_1 A_0}{c \kappa \cdot \mathbf{p}}, \frac{e A_0^2}{8 \kappa \cdot \mathbf{p}} \right). \quad (1.94)$$

We note that in this case the quantities B_n^0 and $\mathbf{B}_n$ appearing in the corresponding reduced transition matrix elements are real and hence the amplitudes $t_{u \rightarrow u}^{(n)}$ and $t_{d \rightarrow d}^{(n)}$ are complex conjugate of each other. Thus, we can readily conclude that for linear polarization of the laser field, the symmetric $u \rightarrow u$ and $d \rightarrow d$ ionization rates must be equal. Similar conclusions hold for the spin-flip transitions $u \rightarrow d$ and $d \rightarrow u$.

Circular Polarization

The same formulas (1.81)–(1.84) for the reduced amplitudes hold except that, since in this case $\xi = \pm\pi/2$,

$$a_p(\xi = \pm\pi/2) = \frac{|\mathbf{p}| \sin \theta_p A_0}{c\kappa \cdot \mathbf{p}}, b_p = 0 \quad (1.95)$$

$$\chi(\xi = \pm\pi/2) = \pm\phi_p, \quad (1.96)$$

therefore the generalized Bessel function of three arguments now simplifies to an ordinary Bessel function with a phase factor, i.e.,

$$J_n = J_n \left(\frac{|\mathbf{p}| \sin \theta_p A_0}{c\kappa \cdot \mathbf{p}} \right) e^{\pm i n \phi_p}. \quad (1.97)$$

In this case, the corresponding expressions of the spin-flip reduced matrix elements $|t_{u \rightarrow d}^{(n)}|^2 \neq |t_{d \rightarrow u}^{(n)}|^2$, which implies an interesting asymmetry for the spin-flip ionization rates with respect to the photon propagation direction.

We may note also that the influence of relativity in the rates of ionization given above arises, first, from the appearance of the relativistic energy-momentum conservation, second, from the nonnegligible momentum of the photon and, third, from the spin degrees of freedom. The first two factors affect the arguments of the Bessel functions in B_n^0 and B_n that determine the reduced amplitudes, $t_{s,s'}^{(n)}$, and in the dressed-momentum $\mathbf{q} = \mathbf{p} + (\lambda_p - n)\kappa$, which is “dressed” by the field strength via the parameter λ_p . The influence of the spin degrees of freedom arises from the 4-component spinor character of both the free-state spinor and the bound-state spinor, and more interestingly depends on the parameter m_2 of the free-electron Dirac spinor, and the parameter β' (that multiply the term $g(p)$) that appears in the “weak” component of the ground-state spinor. They are small in the low velocity nonrelativistic limits but become important in high Z ions and for the inner-shells of heavy atoms.

1.4.4 The Unpolarized Rate of Ionization

The spin-averaged total ionization rate from an unpolarized target atom can be easily obtained by simply adding the four spin-specific rates given above and dividing by 2 (for the average with respect to the two equally probable initially occupied spin states):

$$\frac{dW^{(\text{ion})}}{d\Omega_p} = \sum_{n \geq n_0} \left(\frac{eA_0}{2} N(E) N_{1s} c_0(q) \right)^2 c \frac{p_0 |\mathbf{p}|}{(2\pi)^2} \times \frac{1}{2} \sum_{(s,s')=u,d} \left| t_{s \rightarrow s'}^{(n)}(\mathbf{q}) \right|^2. \quad (1.98)$$

We may note in passing that for the nonrelativistic ground $1s$ -state of H atom, $\gamma' = 1$ and for the ground s -state of a zero-range potential, $\gamma' = 0$. Before ending this section, we should point out that in the nonrelativistic limit (and in the dipole approximation), (1.98) reduces to the corresponding nonrelativistic KFR1 rates (e.g., Equation (44) in [5]).

1.5 Applications

We apply the formulas given above to study the angle and energy differential ionization rates, the total ionization rates, the spin-specific ionization currents, spin-flip asymmetry, spin-averaged asymmetry parameter, and the charge-state (Z) dependence of intense-field ionization process, in the case a circularly polarized incident laser field. This case is particularly interesting from the point of view of the interplay between the spin polarization and the laser helicity. To be specific and to bring out the mutual dependence of helicity of the laser photons and the helicity of the electron spin, we choose the incident laser field to be circularly polarized (cf. Fig. 1.1) with the laser propagation vector, κ , along the z -axis and the unit polarization vector ϵ with orthogonal components along the x and y directions ($\epsilon_1 = \epsilon_x, \epsilon_2 = \epsilon_y$); the momentum $\mathbf{p}$, and the two possible spin states up ($u = \uparrow$) and down ($d = \downarrow$) are also indicated. We choose the laser frequency to be in the infrared, UV and XUV or free-electron laser (FEL) domains or more specifically, a typical Ti-sapphire laser frequency $\omega = 1.55$ eV, as well as a set of higher frequencies, $\omega = 5, 10, 27.2, 54.4$, and 200 eV.

For the purpose of numerical calculations, in this section we conveniently use the atomic units (a.u.): $e_0 = m_0 = \hbar = \alpha c = 1$ and hence in a.u. $e = 1/c = \alpha = 137.036 \dots$, and $m = c$. The peak amplitude of the vector potential A_0 (in a.u.) is related to the peak electric field strength F_0 (in a.u.), and the laser intensity I (in W cm^{-2}) by:

$$A_0 = \frac{c}{\omega} F_0 \quad (1.99)$$

$$F_0 = \sqrt{I/I_a}, \quad (1.100)$$

$$I_a = (3.51 \dots) \times 10^{16} \text{ W cm}^{-2}. \quad (1.101)$$

Fig. 1.1 Geometry indicating the propagation vector κ and the polarization components ϵ_x, ϵ_y of a circularly polarized laser field, as well as the momentum $\mathbf{p}$ and spin states ($\uparrow, \downarrow$) of the emitted electron

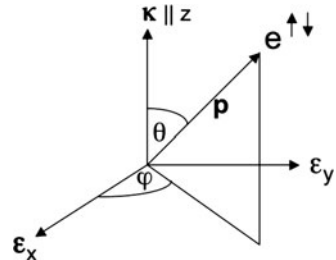
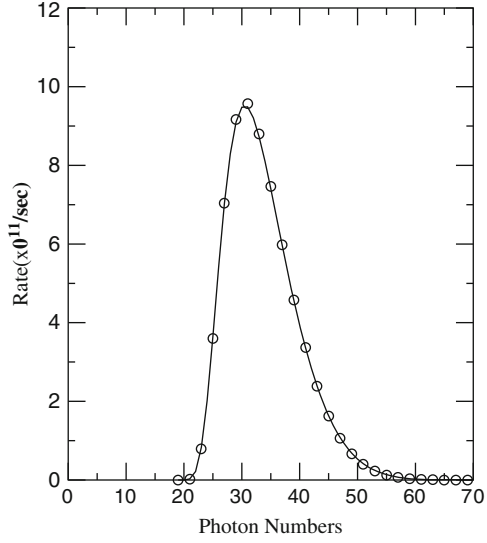


Fig. 1.2 Ionization rate: emitted electron energy spectrum (in units of photon energy) at $\omega = 200$ eV, $I = 10^{21}$ W cm $^{-2}$: intense-field relativistic case (solid line), and nonrelativistic case (open circles)



1.5.1 Ionization Rate: Energy Distribution

In Fig. 1.2, we show the results of calculations for the energy distribution of the electrons from ionization of Hydrogen atom at a free-electron laser frequency $\omega = 200$ eV and an intensity $I = 10^{21}$ W cm $^{-2}$. It can be seen (photoelectron energy in units of the photon energy) that the probability of ionization per second, reaches its maximum at an energy far above the value of $n = 1$ – far above the allowed (relativistic ionization threshold ≈ 13.6 eV) single-photon transition energy. This is a typical characteristic of intense-field ionization in circularly polarized fields, which is accompanied by absorption of a large number of photons *and* angular momenta. In the figure, we have compared the results of the present intense-field relativistic model (solid line) and the corresponding nonrelativistic Schrödinger calculations (open circles) using (44) given in [5]. It is interesting to observe that despite the “high” intensity, the nonrelativistic calculation at this XUV frequency is virtually indistinguishable (in the scale of the figure) from that of the relativistic theory. This implies that effectively this combination of nominally high intensity (higher than most current high intensity infrared or optical laser field) the electron motion throughout is essentially nonrelativistic in nature. This is due to the inverse frequency-square dependence of the characteristic ponderomotive energy $U_p = I/4\omega^2$ (a.u.) (that corresponds to the mean kinetic energy of a free electron in a laser field). The result at a higher intensity of $I = 10^{22}$ W cm $^{-2}$ is shown in Fig. 1.3. It can be seen from the figure that although still the relativistic and the nonrelativistic distributions peak near the same energy, the maximum height is now visibly lower in the relativistic case (solid line) than in the nonrelativistic calculation (open circles). A systematic variation of the peak of the energy distributions as a function of intensity can be seen very clearly in Fig. 1.4 that shows the results of relativistic

Fig. 1.3 Ionization rate: emitted electron energy spectrum (in units of photon energy) at $\omega = 200$ eV, $I = 10^{22}$ W cm $^{-2}$: intense-field relativistic case (solid line), and nonrelativistic case (dashed line)

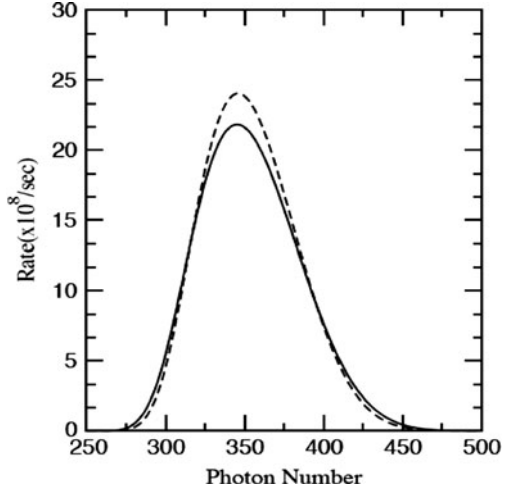
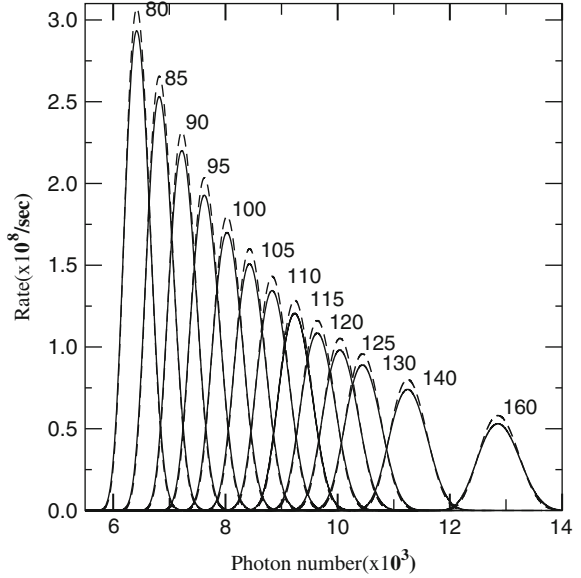


Fig. 1.4 Ionization rate: electron energy spectra (in units of photon energy) at $\omega = 5$ eV and a sequence of intensities $I = (80-160)$ a.u. $= (80-160) \times 3.51 \times 10^{16}$ W cm $^{-2}$. Note how the peak values decrease systematically with increasing intensity for both relativistic (solid line) and nonrelativistic cases (dashed line)



(solid lines) and nonrelativistic calculations (open circles) for a whole sequence of intensities in the range $I = 80-160$ a.u. $= (80-160) \times 3.51 \times 10^{16}$ W cm $^{-2}$, for a fixed laser frequency $\omega = 5$ eV. We note that the sequence of peaks of the distributions occur at an energy of the order of U_p . This is a typical characteristic of electron energy spectrum produced by a circular polarization of the field.

1.5.2 Ionization Rate: Angular Distribution

Next, we compare the angular distributions of the emitted electron in the relativistic and nonrelativistic cases. Typical results are shown in Figs. 1.5 and 1.6. They correspond to the same laser parameters as assumed for the energy distributions shown in Figs. 1.2 and 1.3, respectively. Noting that the angles of emission are measured here from the propagation direction of the laser field, a shift of the relativistic

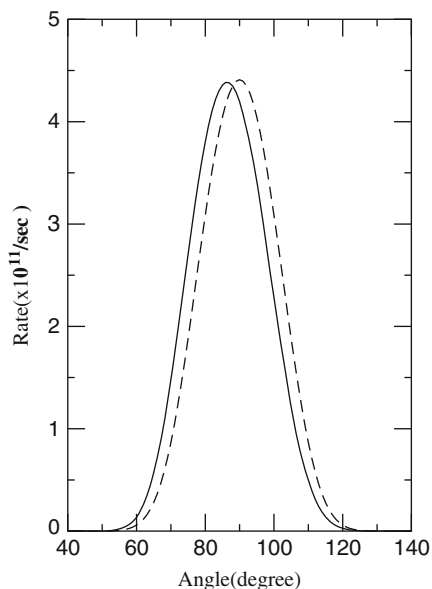


Fig. 1.5 Ionization rate: comparison of angular distributions at $\omega = 200$ eV, $I = 10^{21}$ W cm $^{-2}$: intense-field relativistic Dirac (solid line), nonrelativistic Schrödinger (dashed line)

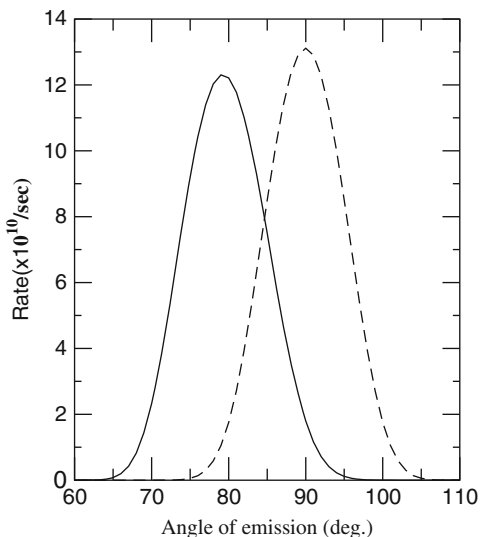


Fig. 1.6 Ionization rate: comparison of angular distributions at $\omega = 200$ eV, $I = 10^{22}$ W cm $^{-2}$: intense-field relativistic Dirac (solid line), nonrelativistic Schrödinger case (dashed line). Note the focusing effect toward the smaller angle

distribution (solid line) toward a smaller angle compared to the nonrelativistic case (dashed line) can be seen to occur in Fig. 1.5. This is a manifestation of the general relativistic *focusing* effect toward the propagation direction (cf., e.g., [5]), that in fact becomes more significant with the increase of the field intensity. This can be seen more prominently from the distribution at a higher intensity ($10^{22} \text{ W cm}^{-2}$) shown in Fig. 1.6. Again one finds that the highest rate is lower in the relativistic case than in the nonrelativistic case.

In Fig. 1.7, we compare the angular distributions calculated at $\omega = 1 \text{ a.u.} = 27.2 \text{ eV}$ and intensity $I = 0.01 \text{ a.u.} = 3.51 \times 10^{14} \text{ W cm}^{-2}$ calculated in three different ways. It can be seen that there is virtually no distinction, at this relatively low intensity, between the intense-field relativistic (solid line) and the nonrelativistic (dashed line) calculations. Moreover, they are essentially equal to the rates given by the single-photon relativistic Born approximation (crosses). The results for the same frequency but for a two orders of magnitude higher intensity ($I = 3.51 \times 10^{16} \text{ W cm}^{-2}$) are shown in Fig. 1.8. It can be seen that the intense-field relativistic (solid line) and the nonrelativistic (dots) distributions are still essentially equal to each other. However, now, they both differ significantly from the results of the single-photon relativistic Born approximation (crosses) and overestimates the peak rate considerably indicating the failure of the perturbation theory.

1.5.3 Total Ionization Rates: Intensity Dependence

We next consider the total ionization probability per unit time (the total rate), which is obtained by integrating the distributions over all angles of emission at a

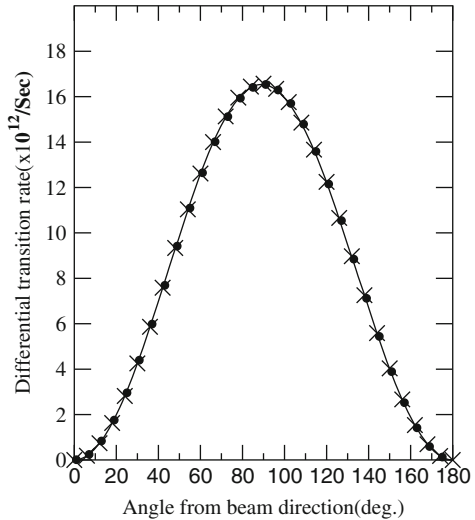


Fig. 1.7 Ionization rate: comparison of angular distributions at $\omega = 27.2 \text{ eV}$, $I = 3.51 \times 10^{14} \text{ W cm}^{-2}$: intense-field Dirac (*solid line*), nonrelativistic Schrödinger (*dots*), and single-photon relativistic Born approximation (*crosses*)

Fig. 1.8 Ionization rate: comparison of angular distributions at $\omega = 1 \text{ a.u.} = 27.2 \text{ eV}$ and $I = 1 \text{ a.u.} = 3.51 \times 10^{16} \text{ W cm}^{-2}$: intense-field Dirac (*solid line*), nonrelativistic Schrödinger (*dots*), and single-photon relativistic Born approximation (*crosses*)

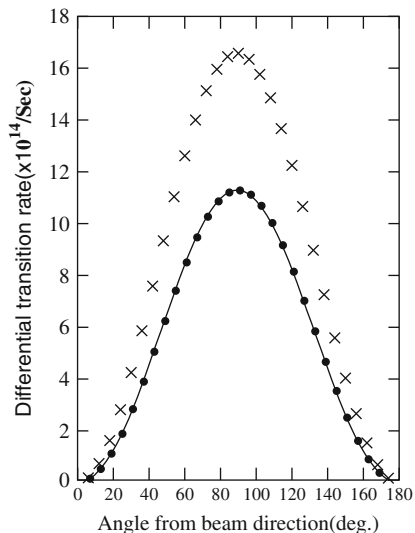
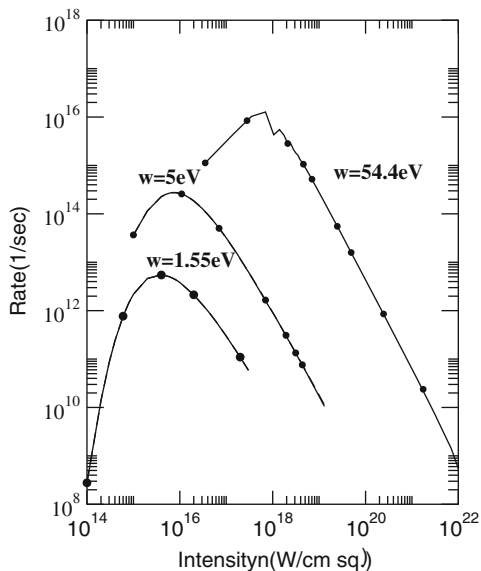


Fig. 1.9 Total ionization rate for H atom at $\omega = 1.55, 5$, and 54.4 eV , as a function of intensity in the range $10^{14} \text{ W cm}^{-2}$ – $10^{22} \text{ W cm}^{-2}$. Note the counter intuitive decrease of ionization probability with increasing intensity above the maxima



given energy followed by a further integration over all the emission energies of the electron, and averaging over the two initial spin states, as well as summing over the two final spin states. In Fig. 1.9, we compare the *total* rates of ionization at three different laser frequencies $\omega = 1.55, 5$, and 54.4 eV , as a function of the laser intensity from 10^{14} to $10^{22} \text{ W cm}^{-2}$. In all cases, the ionization rates show at first an increasing probability with increasing intensity. Then they go through a maximum and *decrease* generally with further increase of intensity. We note that this decrease of ionization rate with the increase of intensity is reminiscent of (but not identical

to) the phenomenon of “adiabatic stabilization” of atoms against ionization in the high frequency limit [17]. In the latter case, the single-photon ionization threshold is supposed to lie much below the laser photon energy. In contrast, in this case, the decreasing rate occurs as well at low frequencies (i.e., below the “high frequency” limit). We note that the maximum rates are higher in strength for the higher frequencies, and they occur at higher intensities. Finally, a modulation of the total ionization rates can be detected below the maximum rate in the top curve – this behavior indicates a *channel closing* effect or the closing of the lowest n -photon ionization channels that can occur with the upward shift of the ionization threshold by the ponderomotive energy $U_p = \frac{I}{4\omega^2}$ (a.u.) with the increase of the intensity (see, e.g., [5]).

1.5.4 Spin Dependence of Ionization Currents

In the cases considered so far we have not resolved the spin degrees of freedom of the electron either in the ground state or in the free state after the ionization. In fact, the distributions shown in the above figures correspond to the usual measurement of the unpolarized ionization signal that is spin-averaged over the initial spin states of the target atom and spin-summed over the final states of the free electron. The present four-component relativistic analysis provides explicit analytic expressions for the individual spin-dependent transition amplitudes (first obtained in [11]). This allows us to further examine the *spin-resolved* electron current, its dependence on polarized or un-polarized target atoms, the spin-flip ionization probabilities, and any asymmetry in the spin currents. We recall, as indicated already in Fig. 1.1, that the field propagation direction (z-axis) is the spin quantization axis and the spin “up” orientation is defined to be along the positive z-direction.

1.5.4.1 Spin-Symmetric Transitions: Angular Distributions

We first consider the spin unchanging (spin symmetric) angular distributions of the ejected electron for $\omega = 27.2 \text{ eV}$ and $I = 3.51 \times 10^{15} \text{ W cm}^{-2}$. Note that at this frequency the single photon ionization channel is open (since the ionization threshold energy of Dirac H-atom $\approx 13.6 \text{ eV}$). In Fig. 1.10 we show the results of intense-field relativistic calculation for the symmetric $u \equiv \uparrow$ to $u \equiv \uparrow$ (solid line) and $d \equiv \downarrow$ to $d \equiv \downarrow$ (crosses) ionization rates. For the sake of comparison, we also show the results of the single-photon relativistic first Born approximation: $\uparrow$ to $\uparrow$ (open circles) and $\downarrow$ to $\downarrow$ (dots). Clearly, there is no significant distinction between the intense-field relativistic and the single-photon Born approximation. Moreover, the rates of $\uparrow$ to $\uparrow$ and $\downarrow$ to $\downarrow$ symmetric transitions are equal as we have discussed analytically in Sect. 4.2. In Fig. 1.11, we present the corresponding results for a higher intensity $I = 3.51 \times 10^{16} \text{ W cm}^{-2}$. Again, the spin-symmetric

Fig. 1.10 Spin symmetric ionization rate: angular distributions at $\omega = 1 \text{ a.u.} = 27.2 \text{ eV}$, $I = 3.51 \times 10^{15} \text{ W cm}^{-2}$ – intense-field relativistic Dirac: $\uparrow$ to $\uparrow$ (solid line), $\downarrow$ to $\downarrow$ (crosses); single-photon relativistic Born approximation: $\uparrow$ to $\uparrow$ (open circle), $\downarrow$ to $\downarrow$ (thick dots)

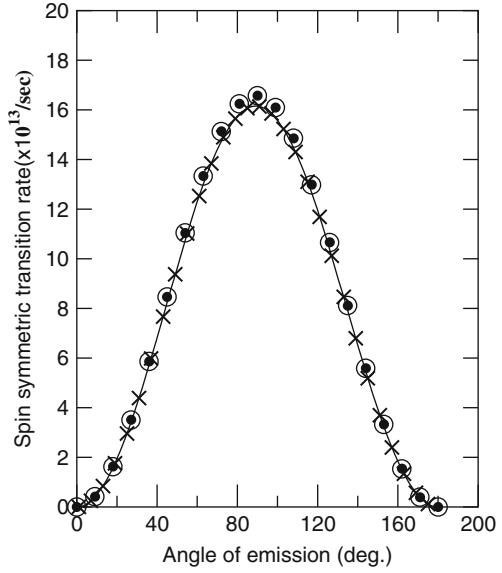
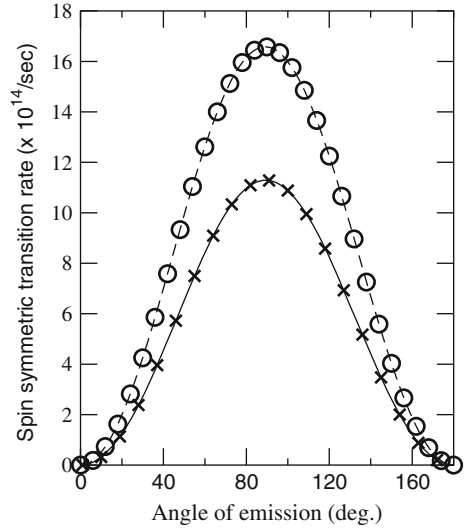


Fig. 1.11 Spin symmetric ionization rate: angular distributions at $\omega = 1 \text{ a.u.} = 27.2 \text{ eV}$, $I = 3.51 \times 10^{16} \text{ W cm}^{-2}$ – intense-field relativistic Dirac: $\uparrow$ to $\uparrow$ (solid line), $\downarrow$ to $\downarrow$ (crosses); single-photon relativistic Born approximation: $\uparrow$ to $\uparrow$ (open circle), $\downarrow$ to $\downarrow$ (dashed line)

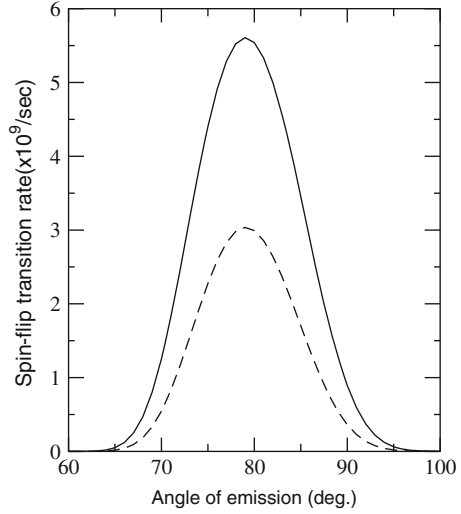


transition rates are equal, however, at the higher intensity the Born approximation results clearly overestimate the corresponding results of intense-field calculations.

1.5.4.2 Spin-Flip Transitions: Angular Distributions

In Fig. 1.12, we show the *spin-flip* ionization rates for an FEL frequency $\omega = 200 \text{ eV}$, at a high intensity $I = 10^{22} \text{ W cm}^{-2}$, calculated using the present

Fig. 1.12 Spin-flip ionization rate: angular distributions for $\omega = 200$ eV, $I = 3.5 \times 10^{22}$ W cm $^{-2}$: intense-field relativistic: $\uparrow$ to $\downarrow$ (solid line), $\downarrow$ to $\uparrow$ (dashed line)



intense-field theory. The $\uparrow$ to $\downarrow$ transition rates (solid line) and the $\downarrow$ to $\uparrow$ transition rates (dashed line) are compared. It can be seen that the $\uparrow$ to $\downarrow$ rates are larger than the $\downarrow$ to $\uparrow$ rates, at all angles of emission. They are, nevertheless, of a comparable *order* of magnitude. This, as we shall see next, is in stark contrast to the *zero* probability for the $\downarrow$ to $\uparrow$ transition given by the relativistic single-photon Born approximation. In Fig. 1.13, we show the results of the angular distributions of the spin-flip ionization rates calculated using the intense-field relativistic approximation ($\uparrow$ to $\downarrow$ (solid line), $\downarrow$ to $\uparrow$ (dashed line)) and, the single-photon relativistic first Born approximation ($\uparrow$ to $\downarrow$ (dots), $\downarrow$ to $\uparrow \equiv 0$ (not shown)). We note that the intense-field $\uparrow$ to $\downarrow$ and $\downarrow$ to $\uparrow$ rates are both finite but the latter is significantly weaker in strength. It is remarkable that the relativistic first Born approximation gives qualitatively different result since (as opposed to the finite $\uparrow$ to $\downarrow$ transition rates) the $\downarrow$ to $\uparrow$ transitions rates vanish (not shown) at all angles.

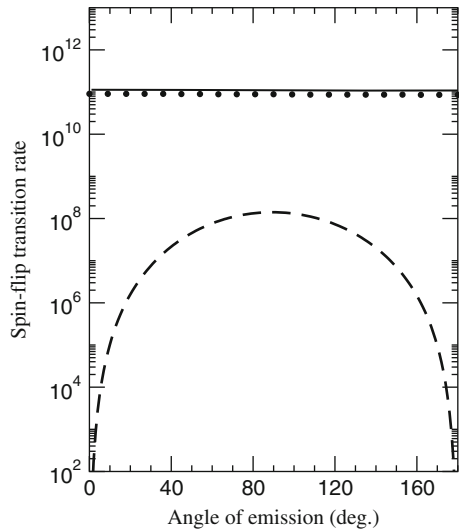
1.5.5 Spin Asymmetry

We may characterize the asymmetry between the spin-flip rates $u \rightarrow d$ and $d \rightarrow u$ quantitatively by an “asymmetry parameter”. To this end, we define the differential up and down spin *currents* (to be calculated from (1.87) and (1.81–1.84)) as:

$$\frac{dW^{\text{up}}}{d\Omega} = \frac{1}{2} \left(\frac{dW_{u \rightarrow u}}{d\Omega} + \frac{dW_{d \rightarrow u}}{d\Omega} \right), \quad (1.102)$$

$$\frac{dW^{\text{down}}}{d\Omega} = \frac{1}{2} \left(\frac{dW_{d \rightarrow d}}{d\Omega} + \frac{dW_{u \rightarrow d}}{d\Omega} \right). \quad (1.103)$$

Fig. 1.13 Spin-flip ionization rate: angular distributions at $\omega = 27.2$ eV, $I = 3.51 \times 10^{16}$ W cm $^{-2}$ – strong-field relativistic: $\uparrow$ to $\downarrow$ (solid line), $\downarrow$ to $\uparrow$ (dashed curve); single-photon relativistic first Born approximation: $\uparrow$ to $\downarrow$ (thick dots), $\downarrow$ to $\uparrow \equiv 0$ (not shown)



We define the *spin-flip* asymmetry parameter:

$$A(\theta) = \left(\frac{dW_{u \rightarrow d}}{d\Omega} - \frac{dW_{d \rightarrow u}}{d\Omega} \right) / \left(\frac{dW^{\text{up}}}{d\Omega} + \frac{dW^{\text{down}}}{d\Omega} \right). \quad (1.104)$$

Similarly, we define the asymmetry parameter for a spin *unpolarized* target as the averaged $\langle A \rangle (\theta)$:

$$\langle A \rangle (\theta) = \left(\frac{dW^{\text{up}}}{d\Omega} - \frac{dW^{\text{down}}}{d\Omega} \right) / \left(\frac{dW^{\text{up}}}{d\Omega} + \frac{dW^{\text{down}}}{d\Omega} \right). \quad (1.105)$$

Figure 1.14 shows the results of angle dependence of the spin-flip asymmetry $A(\theta)$ for a Ti-sapphire laser with $\omega = 1.55$ eV, $I = 10^{16}$ W cm $^{-2}$ (outer curve) and $I = 10^{17}$ W cm $^{-2}$ (inner curve). $A(\theta)$ is positive implying clearly that the $\uparrow$ to $\downarrow$ flip rate is greater than the $\downarrow$ to $\uparrow$ flip rate. We may remark in passing that unlike the spin asymmetry for Fano-effect [18], see also [19] observed in ordinary photoionization in the presence of spin-orbit interaction, the two curves in the figure reveal a strong dependence of $A(\theta)$ on the field intensity, at all angles. The peak value of the asymmetry is seen to lie not on the plane of polarization ($\theta = 90^\circ$) but at a somewhat smaller angle away from it. The peak also moves further away from the polarization plane with increasing intensity. This behavior reveals the change of the electron momentum in an intense field caused by the combined effect of retardation and field intensity. This is seen analytically in the “dressed” momentum $\mathbf{q}$ of the electron in the presence of the field (1.90) where the extra term that adds to $\mathbf{p}$ depends on the photon momentum transfer $n\mathbf{\kappa}$, as well as on the intensity through λ_p . Figure 1.15 shows the spin-flip asymmetry $A(\theta)$ for an FEL frequency $\omega = 20$ eV and intensity $I = 10^{20}$ W cm $^{-2}$. Again, a strong angular dependence

Fig. 1.14 Spin-flip asymmetry parameter $A(\theta)$ vs. angle of electron emission: $\omega = 1.55$ eV, $I = 10^{16}$ W cm $^{-2}$ (outer curve) and $I = 10^{17}$ W cm $^{-2}$ (inner curve)

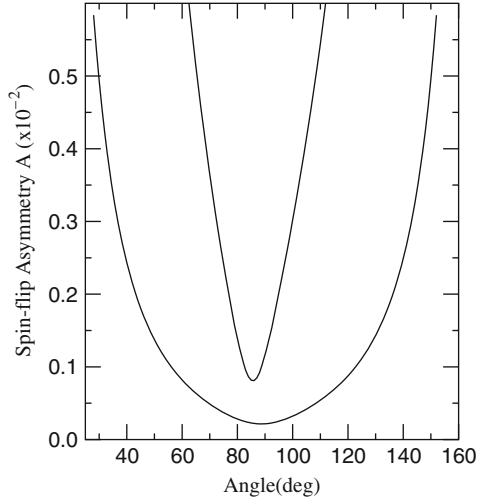
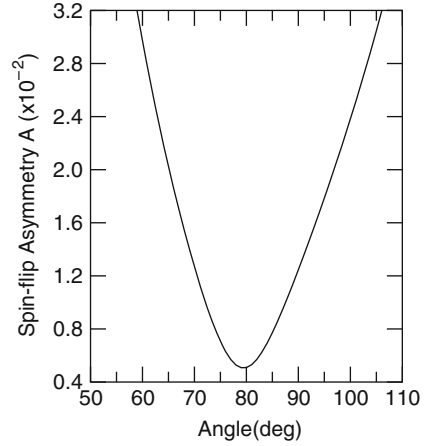
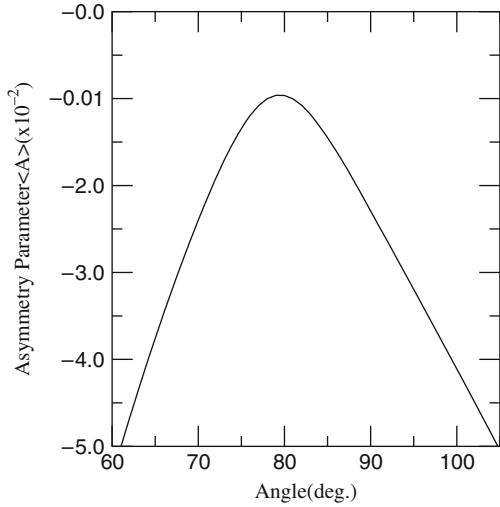


Fig. 1.15 Spin-flip asymmetry parameter $A(\theta)$ vs. angle of electron emission: $\omega = 20$ eV, $I = 10^{20}$ W cm $^{-2}$



of the asymmetry is evident at this higher intensity. One may wonder whether this asymmetry actually would survive for ionization from an *unpolarized* target, which requires a 50/50 weighted average of the up and down spin sub-states of the ground state. In Fig. 1.16 we show the results for the spin current asymmetry parameter $\langle A \rangle(\theta)$ averaged over the ground spin states, as a function of the polar angle of the emitted electrons (measured from the laser field propagation direction) for an $\omega = 20$ eV and an intensity $I = 10^{20}$ W cm $^{-2}$ as in Fig. 1.15. Aside from the change of sign (compare the definitions of the numerators of $A(\theta)$ and $\langle A \rangle(\theta)$ defined by (1.104) and (1.105)), the asymmetry is seen to survive the spin averaging of the target states; it also shows a strong angular dependence as seen already.

Fig. 1.16 Target spin states averaged spin current asymmetry parameter $\langle A \rangle (\theta)$ vs. angle of electron emission:
 $\omega = 20 \text{ eV}$,
 $I = 10^{20} \text{ W cm}^{-2}$



1.5.5.1 Role of Retardation or Finite Photon Momentum

Further insights into the nature of spin-photon interaction in intense fields can be gained by considering the influence of the retardation effect, and hence the influence of the magnetic component of the incident electromagnetic field on the spin-flip probability. Perhaps, the simplest way to examine the necessity or otherwise of the retardation effect for the spin-flip process is to put the light propagation vector κ identically equal to zero in the transition matrix elements $t_{u \rightarrow d}^{(n)}$ and $t_{d \rightarrow u}^{(n)}$ (1.82) and (1.83). Clearly, the argument a_p of the Bessel function $J_n(a_p)$ defined above in the limit of zero retardation ($\kappa = 0$) remains finite and the field-dressed momentum q simply reduces to the free momentum p . Hence, the spin-flip amplitudes do *not* vanish in the limit of zero retardation, i.e., if the magnetic component of the incident laser field is neglected. Note that in the present case there is no spin-orbit interaction in the initial state (ground s-state). So one may rightly enquire: what is the mechanism for the finite spin-flip transition probability in intense fields, in the absence of retardation and the spin-orbit interaction? We first note that (1.82) and (1.83) for the spin-flip amplitudes depend on the parameters m_2 and $g(q)$, which relate to the ‘weak’ components of the Dirac spinor of the free electron and the ground state of Dirac H-atom. Hence, they certainly go beyond the usual Pauli-mechanism of direct coupling of a given external magnetic field to the spin (magnetic moment) of the electron. In fact, further examination of the two equations shows that the non-vanishing coupling occurs through the vector product of the polarization ϵ and the electron momentum p . This along with the outer factor $eA_0 \equiv eF_0/\kappa_0$ (where F_0 is the electric peak field strength of the laser field in the laboratory) in the transition matrix element (see (1.64)) shows that the coupling depends on factors of the form $\mathbf{F}_0 \times \mathbf{p}/c$ (a.u.) $\approx \mathbf{B}'$ (a.u.), where $\mathbf{B}'$ is in fact an effective magnetic field seen by the electron in its own frame of reference. It arises from the Lorentz transformation

(e.g., [20]) of the electric field F_0 of the laser field in the laboratory, contributing to an effective (or ‘motional’) magnetic field in the rest frame of the electron moving with a momentum $\mathbf{p}$ in the laboratory. Therefore, the dominant mechanism that leads to the spin-flip transition in *intense* laser fields is the coupling of the “motional” magnetic field $\vec{B}'$ with the spin σ or magnetic moment $= -\frac{1}{4c^2}\sigma$ (a.u.) of the electron. This is rather analogous but *not* identical to the Lorentz transformation of the electric field associated with the static *atomic* potential $V(r)$ into an effective magnetic field and the resulting spin-orbit interaction responsible for the well-known Fano-effect [18], see also [19] observed in ordinary photoionization in the perturbative domain of intensity. Therefore, we may also add that for a comparatively weak laser field and when the spin-orbit interaction is not negligible an additional contribution to $\mathbf{B}'$, of the order of $-\frac{1}{r} \frac{dV(r)}{dr} \mathbf{r} \times \mathbf{p}/2c$ (a.u.), ought to be taken into account for quantitative accuracy.

1.5.5.2 External Control of the Spin Currents

It is worth noting that the relative dominance of up or down spin electron currents can be *controlled* from outside, for example, completely reversed by changing the *helicity* of the incident light. This can be seen analytically by replacing the right circular polarization vector $\epsilon(\xi = +\pi/2)$ by the left circular polarization vector $\epsilon(\xi = -\pi/2)$ in (1.81)–(1.84) and observing that the following transformations of the amplitudes hold:

$$t_{u \rightarrow u}^{(n)} \rightarrow t_{d \rightarrow d}^{(n)*}, \quad (1.106)$$

$$t_{d \rightarrow d}^{(n)} \rightarrow t_{u \rightarrow u}^{(n)*}, \quad (1.107)$$

$$t_{u \rightarrow d}^{(n)} \rightarrow -t_{d \rightarrow u}^{(n)*}, \quad (1.108)$$

$$t_{d \rightarrow u}^{(n)} \rightarrow -t_{u \rightarrow d}^{(n)*}. \quad (1.109)$$

Hence, the spin-flip rates would exchange their magnitudes and the asymmetries $\langle A \rangle(\theta)$ would change their signs on changing the helicity of the photons from the right circular to the left circular polarization. We may recall that the magnitude of the asymmetry parameters $\langle A \rangle(\theta)$ and $A(\theta)$, in the cases explicitly considered above, are of the orders of 10^{-3} for the near-infrared wavelength and 10^{-2} for the VUV wavelength. These values lie well above the threshold efficiency $\approx 2.4 \times 10^{-4}$ of currently available spin analyzers in the laboratory (e.g., [21, 22]).

We point out that since the spin-flip asymmetry as shown here remains finite and large for weak and long wavelength fields (see, e.g., Fig. 1.13) and can be controlled from outside, it would be useful also for the new science of *spintronics*.

1.5.6 Charge-State Z Dependence and an Anomalous Effect

In this last section, we consider the dependence of the total ionization probability per unit time and its energy and emission angle distributions, on the charge states Z of the target ions, and consider an anomalous behavior of ionization probability vs. ionization potential.

1.5.6.1 Total Ionization Rates Vs. Z

It is known ([4], p. 96) that in the perturbative region of intensities the rate of ionization decreases very rapidly with inverse powers of the ionic charge Z due to the strong increase in the ionization potential of the target ion, $I_p(Z) = \frac{Z^2}{2}$ (a.u.). Thus, it is usually expected that it would be harder to ionize a system (at the same intensity and frequency of the field) if the electron is bound more tightly. However, as noted above, an anomalous *increase* of the ionization probability with the increase of the binding energy of the electron has been predicted [12] to occur, from a model atom investigation based on the relativistic Klein–Gordon equation. This was seen to occur in a certain range of the binding energy. We shall also confirm the anomalous effect of increase of ionization probability of hydrogenic ions with increasing ionization potential in certain domain of the charge states Z . In Fig. 1.17, we show the results of total ionization rates as a function of the charge state Z , from calculations using intense-field KFR1 approximation, for both Dirac relativistic (solid lines) and Schrödinger nonrelativistic (dashed lines) cases, at $\omega = 10$ eV, and $I = 10^{18}$ and 10^{19} W cm $^{-2}$. It can be seen that at an intensity $I = 10^{18}$ W cm $^{-2}$, there is already an indication of the counterintuitive result in which the ionization probability increases for $Z = 2$ than at $Z = 1$ as the ionization potential, $I_p = Z^2/2$, increases from $1/2$ to 2 (a.u.). At a still higher intensity, $I = 10^{19}$ W cm $^{-2}$, the anomalous behavior is clearly exhibited for both the charge states $Z = 2$ and $Z = 3$. We note, in fact, that at this higher intensity the effect is more pronounced in the relativistic calculation than in the nonrelativistic case. It is, therefore, not exclusively a relativistic effect but appears at present to be a manifestation of highly nonlinear dependence of the ionization dynamics on the field intensity, whose *physical* origin remains to be understood.

1.5.6.2 Energy and Angular Distributions Vs. Z

To investigate this anomalous behavior further, we have computed also the energy spectrum as well as the angular distribution of the emitted electrons from different target ions. In Fig. 1.18, the results are shown for $\omega = 10$ eV, at $I = 10^{19}$ W cm $^{-2}$, for a sequence of charge states $Z = 1, 2, 3, 4, 5$. It is clearly seen that the probability of ionization for $Z = 3$ with a higher binding energy $I_p(3) = 9/2$ (a.u.) is *higher* than for the ions with $Z = 2$ that is more weakly bound ($I_p(2) = 2$ (a.u.) $< 9/2$ (a.u.) = $I_p(3)$). This holds essentially throughout the energy distribution. Finally,

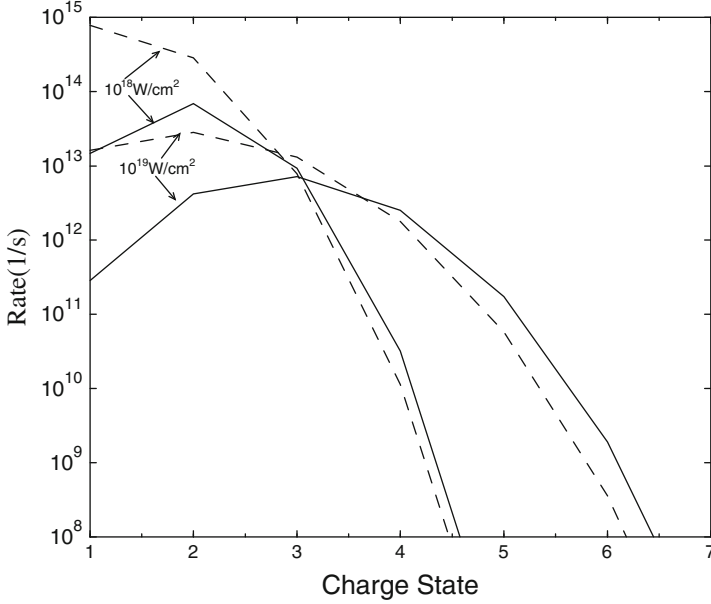


Fig. 1.17 Total ionization rates vs. charge state Z of target ion: $\omega = 10\text{ eV}$, $I = 10^{18}$ and 10^{19} W cm^{-2} – intense-field relativistic Dirac case (*solid lines*), nonrelativistic Schrödinger case (*dashed lines*). Note the anomalous behavior of *larger* rates for higher ionization potentials for the charge states $Z = 2$, and $Z = 2, 3$, than for $Z = 1$, respectively, seen clearly in the relativistic results at the two intensities

in Fig. 1.19, we show the results of calculations for the angular distributions for the ionization of ions with charge states $Z = 1, 2, 3, 4, 5$. It is seen again that the electron emission rates in essentially all directions from the more tightly bound target ions with $Z = 3$ is larger than the rates of emission from the less tightly bound electron in target ions with $Z = 2$ and $Z = 1$. We note that the maximum rate of ionization occurs at about 83° from the direction of propagation of the incident laser beam.

1.6 Summary

To summarize, we have discussed the process of ionization of Dirac Hydrogen atom and hydrogenic ions of increasing charge states, using the four-component relativistic intense-field KFR1 approximation. The energy spectra, the angle dependence, the spin dependence, and the charge-state dependence of the intense-field ionization process are analyzed and discussed in detail. Specific illustrations for the special case of circularly polarized fields allow us to further investigate the helicity dependence of the intense-field ionization process and compare it with ones in

Fig. 1.18 Change-state dependence of ionization rate: energy spectra (in units of photon energy) of H and hydrogenic ions: $Z = 1, 2, 3, 4, 5$; $\omega = 10$ eV, $I = 10^{19}$ W cm $^{-2}$; calculated with present relativistic intense-field ionization model

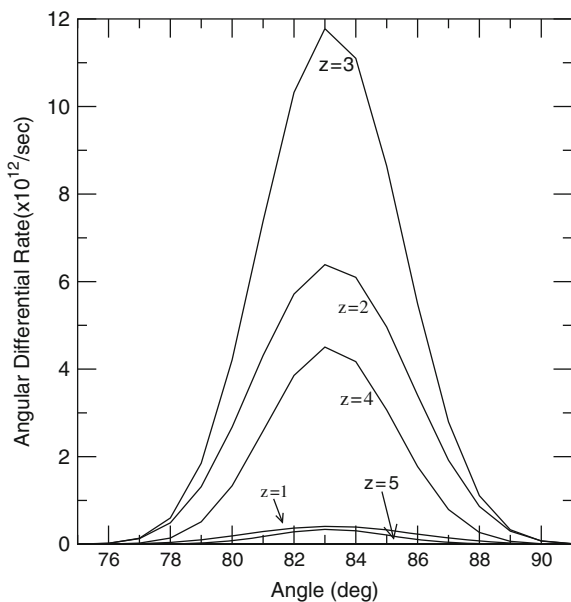
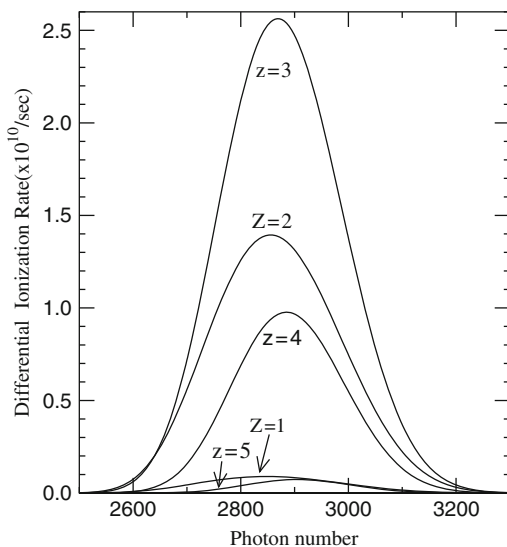


Fig. 1.19 Charge-state dependence of ionization rate: angular distribution for H and hydrogenic ions: $Z = 1, 2, 3, 4, 5$; calculated with present relativistic intense-field ionization model; $\omega = 10$ eV, $I = 10^{19}$ W cm $^{-2}$

the weak and long wavelength fields. To this end, results of calculations from the relativistic and the nonrelativistic intense-field approximations (and in some cases also from the relativistic first Born approximation) are compared. The frequency range is chosen to correspond to the available intense infrared and free-electron

laser fields. Two counterintuitive characteristics of the ionization process predicted earlier: (a) an asymmetry of spin-up and spin-down currents from the unpolarized target and (b) an anomalous increase of ionization probability with the increase of the ionization potential are also confirmed. The strong spin-asymmetry in the case of weak and long wavelength fields and its simple control analyzed here should be of interest for the new field of spintronics.

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