

Chapter 2

Electromagnetic and Network Theory of Waveguide Radiation by Spherical Modes Expansions

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2.1 Introduction

In recent years, modal techniques have been successfully improved and are increasingly used for dealing with design of waveguide discontinuities and passive components [1–8], due to their efficiency and also because they provide rigorous and useful network representations. One distinguished characteristic of modal techniques is to separate the transverse field behavior from the longitudinal one; this decoupling makes it feasible to consider electromagnetic wave propagation inside a waveguide as a superposition of transmission lines (each pertaining to a mode) which couple only at discontinuities. Electromagnetic field representation inside a waveguide with finite cross-section, is therefore achieved by a discrete summation of the relevant waveguide modes.

A similar procedure can be followed when considering free-space as a waveguide, by means of a spherical mode expansion [9, pp. 445–450], [10–14]. Free-space field expansion in terms of spherical modes presents several advantages:

- Straightforward extension of the modal techniques used for waveguide problems also to radiation problems
- Derivation of rigorous equivalent networks

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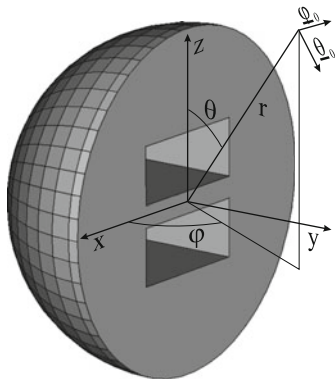
- Derivation of boundary conditions for terminating numerical methods based on space discretization
- Treatment of conformal boundaries, as e.g. finite circular flange, semi-spherical dielectric covering, etc.

While the use of spherical modes for dealing with conformal geometries is quite natural and has received a considerable interest in the past, it is noted here that the first two points are of particular relevance for computer-aided design and have received modest attention. In fact they allow to extend the very successful modal techniques also to free-space radiation problems. In [15–18] a general procedure has been described for the systematic partitioning of complex problems into sub-domains and their rigorous description in terms of networks. By using spherical transmission lines for describing propagation in free-space, one can obtain network representations similar for closed and open problems. In addition it is also feasible to describe the interaction between distant radiators without the necessity of discretizing the region of space between them. The value of the spherical wave expansion for solving antenna problems has been already noted in [19–21]. In these works this approach has been used for coupling the analysis of cavity-backed microstrip antennas performed by a three-dimensional finite element method, to other antennas of the same type but placed far away.

In this paper we describe the electromagnetics and network theory of waveguide radiation by considering as an example an array of waveguides radiating on a *finite* circular flange plane. We rigorously solve the above problem, which to the best of our knowledge has only been solved by discretization methods so far, and we also provide an equivalent network for this type of structures. The problem of flange-mounted array radiation has been considered in the literature, quite some time ago [22]. Fundamental progress has been made in the excellent work of Trevor Bird, of which we cite just a few contributions [24–26]. Further refinements have been proposed in [27] by taking into account the field singularities and by considering strategies for the efficient design with the adjoint network method in [28]. The case of radiation from elliptical horns has been considered in [29] and an efficient scheme for parallel computation has been provided in [30]. In all the above cases the presence of an infinite flange plane has always been assumed. The finite flange problem has been considered in the past in [31] by using a geometrical theory of diffraction (GTD) approximation for dealing with a rectangular flange. In [32] a hybrid technique combining moment method, FEM and GTD has been used to attack the finite flange problem. Notably, the case of a circular flange backed by a metallic semi-sphere (see Fig. 2.1 for a sketch) is amenable of considerable analytical progress when a spherical mode expansion is used. As a result, it is possible to investigate finite flange effects with a modest numerical effort and to provide a rigorous network for this structure.

The paper is structured as follows: in Sect. 2.2 the spherical mode expansion is introduced and is applied to the problem of radiation from an array of flange-mounted rectangular waveguides and its equivalent network is also established. In particular, in this section we start from the statement of the problem (in Sect. 2.2.1), illustrate the analogy with modal techniques for closed waveguides (in Sect. 2.2.2),

Fig. 2.1 Three-dimensional view of a flange-mounted small array of rectangular waveguides radiating into FREE SPACE. Note the finite circular flange; for modeling purposes the backside of the array is considered as a perfectly conducting half-sphere



and we recall the spherical mode expansion (in Sect. 2.2.3) and the generalized transformer (in Sect. 2.2.5). Characterization of the transition region has been outlined in [14] and is not repeated here; furthermore, since description of waveguide regions is well known it has not been considered in the following. Finally, in Sect. 2.3 the rigorous analysis of a small array of rectangular waveguides radiating into a *finite* circular flange plane is considered and some numerical results are provided.

2.2 Generalized Network of Radiating Waveguides in Terms of Spherical Modes

2.2.1 Statement of the Problem

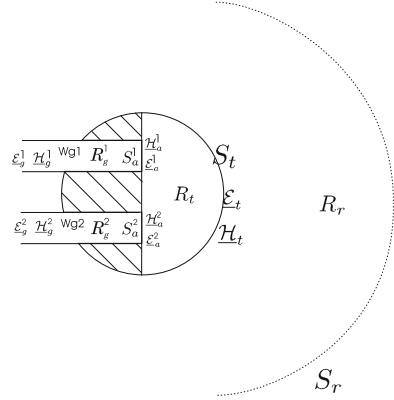
We consider, with reference to Fig. 2.1, an array of rectangular waveguides, mounted on a *finite* circular flange, radiating into free-space. For modeling purposes, it is assumed that the backside of the circular flange is a metallic half sphere (hemisphere).

In order to illustrate the methodology it is sufficient to refer to an array of just two waveguides. In Fig. 2.1 the three-dimensional view of the structure is shown and in Fig. 2.2 is sketched its side-view. From the latter figure we see that different regions of space have been introduced:

- Waveguide regions, denoted as region R_g^i , for the i th waveguide
- Transition Region, denoted by region R_t , i.e. a semi-spherical region of space of the same diameter of the circular flange
- A region of space R_r extending from the end of the transition region up to infinity (and therefore comprising the far-field region)

The apertures, i.e. the boundary between regions R_g^i and R_t , are denoted by S_a . Note that in Fig. 2.2 we have considered a surface S_t separating the transition region

Fig. 2.2 Side view of the structure in Fig. 2.1; different regions of space have been identified: waveguide regions, denoted by R_g^i ; a “Transition Region” denoted as region R_t ; a region of space extending from the end of the transition region up to infinity (and therefore comprising the far-field region) denoted as region R_r . The radius of the Transition region is the same of the finite circular flange



from region R_r and a surface S_r , in region R_t , where the far-field may be evaluated. The Transition Region is bounded on one side by the flange plane on which the surface S_a lies; the surface S_t provides the remaining part of the boundary.

2.2.2 Modal Analysis and Equivalent Networks

As noted before, one significant advantage of using the spherical mode expansion is the fact that it allows to extend modal techniques also to free-space problems. Let us refer to Fig. 2.3, where in the lower right corner is sketched a waveguide problem representing two waveguides (denoted with 1 and 2 respectively), a resonator (region 3), and another waveguide (region 4) attached to the resonator via an iris. On the lower left side of Fig. 2.3 is sketched our problem: the two waveguides radiates into the semi-spherical transition region (region 3) and then into free-space. In the upper part of Fig. 2.3 is sketched the equivalent network representing *both* problems. Note the presence of a generalized transformer between regions 3 and 4. In the waveguiding problem (lower right sketch) the field on the iris between region 3 and 4 is coupled to the waveguide modes of region 4; the coupling matrix corresponds to a generalized transformer. In the case of finite circular flange the field on the semi-spherical surface of the Transition region is coupled to the spherical mode expansion of free-space, taking into account the presence of the metallic semi-sphere.

2.2.3 Spherical Modes

The EM-field in spherical coordinate may be written as superposition of spherical modes as

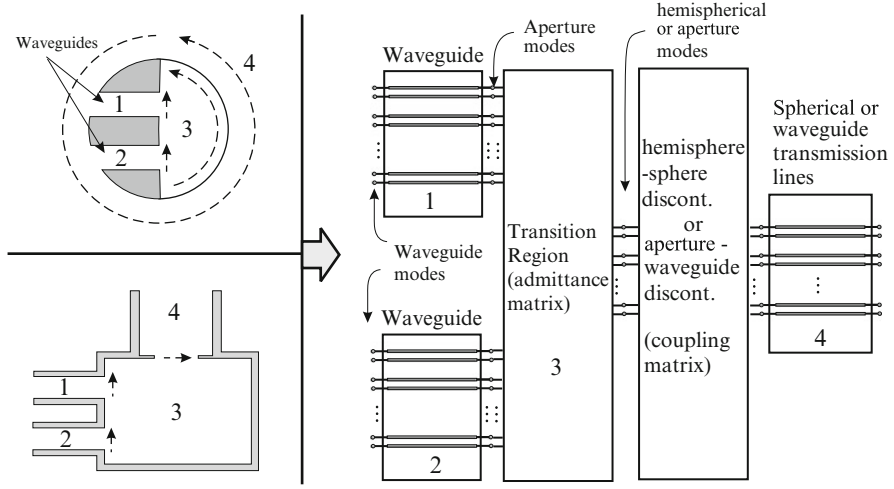


Fig. 2.3 Equivalent network of the two problems sketched in the *left part* of the figure; on the *higher left side* is shown the problem of two waveguides radiating on a finite circular flange; on the *right side* is shown the equivalent waveguiding problem

$$\mathbf{E}_t(r, \theta, \varphi) = \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_j^{TEe, TMe, TEO, TMO} \frac{V_{m,n}^{(j)}(r)}{r} \mathbf{e}_{m,n}^{(j)}(\theta, \varphi) \quad (2.1)$$

$$\mathbf{E}_r(r, \theta, \varphi) = \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_j^{TMe, TMO} \frac{n(n+1)}{-j\omega\epsilon} \frac{I_{m,n}^{(j)}(r)}{r^2} T_{m,n}^{(j)}(\theta, \varphi) \mathbf{r}_0 \quad (2.2)$$

$$\mathbf{H}_t(r, \theta, \varphi) = \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_j^{TEe, TMe, TEO, TMO} \frac{I_{m,n}^{(j)}(r)}{r} \mathbf{h}_{m,n}^{(j)}(\theta, \varphi) \quad (2.3)$$

$$\mathbf{H}_r(r, \theta, \varphi) = \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_j^{TEe, TEO} \frac{n(n+1)}{j\omega\mu} \frac{V_{m,n}^{(j)}(r)}{r^2} T_{m,n}^{(j)}(\theta, \varphi) \mathbf{r}_0 \quad (2.4)$$

with equivalent voltages and currents given by:

$$\begin{cases} V_{m,n}^{(j)}(r) = V_{0m,n}^{(j)+} F_n^{V+}(kr) + V_{0m,n}^{(j)-} F_n^{V-}(kr) \\ I_{m,n}^{(j)}(r) = Y_0 \left[V_{0m,n}^{(j)+} F_n^{I+}(kr) - V_{0m,n}^{(j)-} F_n^{I-}(kr) \right] \end{cases} \quad (2.5)$$

In the above equation V_o^+ and V_o^- represent the incident (toward ∞) and reflected (from ∞) spherical waves amplitude, while F functions are defined in terms of spherical Hankel function as:

	<i>TM</i>	<i>TE</i>
$F_n^{V+}(kr)$	$h_n^{(2)'}(kr)$	$h_n^{(2)}(kr)$
$F_n^{V-}(kr)$	$h_n^{(1)'}(kr)$	$h_n^{(1)}(kr)$
$F_n^{I+}(kr)$	$-j h_n^{(2)}(kr)$	$j h_n^{(2)'}(kr)$
$F_n^{I-}(kr)$	$j h_n^{(1)}(kr)$	$-j h_n^{(1)'}(kr)$

Spherical Hankel functions $h_n^{(\ell)}$ are defined in terms of Hankel functions $H_n^{(\ell)}$ as:

$$h_n^{(\ell)}(x) = \sqrt{\frac{\pi x}{2}} H_{n+\frac{1}{2}}^{(\ell)}(x) \quad \text{where } \ell = 1, 2 \quad (2.6)$$

and we denote with $Y_0 = \sqrt{\frac{\varepsilon}{\mu}}$ and $k = \omega\sqrt{\mu\varepsilon}$ the admittance and the wavenumber, respectively, of the considered medium. Note that from (2.5) we can define the modal admittance for incident (+) and reflected (−) waves as:

$$Y_c^+ = Y_0 \frac{F_n^{I+}(kr)}{F_n^{V+}(kr)} \quad Y_c^- = -Y_0 \frac{F_n^{I-}(kr)}{F_n^{V-}(kr)}. \quad (2.7)$$

The tesseral harmonics $T_{m,n}^{(j)}(\theta, \varphi)$, for both *TE* and *TM* modes, are defined as:

$$\left\{ \begin{matrix} even \\ odd \end{matrix} \right\}_{T_{m,n}} = A \left\{ \begin{matrix} \cos(m\varphi) \\ \sin(m\varphi) \end{matrix} \right\} P_n^m(\cos \theta). \quad (2.8)$$

Finally, the way to evaluate the eigenfunction of electric type $\mathbf{e}_{m,n}(\theta, \varphi)$ depends on the mode type (*TM_e*, *TM_o*, *TE_e*, *TE_o*)

$$\left\{ \begin{matrix} TM_e \\ TM_o \end{matrix} \right\}_{\mathbf{e}_{m,n}} = A \left[\left\{ \begin{matrix} \cos(m\varphi) \\ \sin(m\varphi) \end{matrix} \right\} \frac{d}{d\theta} P_n^m(\cos \theta) \underline{\theta}_0 + \frac{m}{\sin(\theta)} \left\{ \begin{matrix} -\sin(m\varphi) \\ \cos(m\varphi) \end{matrix} \right\} P_n^m(\cos \theta) \underline{\varphi}_0 \right] \quad (2.9)$$

$$\left\{ \begin{matrix} TE_e \\ TE_o \end{matrix} \right\}_{\mathbf{e}_{m,n}} = A \left[\frac{m}{\sin(\theta)} \left\{ \begin{matrix} -\sin(m\varphi) \\ \cos(m\varphi) \end{matrix} \right\} P_n^m(\cos \theta) \underline{\theta}_0 + - \left\{ \begin{matrix} \cos(m\varphi) \\ \sin(m\varphi) \end{matrix} \right\} \frac{d}{d\theta} P_n^m(\cos \theta) \underline{\varphi}_0 \right] \quad (2.10)$$

where m ranges from 0 to n for even modes and from 1 to n for odd modes. The relevant eigenfunction of magnetic type is:

$$\mathbf{h}_{m,n}(\theta, \varphi) = \underline{r}_0 \times \mathbf{e}_{m,n}(\theta, \varphi), \quad (2.11)$$

and the normalization constant A is defined as:

$$\int_0^{2\pi} \int_0^\pi |\mathbf{e}_{m,n}(\theta, \varphi)|^2 \sin \theta d\theta d\varphi = 1. \quad (2.12)$$

The above formulation is referred to free-space modes and the modal eigenfunctions $\mathbf{e}_{m,n}(\theta, \varphi)$ are defined on a spherical surface (spherical modes). In this paper we need also to use modes defined on an hemispherical surface (hemispherical modes). With reference to Figs. 2.2 and 2.3, the free-space region 4 (spherical modes) is connected to the transition region 3 (hemispherical modes).

Hemispherical modes can be considered as modes of the half-space. In fact, by taking a free-space region and dividing it into two parts by using an infinite metal plane passing through the origin of the reference system, we obtain two half-spaces and in each half-space propagate hemispherical modes. Hemispherical modes can be then obtained starting from the spherical modal set by imposing the boundary conditions due to the presence of the electric plane.

In particular if we consider the metal plane lying on the plane xz (the same plane of our circular flange, see Fig. 2.1), the hemispherical modal set is obtained by taking TEe and TMo modes only (discarding TEo and TMe) from the spherical modal set and by multiplying them by $\sqrt{2}$ to re-normalize.

2.2.4 Transition Region

In our problem, according to Fig. 2.4, the transition region is a hemispherical portion of space where a hemispherical port and some rectangular ports are present. For such a region it is possible to find an admittance matrix:

$$\begin{bmatrix} \mathbf{I}_r \\ \mathbf{I}_h \end{bmatrix} = \begin{bmatrix} [Y]_{r,r} & [Y]_{r,h} \\ [Y]_{h,r} & [Y]_{h,h} \end{bmatrix} \begin{bmatrix} \mathbf{V}_r \\ \mathbf{V}_h \end{bmatrix} \quad (2.13)$$

where subscripts r and h stand for rectangular and hemispherical ports, respectively. $\mathbf{V}_r$ ($\mathbf{I}_r$) is a vector containing modal voltages (currents) of all rectangular apertures, while $\mathbf{V}_h$ ($\mathbf{I}_h$) is a vector containing modal voltages (currents) of the hemispherical port. Submatrices $[Y]_{r,r}$ $[Y]_{r,h}$ $[Y]_{h,r}$ $[Y]_{h,h}$ can be found taking advantages from some properties of hemispherical modes, as detailed in [14].

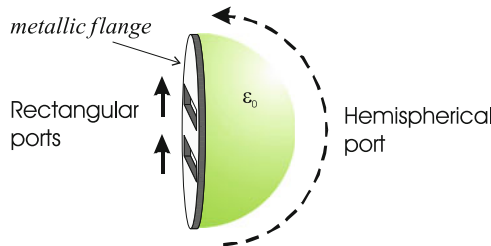


Fig. 2.4 The transition region is a hemispherical portion of space bounded by metal and ports. In particular, the hemispherical surface of the hemisphere is the hemispherical port while the flat surface is composed by a metallic flange with rectangular apertures. Rectangular apertures correspond to rectangular ports

2.2.5 Coupling Between Spherical and Hemispherical Modes

The surface separating region 3 and region 4 on Fig. 2.3 represents the discontinuity between the hemispherical port of region 3 and the spherical port of region 4. Such a discontinuity is a region of zero volume and, similarly to a discontinuity between two waveguides, can be represented by an equivalent network composed solely by transformers [15, 18] and can be studied by applying the mode-matching technique and by evaluating the coupling matrix $[M]$. The coupling matrix representation is:

$$\mathbf{V}_s = [M] \mathbf{V}_h \quad (2.14)$$

$$\mathbf{I}_h = -[M]^T \mathbf{I}_s \quad (2.15)$$

where $\mathbf{V}_s$ ($\mathbf{I}_s$) and $\mathbf{V}_h$ ($\mathbf{I}_h$) are vectors containing equivalent voltages (currents) relating to the spherical modes and hemispherical modes, respectively.

The j th element of the i th row of the coupling matrix ($g_{i,j}$) can be evaluated by the coupling integral:

$$g_{i,j} = \int_0^\pi \int_0^\pi \mathbf{e}_i^{(s)}(\theta, \varphi) \cdot \mathbf{e}_j^{(h)}(\theta, \varphi) \sin \theta d\theta d\varphi \quad (2.16)$$

where the index i (combination of TE_e , TE_o , TM_e , TM_o , m , n) indicate the i -th spherical modes while the index j (combination of TE_e , TM_o , m , n) indicate the j -th hemispherical modes. Obviously, superscript s and h refers to spherical modes and hemispherical modes, respectively.

By inserting (2.9–2.10) into (2.16), it can be noted that the coupling integrals can be conveniently written as product of integrals depending on the variable φ only and integrals depending on the variable θ only. The evaluation of coupling integrals is detailed in the appendix.

2.3 Discussion and Numerical Results

The approach presented in this paper allows rigorous and efficient modeling of a waveguide array mounted on a finite circular flange plane.

In Fig. 2.5 the S_{11} and S_{12} of two rectangular waveguides mounted on a circular flange plane have been computed by using spherical transmission lines and verified against CST simulations, for different values of the flange diameter. Details on the structure geometry are given in the relative caption. The very efficient modeling achieved by using spherical transmission lines represent a considerable advantage when the structure should undergo optimization.

In Fig. 2.6 we have plotted the scattering parameters relative to the case of two rectangular waveguides, placed as shown in the inset, when changing the dimensions of the circular flange. It is apparent that the flange dimension has an effect both on the magnitude and phase of the scattering parameters. It can be seen that

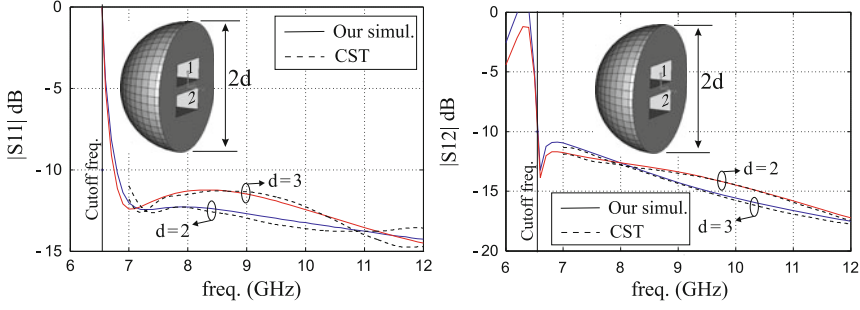


Fig. 2.5 S_{11} and S_{12} of two WR90 rectangular waveguides mounted on a circular flange plane. The distance between waveguide center is 14 mm. The radius ‘d’ of the spherical flange is expressed in cm. The results have been computed by using spherical transmission lines (*continuous line*) and verified against CST simulation (*dashed line*)

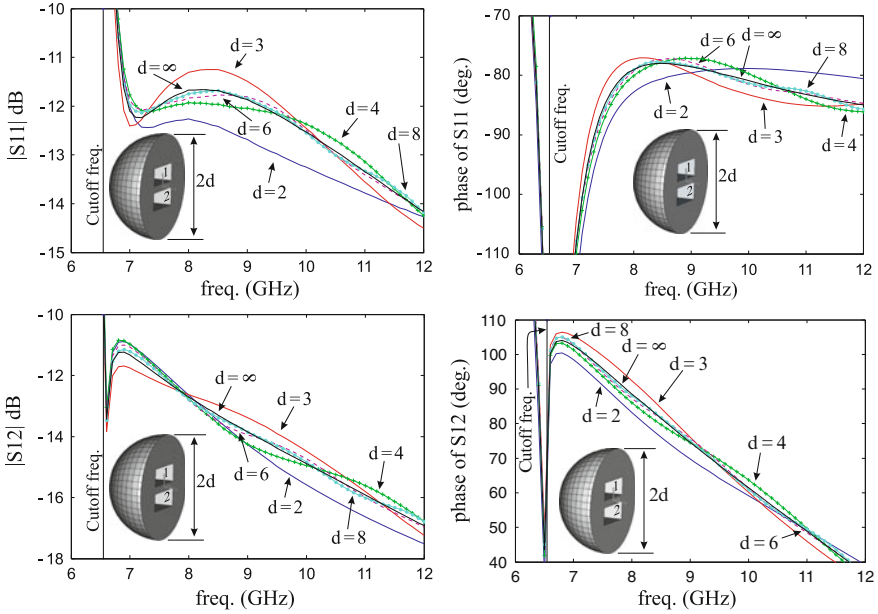


Fig. 2.6 Variation of the scattering parameters with the flange dimensions for the same structure of Fig. 2.5

results obtained by using a finite flange oscillate around the results related to the case of infinite flange and, as expected, increasing the radius of the circular flange, the amplitude of the oscillation decrease and the results tend to those of the infinite flange.

The effect of the finite flange plane on radiation has been investigated in Fig. 2.7 where we have plotted the directivity on the E-plane at 9.5 GHz for flange radius $d = 2$ cm. Since only the upper waveguide has been fed there is a certain

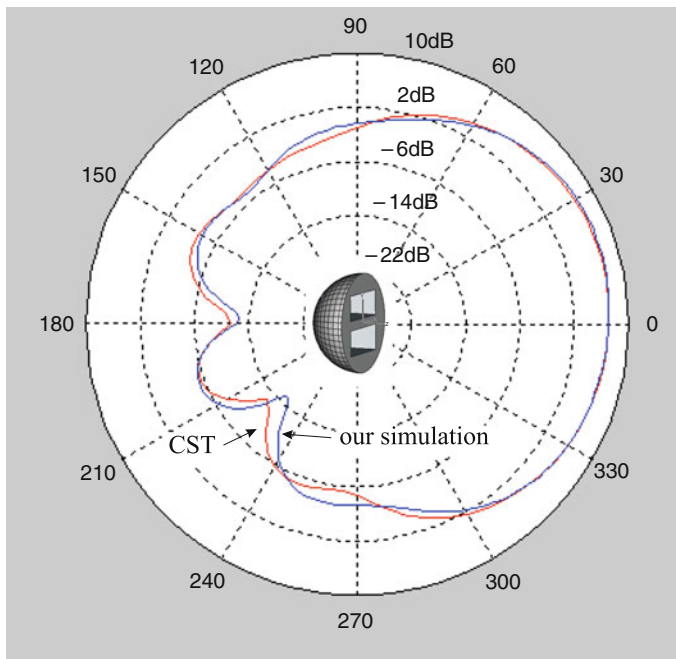


Fig. 2.7 Directivity diagram along the E-plane at 9.5 GHz for the structure of Fig. 2.5 with flange radius $d = 2$ cm when just a waveguide is excited and the other is closed on a matched load. The *continuous line* refers to the spherical mode expansion while the *dashed line* refers to the CST simulation

asymmetry. The result has been checked against CST simulations providing a satisfactory agreement. In particular the max directivity estimated by our program is 7.5 dB, while that estimated by CST is 7.3 dB.

Finally, in Fig. 2.8 we have plotted the three-dimensional radiation diagram for different values of the flange dimensions, as reported in the inset. It is apparent that relatively small values of the flange permit a backside radiation which, as expected, is almost completely eliminated with larger flanges.

2.4 Conclusions

We have considered the problem of an array of rectangular waveguides mounted on a *finite* circular flange plane and radiating into free-space. The use of spherical transmission lines allows a systematic description of the radiation problem and a rigorous network representation. The effect of the *finite* circular flange plane has been rigorously investigated.

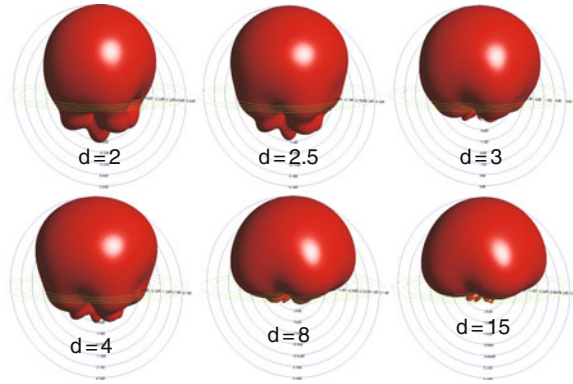


Fig. 2.8 Radiation diagram for the structure of Fig. 2.5 at 9.5 GHz obtained by exciting both waveguides with the same amplitude and phase. The figure refers to a circular flange plane with different radii ‘d’ (expressed in cm) and shows how the radiated field changes with the flange dimensions

Appendix

Considering spherical and hemispherical modes in (2.9)–(2.10), by recalling that superscript (*s*) refers to spherical modes while superscript (*h*) refers to hemispherical modes, the coupling integral (2.16) can be conveniently written as:

$$TE_{e_{m_1, n_1}}^{(s)} - TE_{e_{m_2, n_2}}^{(h)}$$

$$g_{i,j} = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } m_1 = m_2 \neq 0 \text{ and } n_1 = n_2 \\ 0 & \text{Otherwise} \end{cases} \quad (2.17)$$

$$TE_{e_{m_1, n_1}}^{(s)} - TM_{o_{m_2, n_2}}^{(h)}$$

$$g_{i,j} = 0 \quad (2.18)$$

$$TE_{o_{m_1, n_1}}^{(s)} - TE_{e_{m_2, n_2}}^{(h)}$$

$$g_{i,j} = 0 \quad \begin{matrix} \text{for } m_1 + m_2 \text{ even} \\ \text{for } n_1 + n_2 \text{ even} \end{matrix} \quad (2.19)$$

otherwise:

$$g_{i,j} = A^{(s)} A^{(h)} \frac{m_1 [1 - (-1)^{m_1+m_2}]}{(m_1 + m_2)(m_1 - m_2)} \quad (2.20)$$

$$\left\{ m_2^2 \int_0^\pi P_{n_1}^{m_1}(\cos \theta) P_{n_2}^{m_2}(\cos \theta) \frac{d\theta}{\sin \theta} \right.$$

$$\left. + \int_0^\pi \frac{d}{d\theta} P_{n_1}^{m_1}(\cos \theta) \frac{d}{d\theta} P_{n_2}^{m_2}(\cos \theta) \sin \theta d\theta \right\}$$

$$TE_{o_{m_1, n_1}}^{(s)} - TM_{o_{m_2, n_2}}^{(h)}$$

$$g_{i,j} = 0 \quad (2.21)$$

$$TM_{e_{m_1, n_1}}^{(s)} - TE_{e_{m_2, n_2}}^{(h)}$$

$$g_{i,j} = 0 \quad \begin{array}{l} \text{for } m_1 + m_2 \text{ even} \\ \text{for } n_1 + n_2 \text{ odd} \end{array} \quad (2.22)$$

otherwise:

$$g_{i,j} = A^{(s)} A^{(h)} \frac{[1 - (-1)^{m_1+m_2}]}{(m_1 + m_2)(m_1 - m_2)} \quad (2.23)$$

$$\left\{ m_2^2 \int_0^\pi \frac{d}{d\theta} P_{n_1}^{m_1}(\cos \theta) P_{n_2}^{m_2}(\cos \theta) d\theta \right.$$

$$\left. + m_1^2 \int_0^\pi P_{n_1}^{m_1}(\cos \theta) \frac{d}{d\theta} P_{n_2}^{m_2}(\cos \theta) d\theta \right\}$$

$$TM_{e_{m_1, n_1}}^{(s)} - TM_{o_{m_2, n_2}}^{(h)}$$

$$g_{i,j} = 0 \quad \begin{array}{l} \text{for } m_1 + m_2 \text{ even} \\ \text{for } n_1 + n_2 \text{ even} \end{array} \quad (2.24)$$

otherwise:

$$g_{i,j} = A^{(s)} A^{(h)} \frac{m_2 [1 - (-1)^{m_1+m_2}]}{(m_1 + m_2)(m_2 - m_1)} \quad (2.25)$$

$$\left\{ \int_0^\pi \frac{d}{d\theta} P_{n_1}^{m_1}(\cos \theta) \frac{d}{d\theta} P_{n_2}^{m_2}(\cos \theta) \sin(\theta) d\theta \right.$$

$$\left. + m_1^2 \int_0^\pi P_{n_1}^{m_1}(\cos \theta) P_{n_2}^{m_2}(\cos \theta) \frac{d\theta}{\sin(\theta)} \right\}$$

$$TM_{o_{m_1, n_1}}^{(s)} - TE_{e_{m_2, n_2}}^{(h)} \quad g_{i,j} = 0 \quad (2.26)$$

$$TM_{o_{m_1, n_1}}^{(s)} - TM_{o_{m_2, n_2}}^{(h)} \quad g_{i,j} = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } m_1 = m_2 \text{ and } n_1 = n_2 \\ 0 & \text{Otherwise} \end{cases} \quad (2.27)$$

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