

# Contents

<b>Introduction</b> .....	xv
Notation and Conventions .....	xviii
<b>1 Algebraic Homogeneous Spaces</b> .....	1
1 Homogeneous Spaces .....	1
1.1 Basic Definitions .....	1
1.2 Tangent Spaces and Automorphisms .....	3
2 Fibrations, Bundles, and Representations .....	3
2.1 Homogeneous Bundles .....	3
2.2 Induction and Restriction .....	5
2.3 Multiplicities .....	6
2.4 Regular Representation .....	6
2.5 Hecke Algebras .....	7
2.6 Weyl Modules .....	8
3 Classes of Homogeneous Spaces .....	10
3.1 Reductions .....	10
3.2 Projective Homogeneous Spaces .....	11
3.3 Affine Homogeneous Spaces .....	11
3.4 Quasiaffine Homogeneous Spaces .....	13
<b>2 Complexity and Rank</b> .....	15
4 Local Structure Theorems .....	15
4.1 Locally Linearizable Actions .....	15
4.2 Local Structure of an Action .....	16
4.3 Local Structure Theorem of Knop .....	19
5 Complexity and Rank of $G$ -varieties .....	20
5.1 Basic Definitions .....	20
5.2 Complexity and Rank of Subvarieties .....	20
5.3 Weight Semigroup .....	22
5.4 Complexity and Growth of Multiplicities .....	22

6	Complexity and Modality . . . . .	24
6.1	Modality of an Action . . . . .	24
6.2	Complexity and $B$ -modality . . . . .	25
6.3	Adherence of $B$ -orbits . . . . .	26
6.4	Complexity and $G$ -modality . . . . .	27
7	Horospherical Varieties . . . . .	28
7.1	Horospherical Subgroups and Varieties . . . . .	28
7.2	Horospherical Type . . . . .	30
7.3	Horospherical Contraction . . . . .	30
8	Geometry of Cotangent Bundles . . . . .	31
8.1	Symplectic Structure . . . . .	31
8.2	Moment Map . . . . .	31
8.3	Localization . . . . .	32
8.4	Logarithmic Version . . . . .	33
8.5	Image of the Moment Map . . . . .	33
8.6	Corank and Defect . . . . .	35
8.7	Cotangent Bundle and Geometry of an Action . . . . .	36
8.8	Doubled Actions . . . . .	37
9	Complexity and Rank of Homogeneous Spaces . . . . .	39
9.1	General Formulæ . . . . .	39
9.2	Reduction to Representations . . . . .	41
10	Spaces of Small Rank and Complexity . . . . .	43
10.1	Spaces of Rank $\leq 1$ . . . . .	43
10.2	Spaces of Complexity $\leq 1$ . . . . .	44
11	Double Cones . . . . .	46
11.1	HV-cones and Double Cones . . . . .	47
11.2	Complexity and Rank . . . . .	49
11.3	Factorial Double Cones of Complexity $\leq 1$ . . . . .	51
11.4	Applications to Representation Theory . . . . .	52
11.5	Spherical Double Cones . . . . .	55
<b>3</b>	<b>General Theory of Embeddings . . . . .</b>	<b>57</b>
12	The Luna–Vust Theory . . . . .	57
12.1	Equivariant Classification of $G$ -varieties . . . . .	57
12.2	Universal Model . . . . .	58
12.3	Germes of Subvarieties . . . . .	60
12.4	Morphisms, Separation, and Properness . . . . .	61
13	$B$ -charts . . . . .	62
13.1	$B$ -charts and Colored Equipment . . . . .	62
13.2	Colored Data . . . . .	63
13.3	Local Structure . . . . .	65
14	Classification of $G$ -models . . . . .	66
14.1	$G$ -germs . . . . .	66
14.2	$G$ -models . . . . .	67
15	Case of Complexity 0 . . . . .	68

15.1	Combinatorial Description of Spherical Varieties . . . . .	68
15.2	Functoriality . . . . .	70
15.3	Orbits and Local Geometry . . . . .	71
16	Case of Complexity 1 . . . . .	72
16.1	Generically Transitive and One-parametric Cases . . . . .	72
16.2	Hyperspace . . . . .	73
16.3	Hypercones . . . . .	75
16.4	Colored Data . . . . .	77
16.5	Examples . . . . .	80
16.6	Local Properties . . . . .	84
17	Divisors . . . . .	84
17.1	Reduction to $B$ -stable Divisors . . . . .	84
17.2	Cartier Divisors . . . . .	85
17.3	Case of Complexity $\leq 1$ . . . . .	86
17.4	Global Sections of Line Bundles . . . . .	89
17.5	Ample Divisors . . . . .	91
18	Intersection Theory . . . . .	94
18.1	Reduction to $B$ -stable Cycles . . . . .	94
18.2	Intersection of Divisors . . . . .	95
18.3	Divisors and Curves . . . . .	99
18.4	Chow Rings . . . . .	100
18.5	Halphen Ring . . . . .	101
18.6	Generalization of the Bézout Theorem . . . . .	102
<b>4</b>	<b>Invariant Valuations . . . . .</b>	<b>105</b>
19	$G$ -valuations . . . . .	106
19.1	Basic Properties . . . . .	106
19.2	Case of a Reductive Group . . . . .	107
20	Valuation Cones . . . . .	108
20.1	Hyperspace . . . . .	108
20.2	Main Theorem . . . . .	110
20.3	A Good $G$ -model . . . . .	110
20.4	Criterion of Geometricity . . . . .	111
20.5	Proof of the Main Theorem . . . . .	112
20.6	Parabolic Induction . . . . .	114
21	Central Valuations . . . . .	115
21.1	Central Valuation Cone . . . . .	115
21.2	Central Automorphisms . . . . .	116
21.3	Valuative Characterization of Horospherical Varieties . . . . .	118
21.4	$G$ -valuations of a Central Divisor . . . . .	118
22	Little Weyl Group . . . . .	119
22.1	Normalized Moment Map . . . . .	119
22.2	Conormal Bundle to General $U$ -orbits . . . . .	120
22.3	Little Weyl Group . . . . .	121
22.4	Relation to Valuation Cones . . . . .	123

23	Invariant Collective Motion	124
23.1	Polarized Cotangent Bundle	124
23.2	Integration of Invariant Collective Motion	125
23.3	Flats and Their Closures	126
23.4	Non-symplectically Stable Case	128
23.5	Proof of Theorem 22.13	129
23.6	Sources	130
23.7	Root System of a $G$ -variety	131
24	Formal Curves	132
24.1	Valuations via Germs of Curves	132
24.2	Valuations via Formal Curves	133
<b>5</b>	<b>Spherical Varieties</b>	<b>135</b>
25	Various Characterizations of Sphericity	136
25.1	Spherical Spaces	136
25.2	“Multiplicity-free” Property	137
25.3	Weakly Symmetric Spaces and Gelfand Pairs	138
25.4	Commutativity	139
25.5	Generalizations	142
26	Symmetric Spaces	145
26.1	Algebraic Symmetric Spaces	145
26.2	$\theta$ -stable Tori	146
26.3	Maximal $\theta$ -fixed Tori	147
26.4	Maximal $\theta$ -split Tori	148
26.5	Classification	150
26.6	Weyl Group	154
26.7	$B$ -orbits	154
26.8	Colored Equipment	155
26.9	Coisotropy Representation	157
26.10	Flats	157
27	Algebraic Monoids and Group Embeddings	158
27.1	Algebraic Monoids	158
27.2	Reductive Monoids	160
27.3	Orbits	162
27.4	Normality and Smoothness	164
27.5	Group Embeddings	165
27.6	Enveloping and Asymptotic Semigroups	168
28	S-varieties	169
28.1	General S-varieties	169
28.2	Affine Case	170
28.3	Smoothness	173
29	Toroidal Embeddings	173
29.1	Toroidal Versus Toric Varieties	174
29.2	Smooth Toroidal Varieties	174
29.3	Cohomology Vanishing	176

29.4	Rigidity .....	177
29.5	Chow Rings .....	178
29.6	Closures of Flats .....	178
30	Wonderful Varieties .....	179
30.1	Standard Completions .....	179
30.2	Demazure Embedding .....	181
30.3	Case of a Symmetric Space .....	182
30.4	Canonical Class .....	183
30.5	Cox Ring .....	183
30.6	Wonderful Varieties .....	187
30.7	How to Classify Spherical Subgroups .....	188
30.8	Spherical Spaces of Rank 1 .....	189
30.9	Localization of Wonderful Varieties .....	191
30.10	Types of Simple Roots and Colors .....	193
30.11	Combinatorial Classification of Spherical Subgroups and Wonderful Varieties .....	194
30.12	Proof of the Classification Theorem .....	196
31	Frobenius Splitting .....	201
31.1	Basic Properties .....	201
31.2	Splitting via Differential Forms .....	202
31.3	Extension to Characteristic Zero .....	204
31.4	Spherical Case .....	205
<b>Appendices</b> .....		207
A	Algebraic Geometry .....	207
A.1	Rational Singularities .....	207
A.2	Mori Theory .....	208
A.3	Schematic Points .....	211
B	Geometric Valuations .....	212
C	Rational Modules and Linearization .....	214
D	Invariant Theory .....	216
E	Hilbert Schemes .....	220
E.1	Classical Case .....	220
E.2	Nested Hilbert Scheme .....	222
E.3	Invariant Hilbert Schemes .....	223
<b>References</b> .....		227
<b>Name Index</b> .....		239
<b>Subject Index</b> .....		243
<b>Notation Index</b> .....		249



<http://www.springer.com/978-3-642-18398-0>

Homogeneous Spaces and Equivariant Embeddings

Timashev, D.A.

2011, XXII, 254 p., Hardcover

ISBN: 978-3-642-18398-0