Chapter 2
Basic Characteristics of Electromagnetic Radiation

2.1 Radiation Characteristics in the Classical and Quantum Electrodynamics

In case of charged particle motion in an external field, one of the most fruitful approaches allowing to calculate the characteristics of radiation, generated by a particle with charge e, is an approach, where a trajectory $\mathbf{r}(t)$ of the particle in the given field has been found at first, and then the electric and magnetic components of the electromagnetic field are defined according to the rules of classical electrodynamics [1]:

$$\mathbf{E}(t) = \frac{e\left(1-\beta^2\right)(\mathbf{n}-\boldsymbol{\beta})}{R^2(1-\mathbf{n}\,\boldsymbol{\beta})^3} + \frac{e\left[\mathbf{n}\left[(\mathbf{n}-\boldsymbol{\beta})\,\dot{\boldsymbol{\beta}}\right]\right]}{\mathrm{c}R(1-\mathbf{n}\,\boldsymbol{\beta})^3}, \tag{2.1.1a}$$

$$\boldsymbol{H}(t) = [\mathbf{n}(t')\mathbf{E}(t)]. \tag{2.1.1b}$$

In these expressions $c\beta = \dot{r}(t)$, $\mathbf{n}$ is a unit vector in direction connecting the observation point with a charge at the retarded moment of time t',

$$t - t' = \frac{|\mathbf{R}-\mathbf{r}(t')|}{c}. \tag{2.1.2}$$

Here $\mathbf{R}$ is a radius-vector of the observation point.

It is clear, the similar approach gives the reasonable results in a case when it is possible to neglect the particle energy losses due to photon emission (radiation losses), i.e. when the process of radiation has no influence upon a trajectory of the particle.

The first summand term in the formula (2.1.1a) being proportional to R^{-2} does not depend on acceleration of the charge $\dot{\boldsymbol{\beta}}$ and characterizes the quasi-stationary Coulomb field of the moving charge itself (so called "velocity field") while the

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second summand being inversely proportional to the distance R and depending on the charge acceleration, characterizes the radiation wave field ("acceleration field") [1]. The range of the distances R, where the contribution of the first summand is negligible in comparison with the contribution of the second one, refers to the wave (or far-field) zone. In the wave zone both components of the field (2.1.1a) and (2.1.1b) are perpendicular to the vector $\mathbf{n}$ that allows to introduce the Poynting's vector $\mathbf{S} = [\mathbf{EH}]$, directed along a wave vector and describing the density of the energy flow of the electromagnetic wave.

The angular distribution of an energy flow (intensity) in a solid angle $d\Omega$ (the value defined in the observation point) is determined through the Poynting's vector:

$$\frac{dI}{d\Omega} = \frac{cR^2}{4\pi}|\mathbf{S}| = \frac{cR^2}{4\pi}|\mathbf{E}|^2. \tag{2.1.3}$$

The angular distribution of the power of particle radiation losses (with a value determined in a particle position) is connected with intensity (2.1.3) as follows:

$$\frac{dP}{d\Omega} = (1 - \mathbf{n}\boldsymbol{\beta})\frac{dI}{d\Omega}. \tag{2.1.4}$$

Going over to Fourier-components of a field, it is possible to get the expressions

$$\begin{aligned} \mathbf{E}(\omega) &= \frac{e}{cR}e^{ikR}\int \frac{\left[\mathbf{n}\left[(\mathbf{E} - \boldsymbol{\beta})\dot{\boldsymbol{\beta}}\right]\right]}{(1 - \boldsymbol{\beta}\mathbf{n})^2}e^{i(\omega t - kr)}dt, \\ H(\omega) &= [\mathbf{n}\,\mathbf{E}(\omega)]. \end{aligned} \tag{2.1.5}$$

Substituting the received expressions in (2.1.3), it is possible to receive the spectral–angular distributions:

$$\frac{dI}{d\omega\, d\Omega} = \frac{cR^2}{4\pi}|\mathbf{E}(\omega)|^2. \tag{2.1.6}$$

As a rule, the radiation is formed by a source with a finite area S, moreover, this source can emit the electromagnetic waves (the photons) anisotropically. In this case the radiation is characterized by *brightness*

$$L = \frac{dP}{d\Omega\ dS}\left[\frac{W}{\mathrm{sr} \times \mathrm{m}^2}\right] \tag{2.1.7}$$

and *spectral brightness*:

$$\frac{dL}{d\omega} = \frac{dP}{d\omega\ d\Omega\ dS}\left[\frac{W}{\mathrm{s}^{-1} \times \mathrm{sr} \times \mathrm{m}^2}\right]. \tag{2.1.8}$$

For the radiation with frequencies from optical and above ones the spectral brightness is often assigned through the number of photons. Using the Planck's law $\varepsilon = \hbar\omega$ in semi-classical approach, the energy characteristics are expressed through the number of photons N:

$$dP = \varepsilon \frac{dN}{dt}. \tag{2.1.9}$$

Then instead the spectral brightness one may use the *brilliance*

$$\frac{dL}{d\varepsilon} = B = \frac{dN}{dt\; d\Omega\;\; dS\, d\varepsilon/\varepsilon}\left[\frac{\text{photon}}{\text{s} \times \text{sr} \times \text{m}^2 \times d\varepsilon/\varepsilon}\right]. \tag{2.1.10}$$

The spectral–angular density of radiation is got after the integration on the source area

$$I(\theta, \psi, \varepsilon) = \int\limits_S B\, dx\, dy \left[\frac{\text{photon}}{s \times sr \times \Delta\varepsilon/\varepsilon}\right]. \tag{2.1.11}$$

The spectral flux (spectral density) is calculated after the integration over a solid angle

$$\Phi_{\text{S}}(\varepsilon) = \int B\, dx\, dy\, d\Omega \left[\frac{\text{photon}}{\text{s} \times \Delta\varepsilon/\varepsilon}\right]. \tag{2.1.12}$$

And finally, the radiation flux is received via the integration over a spectrum:

$$\Phi = \int \Phi_{\text{S}}(\varepsilon)\, d\varepsilon/\varepsilon \left[\frac{\text{photon}}{\text{s}}\right]. \tag{2.1.13}$$

The field strength of the monochromatic electromagnetic wave (for example, the laser radiation) is characterized by the dimensionless parameter:

$$a_0 = \sqrt{\frac{2e^2\langle A^2\rangle}{(mc^2)^2}} = \frac{e\, E_0}{mc\, \omega}. \tag{2.1.14}$$

In the last formula by $\langle A^2 \rangle$ a mean-square value of an electromagnetic vector potential is designated, E_0 is an amplitude of a wave.

In the majority of experiments the beams of the electromagnetic radiation, formed by means of different optical systems, including, for instance, mirrors, apertures, lenses, etc. are used. In this case, the radiation power can be distributed on the area of the target according to an arbitrary law. Then after the integration with respect to the beam cross-section, we can receive:

$$P = \int\limits_\sigma \frac{dP}{dS} d\sigma = I\sigma_{\text{eff}}, \tag{2.1.15}$$

where $I = \langle dP/dS \rangle$ is an averaged value of the power flux density, σ_{eff} is an effective area of the beam. In the laser physics, the parameter laser field strength [2] is often used

$$I = \frac{P}{\sigma_{\text{eff}}}, \qquad [I] = \text{W/cm}^2, \tag{2.1.16}$$

which can be expressed through the density of the energy of the laser flash ρ:

$$I = \frac{P \times c\tau}{\sigma_{\text{eff}} \times c\tau} = c\frac{W}{V} = c\rho. \tag{2.1.17}$$

In the last expression through τ is designated the flash duration, V is a volume, occupied with laser photons. Then instead of (2.1.14), it is possible to receive a more evident formula:

$$a_0^2 = \frac{2r_0 I \lambda^2}{\pi\, mc^3} = \frac{2r_0 \lambda^2\, \rho}{\pi\, mc^2}, \tag{2.1.18}$$

where $r_0 = 2.82 \times 10^{-13}$ is the classical radius of an electron, as well as the "engineering" formula:

$$a_0 = 0.85 \times 10^{-9} \lambda\ [\mu]\ I^{1/2} \left[\text{W/cm}^2\right]. \tag{2.1.19}$$

In formulas (2.1.18) and (2.1.19), λ is a length of a monochromatic wave.

Going from the energy density to the concentration of photons per volume unit n: $n = \rho/\hbar\omega$, it is possible to receive the estimation of (2.1.18) through the number of photons in a volume $4\alpha\, \lambda_e^2\, \lambda$, i.e. in a parallelepiped with transverse cross section $\bar{\lambda}_e^2$ ($\bar{\lambda}_e$ is the Compton wavelength of the electron) and length $4\alpha\lambda$:

$$a_0^2 = 4\alpha\, \lambda_{\text{e}}^2\, \lambda\, n, \tag{2.1.20}$$

$\alpha = 1/137$ is the fine structure constant.

For a field strength parameter $a_0 \geq 1$, it is spoken about the "strong" electromagnetic wave, whereas the "linear" model of the classical electrodynamics remains valid for $a_0 \ll 1$.

2.2 Polarization Characteristics of Radiation

Hereinafter, the usage of the term "the photon beam" supposes that it concerns the electromagnetic radiation propagating along the fixed direction with a negligibly small angular divergence, the characteristics of which (intensity, polarization, position of maximum in spectrum, temporal modulation, etc.) are possible to adjust in a rather large range.

A single photon, i.e. an elementary particle with a spin equal to 1, definitionally exists in a pure spin state (just as the flat monochromatic electromagnetic wave—a classical analogue of a photon—is always completely polarized). There is a whole ensemble of photons in a real beam, therefore, for the description of a beam polarization as a whole (after averaging on ensemble), the matrix of the density ρ_{ij} (Hermitian tensor of the second rank determined in a plane, which is perpendicular to a direction of photon beam propagation) is used:

$$\rho_{ij} = \frac{1}{2}\begin{pmatrix} 1+\xi_3 & \xi_1 - i\xi_2 \\ \xi_1 + i\xi_2 & 1-\xi_3 \end{pmatrix} = \frac{1}{2}\left(\delta_{ij} + \boldsymbol{\xi\sigma}\right), \tag{2.2.1}$$

where $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are the Pauli matrices.

Three real-valued parameters ξ_i ($i = 1,\ 2,\ 3$)—so-called Stokes parameters completely describe a polarization state of a photon beam. The Stokes parameters ξ_1, ξ_3 characterize the linear polarization of a beam, and ξ_2 the circular one. The values $\xi_1^2 + \xi_3^2$ and ξ_2 are the Lorentz-invariants. Parameters ξ_1, ξ_3 are scalars, and ξ_2 is pseudo-scalar.

In case when none of Stokes parameters is equal to zero, it is spoken about elliptic polarization, and when $\xi_2 = 0$—about linear polarization of the radiation.

In the last case, the following values are often used instead of the Stokes parameters:

$$P = \sqrt{\xi_1^2 + \xi_3^2} \tag{2.2.2}$$

is a degree of polarization;

$$\varphi_0 = (1/2)\,\mathrm{arctg}\,(\xi_1/\xi_3) \tag{2.2.3}$$

—the inclination angle of a plane of the maximal linear polarization concerning the chosen system of basis vectors (for instance, concerning a plane *XZ*, if *Z*-axis is directed along a photon beam direction).

The degree of linear polarization P can be determined as follows:

$$P = \left(N_{||} - N_{\perp}\right)/\left(N_{||} + N_{\perp}\right), \tag{2.2.4}$$

where $N_{||(\perp)}$ is the number of the photons polarized parallel (perpendicularly) to a plane of the maximal linear polarization.

Reverse transition to the Stokes parameters follows from (2.2.2), (2.2.3):

$$\xi_1 = P\,\sin(2\varphi_0); \quad \xi_3 = P\,\cos(2\varphi_0). \tag{2.2.5}$$

An unpolarized beam can be always presented as superposition of two non-interacting completely polarized beams of photons with identical intensity and with mutually perpendicular planes of polarization. Similarly, it is possible to present a partly polarized photon beam (for which $0 < \xi_1^2 + \xi_2^2 + \xi_3^2 < 1$) as superposition of completely polarized and non-polarized beams with various intensities.

In the classical electrodynamics, the Stokes parameters are calculated as follows:

$$\xi_1 = \frac{E_1^* E_2 + E_1 E_2^*}{|E_1|^2 + |E_2|^2}, \quad \xi_2 = i\frac{E_1^* E_2 - E_1 E_2^*}{|E_1|^2 + |E_2|^2}, \quad \xi_3 = \frac{|E_1|^2 - |E_2|^2}{|E_1|^2 + |E_2|^2}. \tag{2.2.6}$$

The components of the field are calculated in a system, where the third axis coincides with the direction of a wave vector. If the task has any chosen plane, the coordinate system is assigned via basis vectors. If in problem there is a chosen plane, then the coordinate system is

$$\mathbf{e}_1 = c_1[\mathbf{n}, \mathbf{b}]; \quad \mathbf{e}_2 = [\mathbf{e}_1, \mathbf{n}], \quad \mathbf{n} = \mathbf{k}/\omega, \tag{2.2.7}$$

where $\mathbf{b}$ is the vector, perpendicular to the chosen plane; $\mathbf{k}$ is a wave vector; ω is a frequency; c_1 is a normalization factor.

For the radiation of ultrarelativistic particles the cone of outgoing photons has an opening of order γ^{-1} (γ is the Lorentz-factor) relative to the average value of the electron momentum. Therefore, it is possible to speak about the mean polarization of a beam (with accuracy to γ^{-2}) if the radiation cone is formed by the aperture with opening $\Delta\Omega \sim \gamma^{-2}$. In this case, for calculation of average Stokes parameters in (2.2.6), it is necessary to use the bilinear combinations of fields $\langle E_i^* E_k \rangle$, averaged on the given angular interval:

$$\langle E_i^* \, E_k \rangle = \int\limits_{\Delta\Omega} d\Omega \, E_i^* \, E_k, \quad i, k = 1, 2. \tag{2.2.8}$$

Generally speaking, the averaging similar to (2.2.8) can be carried out not only by the angular variables but also by any other non-observable kinematic ones. Thus, during the calculation of polarization characteristics of coherent bremsstrahlung, the averaging similar to (2.2.8) is carried out by the momentum of a final electron [3].

2.3 The Formation Length of Radiation by a Charged Particle

Ter-Mikaelyan in his monograph [4] considering the spatial region, in which the bremsstrahlung is generated by ultrarelativistic electron moving in a medium, has shown that the longitudinal size of this region (along the direction of the initial electron) sharply increases with the growth of the electron Lorentz-factor and with decrease of the photon energy. This spatial scale, which was named “formation length” ℓ_f, can have macroscopic sizes greatly exceeding the wavelength of the bremsstrahlung photon. After the passage of the length ℓ_f, the electron and emitted photon can be considered as independent particles.

The estimation of this spatial scale can be found from classical electrodynamics (see, for example, [5]). In this approach, the charge, which passes through a rather small area and where external fields are concentrated, is emitted an electromagnetic wave with the length λ without appreciable distortion of a charge trajectory and the change of its energy (see Fig. 2.1).

The determination of the formation length follows from the phase relationships: on the length ℓ_f, which a charge passes after the area of a field at velocity β, the front of a wave, emitted in angle θ, should "lag behind"a charge for a wave length:

$$\frac{\ell_\mathrm{f}}{\beta} - \ell_\mathrm{f}\cos\theta = \lambda, \tag{2.3.1}$$

and (2.3.1) directly results in the formula for the formation length:

$$\ell_\mathrm{f} = \frac{\lambda}{1/\beta - \cos\theta}. \tag{2.3.2}$$

In the ultrarelativistic approach $(1/\beta \approx 1 + \gamma^{-2}/2)$for the "straightforward" radiation we have

$$\ell_\mathrm{f} = 2\gamma^2\lambda. \tag{2.3.3}$$

If the following area of a field concentration is located along a trajectory on the distance $L < \ell_f$ (see Fig. 2.1), then in this case the electromagnetic waves, emitted by a charge in two areas of an external field, will interfere in a destructive manner, i.e. the intensity of resulting radiation will be less than the sum of intensities from two independent sources.

Let carry out the quantum consideration of the formation length problem on an example of bremsstrahlung, following to Ter-Mikaelyan [4].

We shall estimate the minimal value of a longitudinal recoil momentum q_l, which is transferred to a nucleus, during the process of bremsstrahlung of the ultrarelativistic electron with energy ε_1. Such situation is realized for collinear geometry, when the final electron with energy ε_2 and a photon with energy $\hbar\omega$ move along the direction of the initial electron:

$$q_{l\ \min} = p_1 - p_2 - k. \tag{2.3.4}$$

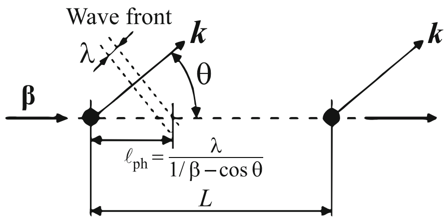

Fig. 2.1 The scheme illustrates the concept of the formation length

Here p_1, p_2, k are momenta of initial and final electrons and photons, accordingly. Neglecting the energy transferred to a nucleus (i.e. in case of fulfillment of a condition $\varepsilon_1 = \varepsilon_2 + \hbar\omega$), momentum p_i in the ultrarelativistic approach becomes

$$p_1 = \frac{\varepsilon_1}{c}\left(1 - \frac{1}{2\gamma_1^2}\right), \quad p_2 = \frac{\varepsilon_1 - \hbar\omega}{c}\left(1 - \frac{1}{2\gamma_2^2}\right),$$

and (2.3.4) results in

$$q_{l\,\min} = \frac{mc}{2\gamma_1}\frac{\hbar\omega}{\varepsilon_2}. \tag{2.3.5}$$

From the uncertainty principle it follows that the last expression defines the length:

$$\ell = \frac{h}{q_{l\,\min}} = 2\gamma_1 \lambda\!\!\!^{-}_e \frac{\varepsilon_2}{\hbar\omega}, \tag{2.3.6}$$

where $\lambda\!\!\!^{-}_e$ is the Compton wavelength of an electron. It is clear that for the case $\hbar\omega \ll \varepsilon_1, \varepsilon_2$ (i.e. $\varepsilon_2 \approx \varepsilon_1$) from the formula (2.3.6) follows the expression (2.3.3):

$$\ell = 2\gamma^2 \lambda = \ell_{\mathrm{f}}$$

that illustrates the generality of the concept of the formation length both for quantum consideration, where recoil effects are important, and for classical one.

The concept of the formation length plays an important role in considering of various physical effects (see in detail the review [6]). With regard to the radiation in periodic structures, where a constructive interference is the reason of monochromaticity of the radiation spectrum (for the fixed radiation angle θ), the wavelength corresponding to the spectral line with minimal frequency (so-called “fundamental” harmonic), is defined from the relationship

$$\ell_{\mathrm{f}} = d, \tag{2.3.7}$$

where d is a period of the structure.

Expression (2.3.7) does not depend on the radiation mechanism and is applicable both in classical electrodynamics (for instance, for undulator radiation or Smith–Purcell radiation), and in quantum one (the typical example is the coherent bremsstrahlung). The mentioned mechanisms, as well as some others, are considered in the following chapters of this book.

2.4 Interference Factor and the Resonance Condition

Let us consider the electromagnetic radiation of the charge moving on a flat periodic trajectory (Fig. 2.2). Let us designate through $\mathbf{E}_1(\mathbf{k})$ the radiation field on the first period, where $\mathbf{k}$ is a wave vector; $\Delta t_e = d/\beta_{\|}c$ is time of the electron

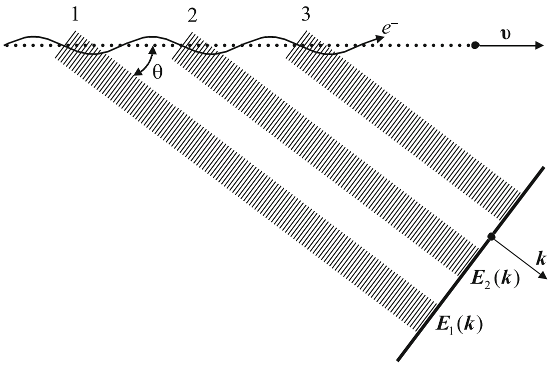

Fig. 2.2 Constructive interference of the electromagnetic radiation in the periodic structure

passing with velocity $\beta_{||}c$ through the first period; $\Delta t_K = d\cos\theta/c$ is time of the wave front passing from the first period till identical position on the second period.

The phase difference of two wave packages generated by electron on the first and second periods are the follows:

$$\Phi = \omega(\Delta t_e - \Delta t_K) = \frac{2\pi}{\lambda}\beta_{||}c\left(\frac{d}{\beta_{||}c} - \frac{d\cos\theta}{c}\right) = 2\pi\frac{d}{\lambda}\left(1 - \beta_{||}\cos\theta\right). \quad (2.4.1)$$

Thus, the field of radiation on the second period is defined by the expression

$$\mathbf{E}_2(\mathbf{k}) = \mathbf{E}_1(\mathbf{k})\ \exp(i\Phi). \quad (2.4.2)$$

Reasoning by analogy, it is possible to express the radiation field for the nth period as:

$$\mathbf{E}_n(\mathbf{k}) = \mathbf{E}_1(\mathbf{k})\ \exp(i(n-1)\Phi). \quad (2.4.3)$$

Then the total field from the periodic structure containing N elements is represented as the sum

$$\begin{aligned}\mathbf{E}_\Sigma(\mathbf{k}) &= \mathbf{E}_1(\mathbf{k}) + \mathbf{E}_2(\mathbf{k}) + \mathbf{E}_3(\mathbf{k}) + \cdots + \mathbf{E}_\mathrm{N}(\mathbf{k}) \\ &= \mathbf{E}_1(\mathbf{k})\{1 + \exp(i\Phi) + \exp(i\,2\,\Phi) + \cdots + \exp(i(N-1)\Phi)\}. \quad (2.4.4)\end{aligned}$$

Having designated ($\exp(i\Phi) = q$), we shall receive an expression for the total intensity of the field:

$$\begin{aligned}\mathbf{E}_\Sigma(\mathbf{k}) &= \mathbf{E}_1(\mathbf{k})\ \left\{1 + q + q^2 + \cdots + q^{N-1}\right\} \\ &= \mathbf{E}_1(\mathbf{k})\frac{1-q^N}{1-q} = \mathbf{E}_1(\mathbf{k})\ \frac{1-\exp(iN\Phi)}{1-\exp(i\Phi)}, \quad (2.4.5)\end{aligned}$$

using the well-known formula for a geometric progression.

The spectral–angular distribution of the radiation intensity can be calculated, knowing the field intensity:

$$\frac{d^2W_\Sigma}{d\omega\, d\Omega} = \text{const}\,|\mathbf{E}_\Sigma(\mathbf{k})|^2$$
$$= \underbrace{\text{const}\,|\mathbf{E}_1(\mathbf{k})|^2}_{\frac{d^2W}{d\omega\, d\Omega}} \frac{|1-\exp\,(iN\Phi)|^2}{|1-\exp\,(i\Phi)|^2} = \frac{d^2W}{d\omega\, d\Omega}F_N. \tag{2.4.6}$$

Here $\frac{d^2W}{d\omega\, d\Omega} = \text{const}\,|\mathbf{E}_1(\mathbf{k})|^2$ describes the radiation "collected" from one period of a trajectory, and a multiplier

$$F_N = \left|\frac{1-\exp(iN\Phi)}{1-\exp(i\Phi)}\right|^2 \tag{2.4.7}$$

refers to as an interference factor, since it describes the interference from N identical radiators.

Using known trigonometric rules, the last formula can be rewritten as

$$F_N = \frac{\sin^2(N\Phi/2)}{\sin^2(\Phi/2)}. \tag{2.4.8}$$

The function F_N has a set of sharp maxima for the values of an argument, which makes a denominator zeroth:

$$\frac{\Phi}{2} = \pi\frac{d}{\lambda_m}\left(1-\beta_\parallel\cos\theta\right) = m\pi, \quad m\,\text{is an integer.}$$

The last formula is reduced to the following expression for the case $\beta \approx \beta_\parallel$

$$\lambda_m = \frac{d}{m}(1-\beta\cos\theta), \tag{2.4.9}$$

which was received regardless to any fixed radiation mechanism and can be applied to any type of radiation, which is characterized by the periodic disturbance of a trajectory. The received relationship is generalization of the resonance condition (2.3.7) for $m \neq 1$.

Frequently, the index $m = 1, 2, 3, \ldots$ refers to harmonic number. The harmonic $m = 1$ for ultrarelativistic particles with frequency

$$\omega_1 = \frac{4\pi\gamma^2 c}{d\left(1+\gamma^2\theta^2\right)} = \frac{2\gamma^2\omega_0}{1+\gamma^2\theta^2} \tag{2.4.10}$$

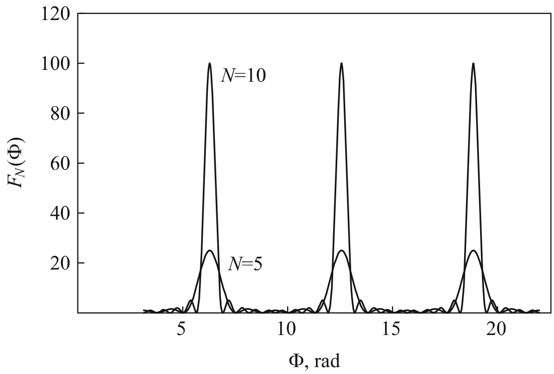

Fig. 2.3 An interference factor for periods $N = 5, 10$

is identified as fundamental. The resonance condition brings to the following conclusion: the frequencies of the higher harmonics in m time differ from fundamental ones:

$$\omega_m = m\,\omega_1. \tag{2.4.11}$$

The diagram of the function F_N is presented in Fig. 2.3 for $\theta = 0$ at $N = 5$ and 10.

As expected, the function F_N differs from zero in a small range of frequencies close by ω_m, and the width of this range is defined by a number of the periods:

$$\frac{\Delta\omega_m}{\omega_m} \sim \frac{1}{N}. \tag{2.4.12}$$

As it follows from the picture, the maximal value of the function is

$$F_{N\ \max} = N^2. \tag{2.4.13}$$

From (2.4.12) and (2.4.13) it follows that the area under the peak is

$$S \sim \Delta\omega_m \times F_{N\ \max} = N\omega_m \tag{2.4.14}$$

and linearly increases with a number of periods.

For big values $N \gg 10$ the function F_N (2.4.7) is approximated well by δ-function:

$$F_N \approx 2\pi N\delta(\Phi - 2m\,\pi) = \frac{N}{m}\delta\left(\frac{\omega}{m\,\omega_0}(1 - \beta\cos\theta) - 1\right). \tag{2.4.15}$$

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