Preface

Computers play an increasingly important role in our society. A breakdown of all computer systems would cause a breakdown of almost all activities of daily life. Furthermore, personal computers are available in almost every home in the industrialized world. But there is one sector where computers have a more strategic role, and that is in science and technology. A large number of physical and engineering problems are solved by the use of advanced computers. The first aircraft were designed by very clever individuals who understood the basic principles of aerodynamics, but today this is not enough. No manufacturer would start building a new aeroplane without extensive computer simulations of various models. Another example where computer simulation is a necessary tool is weather prediction. We know that these predictions are not completely accurate, but are still good enough to get a fairly good idea about the weather for the next few days. The question then is: how is it at all possible to predict the future of a physical system like the atmosphere around the globe? Or in the first example: how is it possible to predict the flight properties of an aircraft that has not yet been built, and where not even a model of the aircraft is available to put in a wind tunnel? No matter how powerful the computers are, we have to provide them with a program that tells them how to carry out the simulation. How is this program constructed?

The fundamental basis for these algorithms is a mathematical model of some kind that provides certain relations between the state variables. These relations lead to a set of equations, and in most cases these equations are *differential equations*. The problem is that these differential equations must be solved, and in most cases they are too difficult to be solved by any mathematician, no matter how sharp. Unfortunately, this is true even for the most powerful computer. This difficulty is overcome by constructing an approximation to the mathematical model, arriving at a numerical model that has a simpler structure based on simple operations like addition and multiplication. The problem usually requires an enormous number of such operations, but nowadays we have access to very fast computers. The state variables, like air pressure and velocity for the weather prediction, are computed by using the numerical model and, if the computer is faster than the weather proceeds in real time, a forecast can be presented for the general public.

This book is about the principles of mathematical and numerical models. We shall put most emphasis on the construction of numerical models, and how it leads to computational mathematics and scientific computing, which is now an indispensable tool in science and engineering. For many applications, the mathematical models were developed one or two centuries ago. There were also numerical models long ago, but the more powerful and robust methods didn't arise until there were electronic computers that could carry out the arithmetic fast enough. Mathematical modeling is usually called applied mathematics, and there are many new areas where this type of mathematics comes into use and is further developed. Numerical modeling is called numerical analysis, or numerical mathematics, and the development is very fast. The success of computer simulation should in fact be credited in equal parts to the development of fast computers and to the development of new numerical methods.

The famous mathematician, physicist and astronomer (and musician) Galileo Galilei (1564–1642) said that "the book of nature is written in the language of mathematics". This is certainly true but, in order to make practical use of it, we also need numerical analysis. By using simplified examples, it is our hope to catch the basics of mathematical/numerical modeling, and in that way explain how the more complicated problems can be solved. In this book we will explain the underlying mathematics in more detail than is usually done in textbooks. Anybody with senior high school mathematics should be able to understand most of the material, but it helps to have basic college mathematics. Scientists and engineers with no or little knowledge about computational mathematics is another group that hopefully will benefit from reading this book as an introduction to the topic. But they can skip the part about basic calculus in Chaps. 2–5. These chapters have a more tutorial style, and are written for those who have forgotten their calculus, or maybe never had much of it.

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