Colliding Hadrons as Cosmic Membranes and Possible Signatures of Lost Momentum

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Abstract We argue that in the TeV-gravity scenario high energy hadrons colliding on the 3-brane embedded in $D = 4+n$-dimensional spacetime, with n dimensions smaller than the hadron size, can be considered as cosmic membranes. In the 5-dimensional case these cosmic membranes produce effects similar to cosmic strings in the 4-dimensional world. We calculate the corrections to the eikonal approximation for the gravitational scattering of partons due to the presence of effective hadron cosmic membranes. Cosmic membranes dominate the momentum lost in the longitudinal direction for colliding particles that opens new channels for particle decays.

1 Introduction

In recent years the study of transplanckian scattering[1] within the TeV-gravity scenario [1] has attracted significant theoretical and phenomenological interest. Within the TeV-gravity scenario [1] transplanckian scattering could be observed at the LHC and other future colliders [2, 3, 4, 5, 6, 7, 8], as well as in collisions of high-energy cosmic neutrinos with atmospheric nucleons [9, 10].

Different physical pictures are expected for different ranges of impact parameters b. For impact parameters b of the order of the Schwarzschild radius R_S of a black hole of mass $\sqrt{s}$, microscopic black hole formation and its subsequent evaporation is expected [11, 12, 13, 14] [2], while for large impact parameters $b \gg R_S$ the eikonal picture given by eikonalized single-graviton exchange is expected [19, 20, 21, 22].

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[1] Scattering at center-of-mass (CM) energies exceeding the quantum gravity scale.

[2] See also [15] and references therein; there are also proposals concerning the production of more complicated objects such as wormholes, or time machines [16, 17, 18].

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Corrections in R_S/b to the elastic eikonal scattering have been studied [23, 24, 25, 26].

To study high-energy scattering of the hadrons one usually deals with the parton picture. In the case of a 3-brane embedded in $D = 4+n$-dimensional spacetime for large impact parameters graviton exchanges dominate in parton amplitudes [19, 21, 23, 22, 3]. In all D dimensions the graviton is supposed to be propagated freely. Since D-dimensional gravity is strong it would be interesting to calculate the modification of the graviton propagator due to a presence of matter. This is difficult problem, however it can be solved in 2+1 gravity, where we know analytically the modification of the spacetime due to the presence of pointlike matter. We know also the modification of the spacetime metric by a cosmic string in 4-dimensional spacetime and by a cosmic membrane in 5-dimensional spacetime.

Due to Lorentz contraction we can treat colliding hadrons in the laboratory frame as membranes with the transversal characteristic scale of order of the hadron and a negligible thickness. These membranes are located on our 3-brane. Since $4+n$ gravity is strong enough we can expect that hadron membranes modify the $4+n$-spacetime metric.

Only for the case of $n=1$ we know explicitly the modified metric and we can estimate explicitly an influence of this modification on the parton and other particle scattering. It is known that the 5-dimensional ADD model with $M_{Pl,5} \sim$ TeV is not phenomenologically acceptable and we can deal with the RS2 model [27] or with the DGP model [28]. In all these cases we treat a moving hadron as an infinite moving membrane in the 5-dimensional world with location on the 3-brane (our world). In other words, we deal with an effective 3-dimensional picture in the high-energy scattering (compare with the usual effective 2-dimensional picture in 4-dimensional spacetime, see [29, 30] and references therein).

In the framework of the picture described above, we can consider the influence of the matter on graviton propagation. Due to the presence of the hadron membrane the gravitational background is nontrivial and describes a flat spacetime with a conical singularity located on the hadron membrane. This picture is a generalization of the cosmological string picture in the 4-dimensional world to the 5-dimensional world. The deficit angle is proportional to the product of the hadron matter density on the membrane and the 5-dimensional gravitational coupling. This is a rather small number [3], $\delta_{h_0} \sim \frac{1}{M^3_{Pl,5}} \frac{M_{hadron}}{l^2_{hadron}} \sim 10^{-9}$. Since the hadrons collide with Lorentz boost factor, $\gamma = 1/\sqrt{1-v^2}$, about $\gamma \sim 10^4$, we have $\delta_h \sim 10^{-5}$. For heavy ions composed of A hadrons, this number is near $\delta_{Ion} \sim A^{1/3}_{Ion}\delta_h$.

We can take into account corrections to the graviton propagation. A study of these corrections and their physical consequences is the subject of the present letter. A more detailed discussion of the topological defects in TeV-gravity including the RS2 and DGP models and will be presented in [33]. As to higher dimensional cases we can just expect that numerical calculations could exhibit similar qualitative results.

[3] One can compare this number with an estimate of the deficit angle $\delta_{cs} \sim 10^{-6}$ for a cosmic string in 4-dimensional spacetime with the Newtonian gravitational constant $G_{N,4}$ and the density $\rho = \frac{m}{l} = 10^{33} GeV^2$, that corresponds to the Earth mass distributed on a length of about $l = 9$ km.

The paper is organized as follows. In Section 2 we present our setup and argue why in the TeV-gravity scenario the high energy hadrons colliding on the 3-brane embedded in $4+n$-dimensional spacetime with n dimensions smaller than the hadrons size, can be considered as cosmic membranes in the $4+n$-dimensional world. We recall basic facts about eikonalization of graviton exchanges and the form of the spacetime metric with a cosmic membrane. In Section 3 we present corrections to the eikonal phase due to a conical singularity. We restrict ourself here to a flat bulk for simplicity. The AdS case corresponding to the RS2 model can be investigated in a similar way. Others possible effects related with cosmic membranes and their signatures are briefly discussed in the conclusion.

2 Setup

It is known that for large impact parameters $b \gg R_S$ (elastic small-angle scattering) the transplanckian amplitude is dominated by eikonalized single-graviton exchange [19],[20],[21],[22]. The eikonal amplitude has been used in [10] to compute the differential cross section for neutrino-nucleon scattering and in [4] to compute the close to beam jet-jet production at the LHC. For small impact parameters $b \ll R_S$ the nonlinear effect are important and within the classical gravity one can expect the black hole formation.

2.1 Hadron as a membrane in 5-dimensional world

The graviton exchange is supposed to take place in the $4+n$-dimensional spacetime. In the total transplanckian cross section there is a factor, describing dependence on n and on the form of the background in the extra dimensional spacetime. In all previous considerations [10, 4, 8] the graviton is supposed to propagate freely in extra dimensions. It would be interesting to be able to calculate the modification of the propagator due to the presence of the hadron matter. This can be done for example in the 2+1 gravity, where we know analytically the modification of the spacetime due to the present of pointlike matter.

In $2+1$ dimensions, solutions to Einstein's equation with point masses are flat metrics except conical singularities at the location of the masses. In $3+1$ dimensions, there are solutions with singularities on the worldsheets of the strings. The deficit angle of the conical singularity is proportional to the mass in the $2+1$ case and the mass per length μ in the $3+1$ case [36]. In $4+1$ dimensions, there is a solution with singularity on the worldsheet of the membrane. One can imagine this membrane as high velocity moving hadron, that in the rest frame is tried as a ball. If we have extra dimensions, they are not available for the hadron and the hadron membrane cannot stretch in these dimensions. Hence, we get the 2-dimensional hadron membrane propagated on the 3-brane embedded in $4+n$-dimensional spacetime.

We know explicit solutions to Einstein's equations with the hadron membrane in the 5-dimensional ADD and RS2 models. The first case is simpler and in spite of it is not phenomenologically acceptable, we consider this case for simplicity [4].

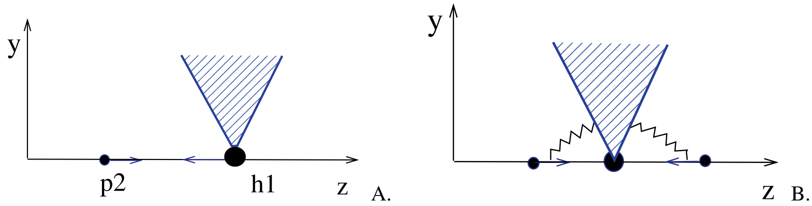

Fig. 1 A. Ultra relativistic colliding hadron h_1 as it seen by the parton p_2. B. The graviton exchange with modified propagator between partons.

2.2 *Bulk with conical singularities*

In the ADD model the metric in the bulk is flat,

$$ds^2 = -dt^2 + dx_\perp^2 + d\rho^2 + \rho^2 d\Omega^2, \; \rho^2 = \sum_1^n y_i^2 + z^2, \; x_\perp = (x_1, x_2) \tag{1}$$

here x_1, x_2, y_i, z are coordinates in the bulk and $d\Omega^2$ is the metric on the unit sphere S^n. However, the hadron membrane produces a nontrivial background. We know this background explicitly for the case of $n = 1$. In this case the bulk metric remains locally flat, $d\Omega^2 = d\phi^2$ and the hadron membrane produces only the conical singularity, i.e. the range of the angle is $0 < \phi < \alpha$. The angle α defines the deficit angle δ

$$\alpha = 2\pi - \delta, \tag{2}$$

where

$$\delta = 8\pi G_5 \frac{m_h}{S_h} = \frac{32}{M_{Pl,5}^3} \frac{m_h}{l_h^2}. \tag{3}$$

Here m_h is the hadron mass and l_h is the hadron size, $S_h = \pi l_h^2/4$. The top of the cone is located on the brane.

The gravitational effect of the hadron membrane in the RS2 model is convenient to present in the Poincaré coordinates. Starting from the metric

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{4}$$

[4] One can assume an anisotropic compactification with essentially suppressed $n-1$ dimensions (in this case $M_{Pl,D} \sim$ TeV and $M_{Pl,5} \sim 10^3$ TeV), or just consider a toy model with $M_{Pl,5toy} \sim 10^3$ TeV.

where $\eta_{\mu\nu}$ is the 4-dimensional Minkowski metric and the warp factor $a(z)$ has the form [27]

$$a(y) = e^{-k|y|}, \tag{5}$$

$1/k$ is the radius of 5-dimensional AdS spacetime, we get the metric in the Poincaré coordinates after the following change of variable, $y \to w$, $w = r_0 e^{y/r_0}$,

$$ds^2 = \frac{r_0^2}{w^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dw^2) \tag{6}$$

[31]. According to the usual prescription to incorporate a membrane we cut a wedge. This can be done by reducing the range of a suitable angular coordinate. For example, for AdS_5

$$\frac{R_5^2}{w^2}\left[dw^2 - dt^2 + dz^2 + d\rho^2 + \rho^2 d\phi^2\right] , \tag{7}$$

and the range of the angle is $0 < \phi < 2\pi - \delta$ where δ is given by (3).

2.3 Eikonalization of graviton exchanges

The parton-parton elastic forward scattering amplitude for a large center of mass energy is given by the eikonal technique [34],[35]. In the transplanckian regime the graviton exchanges [22, 4] dominate and define the amplitude

$$\mathscr{A}_{\rm eik}(\mathbf{q}) = \mathscr{A}_{\rm Born} + \mathscr{A}_{1-loop} + \ldots = -2is\int d^2\mathbf{b}\, e^{-i\mathbf{q}\cdot\mathbf{b}}(e^{i\chi(\mathbf{q})} - 1), \tag{8}$$

where the eikonal phase χ is given by the Fourier transform of the Born amplitude in the transverse plane

$$\chi(\mathbf{b}) = \frac{1}{2s}\int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} A_{Born}(s,\mathbf{q}). \tag{9}$$

The $4+n$-dimensional Born amplitude for the exchange of the graviton, which does not get any transferred momenta in the direction transversal to the brane, is given by

$$\mathscr{A}_{Born}(s,q) = \frac{-s^2}{M_D^{n+2}}\int \frac{d^n l}{\mathbf{q}^2 + l^2},\ |\mathbf{q}| = q. \tag{10}$$

The expression for the eikonal amplitude [10, 4] is

$$\mathscr{A}_{\rm eik} = 4\pi s b_c^2 F_n(b_c q), \tag{11}$$

$$F_n(y) = -i\int_0^\infty dx x J_0(xy)\left(e^{ix^{-n}} - 1\right), \tag{12}$$

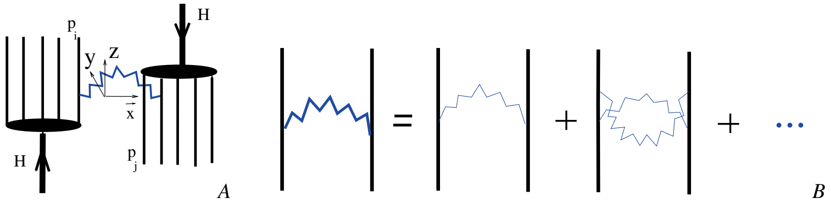

Fig. 2 A. Collision of hardons with a large impact parameter in $(x_\perp, z)$ coordinates is presented as an elastic scattering between partons due to a free graviton exchange. y-coordinate schematically presents extra dimensions. B. The $2 \to 2$ small angle T-scattering amplitude is given by a sum of crossed-ladder graviton exchanges.

where the integration variable is related with the impact parameter, $x = b/b_c$ and in (12) we take into account that the eikonal phase has the power dependence on the impact parameter

$$\chi(b) = \left(\frac{b_c}{b}\right)^n, \text{ where } b_c \equiv \left[\frac{(4\pi)^{\frac{n}{2}-1} s\Gamma(n/2)}{2M_D^{n+2}}\right]^{1/n}. \tag{13}$$

Functions F_n, $n > 1$, when $y \gg 1$ oscillate around their asymptotic values given by $F_{n,as}(y) = \frac{-in^{\frac{1}{n+1}} y^{-\frac{n+2}{n+1}}}{\sqrt{n+1}} \exp\left[-i(n+1)\left(\frac{y}{n}\right)^{\frac{n}{n+1}}\right]$ [4]. Within the TeV-gravity scenario [1] the total transplanckian cross section is finite, grows with energy, and is dominated by small-angle scattering between partonic constituents [10],[4].

The real and imaginary parts of the function F_1 are shown in Fig. 3.A, and we also see the oscillations of the real part of the function F_1.

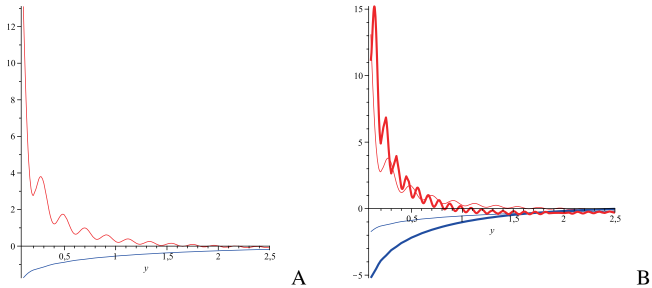

Fig. 3 A. The real (red) and imaginary (blue) parts of the eikonal amplitude F_1. B. Thick lines represent the real and imaginary parts of the eikonal amplitude with doubling eikonal phase in the toy model with the deficit angle equal to π.

3 Eikonal in the conical spacetime

The goal of this section is to estimate the influence of the hadron membrane on the forward scattering of the partons.

3.1 Graviton exchange with modified graviton propagator

The tree level 2 partons $\rightarrow$ 2 partons S-matrix element corresponding to one graviton exchange in the $C_\alpha \times M^3$ spacetime,

$$< p_1, p_2|S|p_3, p_4 >_{1\text{graviton}} \equiv \mathscr{S}_{\text{graviton},\alpha}(p_1, p_2, p_3, p_4), \tag{14}$$

is given by the linearization of gravity [22] and in $s \gg t$ regime is

$$\mathscr{S}_{\text{graviton},\alpha}(p_1, p_2, p_3, p_4) \approx -16\pi G\gamma(s)\,\mathscr{S}_{scalar,\alpha}, \tag{15}$$

here $\approx$ means that we ignore the recoil of the matter field and take the prefactor $\gamma(s)$ the same as for the flat case, $\gamma(s) = ((s-2m^2)^2 - 2m^4)/2$.

In the flat spacetime

$$\mathscr{S}_{\text{graviton,flat}}(p_1, p_2, p_3, p_4) = i(2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4)\,\mathscr{A}_{\text{Born}}(s,t), \quad t \approx -\mathbf{q}^2. \tag{16}$$

In what follows, $\mathscr{S}_{scalar,\alpha} \equiv \mathscr{S}_\alpha$ is the Born amplitude for the scalar particles scattering due to the scalar exchange in the $C_\alpha \times M^3$ spacetime. It can be written (after the Euclidean rotation) in the Schwinger representation as

$$\mathscr{S}_\alpha = \int d^4Xd^4X' e^{i(p_1-p_3)X+i(p_2-p_4)X'} \int d\tau e^{-m^2\tau} K(t,x_\perp;t',x'_\perp;\tau) K_\alpha(z,0;z',0;\tau),$$

here $X = (t, x_\perp, z) \equiv (x^{\check{\mu}}, z)$ and $K(t,x_\perp;t',x'_\perp;\tau)$ is the heat kernel on the 3-dimensional plane and $K_\alpha(z,y;z',y';\tau)$ is the heat kernel on the 2-dimensional cone C_α. K_α has a representation [37, 38, 39]

$$K_\alpha(z,y;z',y';\tau) = \frac{i}{2\alpha}\int_\gamma dw\, \text{ctg}\left(\frac{\pi w}{\alpha}\right) K(z(w),y(w);z',y';\tau). \tag{17}$$

Here $(z(w), y(w)) = (r\cos(\theta + w), r\sin(\theta + w))$, (r,θ) are related with coordinates (z,y) as $(z,y) = (r\cos(\theta), r\sin(\theta))$, $K(z,y;z',y';\tau)$ is the heat kernel on the 2-dimensional plane

$$K(z,y;z',y';\tau) = \frac{1}{4\pi\tau}\exp\{-\frac{(z-z')^2 + (y-y')^2}{4\tau}\}, \tag{18}$$

and γ is a characteristic contour presented in Fig. 4, where $\Delta\theta = \theta' - \theta$ and θ' is related with z'.

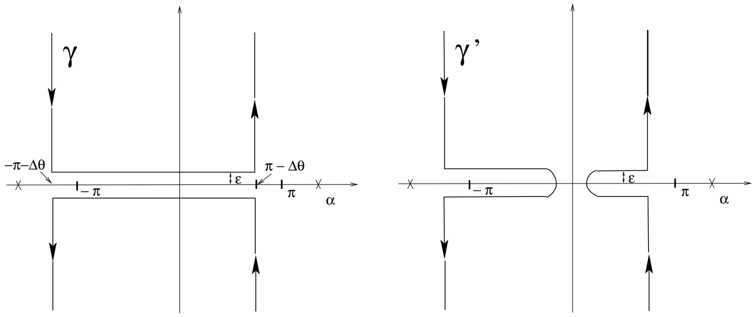

Fig. 4 Contours γ and γ'.

Under assumption that we are on the brane, $\theta = 0$ and $\theta' = 0$ (or $\theta, \theta' = \pi$) we have

$$(z(w), y(w))|_{\text{on brane}} = (z\cos(w), z\sin(w)), \ \ (z', y')|_{\text{on brane}} = (z', 0) \tag{19}$$

and

$$K_\alpha(z, 0; z', 0; \tau) = \frac{i}{2\alpha} \int_\gamma dw \,\text{ctg}\left(\frac{\pi w}{\alpha}\right) \mathcal{K}_w(z, z'; \tau) \tag{20}$$

where

$$\mathcal{K}_w(z, z'; \tau) \equiv \frac{1}{4\pi\tau} \exp\{-\frac{z^2 + z'^2 - 2zz'\cos w}{4\tau}\} \tag{21}$$

We can define the Fourier transformation of the propagator associated with (21) as

$$\mathcal{D}(r, v) = \int\int e^{ir(z-z')+iv(z+z')} e^{-m^2\tau} \mathcal{K}_w(z, z'; \tau) dz dz' \frac{d\tau}{4\pi\tau} \tag{22}$$

and find

$$\mathcal{D}(r, v) = \frac{2}{\sin w} \frac{1}{\frac{r^2}{\sin^2 \frac{w}{2}} + \frac{v^2}{\cos^2 \frac{w}{2}} + m^2}. \tag{23}$$

Finally, we get

$$\mathcal{S}_\alpha = i(2\pi)^3 \delta^3 \left((p_1 + p_2 - p_3 - p_4)_{\check{\mu}}\right) \mathcal{M}_\alpha,$$

$$\mathcal{M}_\alpha = \frac{i}{2\alpha} \int_\gamma dw \,\text{ctg}\left(\frac{\pi w}{\alpha}\right) \frac{2}{\sin w} \frac{1}{\frac{Q^2}{\sin^2 \frac{w}{2}} + \frac{P^2}{\cos^2 \frac{w}{2}} + q_{\check{\mu}}^2 + m^2}, \tag{24}$$

here and below $q_{\check{\mu}} = (q_0, q_1, q_2)$, $\check{\mu} = 0, 1, 2$, $q = (q_{\check{\mu}}, q_z)$, $q_\perp = (q_1, q_2)$,

$$Q = \frac{1}{2}(p_1 - p_2 - p_3 + p_4)_z, \ P = \frac{1}{2}(p_1 + p_2 - p_3 - p_4)_z, \ q_{\check{\mu}} = (p_1 - p_3)_{\check{\mu}}. \tag{25}$$

Q and P are related as $Q = q_z - P$. In the eikonal regime $Q \approx -P$ and this gives a simplification of (24)

$$\mathscr{M}_\alpha \approx \frac{i}{2\alpha}\int_\gamma dw \operatorname{ctg}\left(\frac{\pi w}{\alpha}\right)\mathscr{B}_w(q_\perp, P), \tag{26}$$

where

$$\mathscr{B}_w(q_\perp, P) = \frac{2}{\sin w}\,\frac{1}{q_\perp^2 + m^2 + \frac{4P^2}{\sin^2 w}}. \tag{27}$$

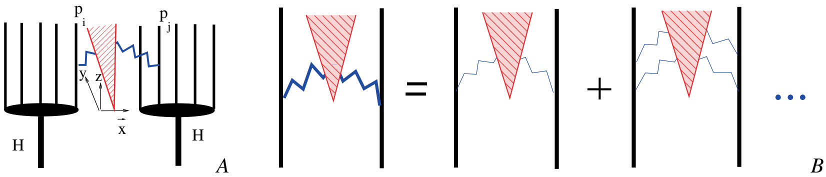

Fig. 5 A.Collision of hadrons with a large impact parameter is presented as an elastic scattering between partons due to a graviton exchange in the space $(x_\perp, z, y)$ with the conic point in the (z, y) section. B. The $2 \to 2$ small angle T-scattering amplitude is given by a sum of crossed-ladder graviton exchanges in the space with the hadron membrane.

Let now define the w-eikonal phase χ as the Fourier transform of (27)

$$\mathscr{X}_w(\mathbf{b}, P) = \frac{1}{2s}\int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}.\mathbf{b}}\mathscr{B}_w(q_\perp, P). \tag{28}$$

The total eikonal phase is given by the integral over the contour γ

$$\chi_\alpha(\mathbf{b}, P) = \frac{i}{2\alpha}\int_\gamma dw \operatorname{ctg}\left(\frac{\pi w}{\alpha}\right)\mathscr{X}_w(\mathbf{b}, P) \tag{29}$$

Using the explicit expression for the eikonal phase for a massive particle we get

$$\mathscr{X}_w(\mathbf{b}, P) = \frac{1}{2\tau}\frac{1}{\pi \sin w} K_0\left(|\mathbf{b}|\sqrt{m^2 + P^2\frac{4}{\sin^2 w}}\right). \tag{30}$$

In the case of $m \approx 0$

$$\mathscr{X}_w(\mathbf{b}, P) = \frac{1}{2\tau}\frac{1}{\pi \sin w} K_0(2|\mathbf{b}||\frac{P}{\sin w}|). \tag{31}$$

and for small w we have

$$\mathscr{X}_w(\mathbf{b},P) \approx \frac{1}{4\tau}\frac{e^{-2|\mathbf{b}||\frac{P}{\sin w}|}}{\sqrt{\pi|\mathbf{b}||P\sin w|}} \tag{32}$$

It is known that the propagator in the conic space can be present as a sum of two terms [38, 39]

$$K_\alpha(z,y;z',y';\tau) = K(z,y;z',y';\tau) + K'_\alpha(z,y;z',y';\tau), \tag{33}$$

where

$$K'_\alpha(z,y;z',y';s) = \frac{i}{2\alpha}\int_{\gamma'} dw \operatorname{ctg}\left(\frac{\pi w}{\alpha}\right) K(z(w),y(w);z',y';s), \tag{34}$$

with a modified contour γ' presented in Fig. 4.

Therefore, the eikonal matrix element can be written as

$$\begin{aligned}\mathscr{S}_{\text{eik},\alpha}(p_1,p_2,p_3,p_4) &= i(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\mathscr{A}_{\text{eik,flat}}\\ &+ i(2\pi)^3\delta^3\left((p_1+p_2-p_3-p_4)_{\check{\mu}}\right)\mathscr{M}_{\text{eik},\alpha},\end{aligned} \tag{35}$$

where

$$\mathscr{M}_{\text{eik},\alpha} = -2i\tau\int d^2b_\perp e^{iq_\perp b_\perp} e^{i\chi_{plane}(b_\perp)}\left[e^{\Delta\chi_\alpha(b_\perp,P)}-1\right], \tag{36}$$

where $\chi_{\text{plane}}(b_\perp)$ is given by (13) for $n=1$ and

$$\Delta\chi_\alpha(b_\perp,P) = \frac{1}{2s}\int\frac{d^2q_\perp}{(2\pi)^2}e^{-iq_\perp b_\perp}B_\alpha(q_\perp^2,P), \tag{37}$$

where

$$B_\alpha(q_\perp,P) = \frac{i}{2\alpha}\int_{\gamma'} dw \operatorname{ctg}\left(\frac{\pi w}{\alpha}\right)\mathscr{B}_w(q_\perp,P). \tag{38}$$

Now if we take this correction perturbatively we get

$$\mathscr{M}_{\text{eik},\alpha} \approx -2is\int d^2b_\perp\,\Delta\chi_\alpha(b_\perp,P)\,e^{iq_\perp b_\perp + i\chi_{\text{plane}}(b_\perp)}. \tag{39}$$

We can analyze the correction for arbitrary angle α only numerically.

3.2 Correction to the eikonal amplitude for toy model $\alpha = \pi/N$

It is known, that the propagator in the conic space with $\alpha = \pi/N$ can be present as a finite sum of propagators

$$K_{\pi/N}(z,z',\tau) = \sum_{n=0}^{N}\mathscr{K}_{n\pi/N}(z,z',\tau), \tag{40}$$

where

$$\mathscr{K}_{n\pi/N}(z,z',\tau) \equiv \frac{1}{4\pi\tau}\exp\{-\frac{z^2+z'^2-2zz''\cos(\frac{n\pi}{N})}{4\tau}\}. \tag{41}$$

We can calculate the contour integral in (24) explicitly to get

$$\mathscr{S}_{\pi/N} = i(2\pi)^3\delta^3\left((p_1+p_2-p_3-p_4)_{\check{\mu}}\right)\left[\sum{}'\frac{2}{\sin\frac{\pi n}{N}}\frac{1}{\frac{Q^2}{\sin^2\frac{\pi n}{2N}}+\frac{P^2}{\cos^2\frac{\pi n}{2N}}+q_{\check{\mu}}^2+m^2}\right.$$
$$\left. +\ \delta(Q)\frac{\pi}{\sqrt{P^2+q_{\check{\mu}}^2+m^2}}+\delta(P)\frac{\pi}{2\sqrt{q_{\check{\mu}}^2+m^2}}\right]. \tag{42}$$

Here the prime in the sum means that we do not take into account $n=0$ and $n=N$.

If we consider $N=1$ we get just one new term as a correction to the usual Born amplitude

$$\mathscr{S}_{\pi} = \mathscr{S}_{\text{flat}}+\Delta\mathscr{S}_{\pi}, \tag{43}$$

$$\mathscr{S}_{\text{flat}} = \delta^4(p_1+p_2-p_3-p_4)\frac{i(2\pi)^4}{2\sqrt{q_{\check{\mu}}^2+m^2}}, \tag{44}$$

$$\Delta\mathscr{S}_{\pi} = \delta^3\left((p_1+p_2-p_3-p_4)_{\check{\mu}}\right)\delta\left((p_1-p_3-p_2+p_4)_z\right)\frac{i(2\pi)^4}{2\sqrt{q^2+m^2}}. \tag{45}$$

In the eikonal regime $Q\approx P$ and both terms (44) and (45) give the same contribution and we get a doubling of the eikonal phase.

4 Conclusion and Discussion

In this paper we have argued that in the TeV-gravity scenario high energy hadrons colliding on the 3-brane embedded in $D=4+n$-dimensional spacetime, with n dimensions smaller than the hadrons size, can be considered as cosmic membranes. In the 5-dimensional case this consideration leads to the 3-dimensional effective model of high energy collisions of hadrons. The cosmic membranes in the 5-dimensional case are similar to cosmic strings in the 4-dimensional world.

It is well known that, the cosmic strings give rise to remarkable classical gravitational and quantum phenomena. In particular, the cosmic string acts as a gravitational lens [31]. This effect becomes manifest when two particles move along opposite sides of the string. Also there is a self-force acting on a test charged particle around the cosmic string [40] and a freely moving charged particle radiates near the cosmic string [41, 42]. This is an analogue of the radiation by the charged particle when it suffers the Aharonov-Bohm scattering [43] and this radiation occurs due to the fall down of the Huygens principle in curved spacetime.

There are also quantum effects. The presence of the cosmic string allows effects such as particle-antiparticle pair production by a single photon and bremsstrahlung radiation from charged particles [44, 45] which are not possible in empty Minkowski space, due to conservation of linear momentum. The conical structure of the cosmic string spacetime is the source of momentum non-conservation in the plane perpendicular to the string, which permits pair production by a single photon. The gravitational mechanism that permits pair production by a single photon around a cosmic string has common topological features with the Aharonov-Bohm effect [43]. The absence of global momentum conservation was already stressed for gravity in 2+1 dimensions by Henneaux [46] and Deser [48]. It is worth also to mention that the string polarizes the vacuum around it, in a way similar to the Casimir effect between two conducting planes forming a wedge [49, 50]. The study of quantum field theories the spasetime with conic singularities requires a regularization [51]. Among possible regularizations the zeta-function regularization is more convenient [52].

Our specifics is that not all process mentioned above can be realized for particles attached to the 3-brane. In particular, to see the lens effect we have to deal with the motion of particles in the 2-plane that is perpendicular to the hadron membrane. But only gravitons can move in this plane in any direction. However one can estimate the self-force effect.

The same concerns also the quantum effects. From one side, only the graviton can propagate in the 2-plane perpendicular to the hadron membrane and feel the deficit angle. From other side, the above mentioned quantum processes are available for other particles if their have not to abandon the 3-brane to participate in the processes.

In this paper we have estimated corrections to the eikonal scattering amplitude due to the hadron membrane.

Similar to the case of cosmic string [44], one can also estimate the decay of a light ultra-relativistic particle on two heavy particles with mass M. For large longitudinal momentum of the light particle, $k_z >> 2M\delta^{-1}$, the cross-section does not depend on k_z and is defied only by the coupling g of these 3 particles and heavy mass

$$\sigma_{1\,\text{light}\to 2\,\text{heavy}} \approx \frac{g^2}{M^3} \tag{46}$$

To realize the condition $k_z >> 2M\delta$ it is enough to take $k_z \sim 1TeV$ and M of the order of the few MeV's.

Other processes we are going to estimate in the separate work [33].

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