2 Basics

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2.1 The physics of wound ballistics

2.1.1 Preliminary remarks

Wound ballistics is an inter-disciplinary science, involving a wide range of specialists – doctors, physicists, lawyers, weapons experts, etc. There is therefore a need for a concise introduction to the basic physics involved. Readers with a good knowledge of physics may wish to skip Section 2.1.

2.1.2 Coordinates, systems of units and notation

To describe physical phenomena simply, one needs a suitable system of coordinates. For ballistics, we generally use the ballistic coordinate system: the x- and y-axes between them define a vertical plane, with the y-axis pointing in the opposite direction to the earth's gravitational pull. The z-axis completes the system, creating a right-handed three-dimensional system (see Fig. 2-1). The movements of a body (a bullet in this instance) are described by reference to a Cartesian system fixed in the body. The origin of that system is located at the centre of gravity of the body concerned and its principal axis is aligned with the direction of movement of the centre of gravity at any given point in time.

The units are those of the SI system (Système International d'Unités), which is the official system in many countries. Length is measured in metres, mass in

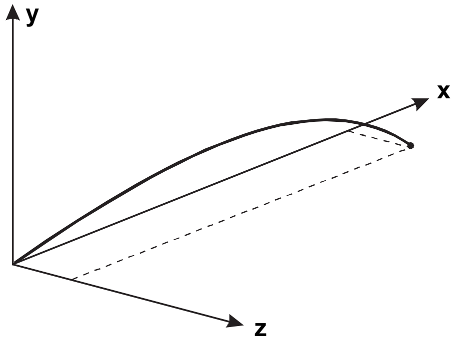

Fig. 2-1. Ballistic coordinate system x-axis in the direction of the shot, y-axis upwards, z-axis to the right.

kilograms and time in seconds, often modified by prefixes such as milli-, kilo- or mega-. A summary of the decimal prefixes appears at the end of the table of symbols (p. XXIII). Other, derived units will be explained the first time the corresponding parameter occurs.

Imperial/US units are still widely used in the fields of weapons, ammunition and ballistics. Again, the formulas for converting between these units and SI units are at the end of the table of symbols (p. XXIII).

Most phenomena of interest to us are three-dimensional, and must therefore be described in terms of three components. For the sake of clarity, however, we shall in many cases use only one component. This makes little difference in practice, as it is possible to study many processes one-dimensionally by selecting a system of coordinates appropriately.

Some definitions and equations include differential quotients. In such cases, we shall follow the usual convention: derivatives over time are written with a dot over the symbol, while derivates over distance are indicated using a prime, thus:

(2.1:1) $$\frac{dx}{dt} \Leftrightarrow \dot{x}, \quad \frac{dv_x}{dt} \Leftrightarrow \dot{v}_x, \quad \frac{dv_x}{dx} \Leftrightarrow v'_x \, .$$

2.1.3 Mechanics

2.1.3.1 Kinematics

Kinematics is the study of the motion of a body in space, disregarding the specific characteristics of the body concerned. The primary role of kinematics is to describe the path, or trajectory of the body. For these purposes we ignore the size of the body, treating it as a point. The most important kinematic parameter is velocity. Velocity is a vector value, of which the components are defined as distance covered per unit time along three axes:

(2.1:2) $$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt} \, . \qquad [\mathrm{m/s}]$$

Speed (as opposed to velocity) is given by:

(2.1:3) $$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \, . \qquad [\mathrm{m/s}]$$

Table 2-1 gives a rough indication of typical speeds encountered in ballistics.

Change in velocity per unit time is termed *acceleration* if velocity increases and *negative acceleration*, or *deceleration*, if velocity decreases:

(2.1:4) $$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt} \, . \qquad [\mathrm{m/s^2}]$$

Table 2-1. Typical velocities

Object/medium		Velocity	
		[m/s]	[ft/s]
Projectile	Air rifle/air pistol	100 ... 250	330 ... 820
	Handguns	250 ... 400	820 ... 1,310
	Rifles	600 ... 1,000	1,970 ... 3,280
	Flechette	1,500 ... 1,800	4,920 ... 5,900
	Fragment	< 2,000	< 6560
Sound waves	In air (15 °C)	340	1,115
	In water (20 °C)	1,483	4,865
	In steel	5,180	16,995
	In glass	5,225	17,142

We can express the magnitude of acceleration in a manner similar to that already discussed for speed:

$$(2.1{:}5) \qquad a = \sqrt{a_x^2 + a_y^2 + a_z^2}\ . \qquad [\mathrm{m/s^2}]$$

Table 2-2 lists some typical acceleration values encountered in ballistics.

Because velocity is a vector, a change in direction is also an acceleration, even in cases where speed does not change.

If acceleration is constant, we can readily calculate speed and distance using the following formula:

$$(2.1{:}6) \qquad v = v_0 + a \cdot t\ , \qquad [\mathrm{m/s}]$$

$$(2.1{:}7) \qquad x = x_0 + v_0 \cdot t + \tfrac{1}{2} \cdot a \cdot t^2\ , \qquad [\mathrm{m}]$$

where v_0 is the speed at time $t = 0$ and x_0 is the distance that the object has already moved at that moment in time.

Movement along a curved path can always be expressed as movement along a segment of a circle. The rotational movement is described in kinematic terms by

Table 2-2. Typical acceleration values

Object	Acceleration	
	[m/s^2]	[ft/s^2]
Acceleration due to gravity (standard value)	– 9.80665	– 32.1740
Pistol bullet in air	– 200	– 6,560
Rifle bullet in air	– 400	– 1,310
Fragment in air	– 60,000	– 197,000
Rifle bullet in barrel on firing, max.	100,000	– 328,100
Deformation projectile penetrating a dense medium (mean value) up to	– 800,000	– 2,625,000

the angular velocity, with angular displacement φ measured in radians (one complete revolution corresponds to 2π radians, which equates to 360°).

(2.1:8) $$\omega = \frac{d\varphi}{dt} . \qquad [rad/s]$$

We obtain the rotational speed ν (the number of revolutions per second) from the angular velocity, thus:

(2.1:9) $$\nu = \frac{\omega}{2 \cdot \pi} . \qquad [1/s]$$

Rotation at a constant angular velocity is known as *uniform circular motion*. The tangential velocity is given by:

(2.1:10) $$v = r \cdot \omega , \qquad [m/s]$$

where r is the radius of rotation, i.e. the distance between the point under consideration and the centre of rotation.

This type of motion involves acceleration, because while the speed of the object does not change, its direction is continuously changing. The acceleration is perpendicular to the direction of travel at any given moment, and is directed away from the centre of the circle. Acceleration is given by the following equation:

(2.1:11) $$a = v \cdot \omega = \frac{v^2}{r} = r \cdot \omega^2 . \qquad [m/s^2]$$

Conversion between the various representations is performed using Eqn 2.3:9. Changes in angular velocity are expressed as angular acceleration α:

(2.1:12) $$\alpha = \frac{d\omega}{dt} . \qquad [rad/s^2]$$

One case of rotation in ballistics is that of a spin-stabilized bullet, which rotates about its longitudinal axis. The angular velocities and rotational speed involved are quite high, and although the radius of a bullet is small, the circumferential velocities attained are considerable (see Table 2-3).

Table 2-3. Typical values for angular velocity ω, rotational speed ν and circumferential velocity v

Type of bullet	ω	ν	v	
	[rad/s]	[1/s]	[m/s]	[ft/s]
Pistol (9 mm Luger)	8,800	1,400	40	130
Revolver (.44 Rem. Mag.)	5,440	866	30	100
Rifle (7.62 mm NATO)	17,100	2,721	67	220

2.1.3.2 Mass, momentum and force

Mass is one of the basic characteristics of a body. It expresses itself in two ways:

- a body resists any change in state of movement (inertia);
- a body alters the state of movement of other bodies (gravitation).

Physical phenomena are often independent of the dimensions of a body. In such cases, it is easier to describe the phenomenon if we treat the body as a point, with the entire mass of the body concentrated in that point. This point – the centre of gravity – is chosen such that even when forces act upon the body there is no difference between the path of the actual body (of non-zero dimensions) and that of the point mass.

In SI units, mass is expressed in kilograms or grams. Grains and pounds are still used in some English-speaking countries (see conversion factors on p. XXIII).

The velocity of a body, v, multiplied by its mass, m, is its *momentum*, M. Momentum is a vector, of which the direction is identical to that of the body's velocity:

(2.1:13) $$I = m \cdot v \,. \qquad [\mathrm{kg{\cdot}m/s}]$$

The physical parameter responsible for a change in the state of movement of a body, or deformation of that body, is known as *force*. Force is defined by *Newton's laws*.

1. The velocity of a body (or, more accurately, its momentum) remains constant unless a force acts upon it.
2. Force equals change in momentum over time, so:

(2.1:14) $$F = \frac{dI}{dt} = \frac{d}{dt}(m \cdot v) \,. \qquad [\mathrm{N}]$$

The unit of measurement for force is the Newton, and 1 Newton [N] = 1 $\mathrm{kg{\cdot}m/s^2}$.

Though not an SI unit, the kilopond [kp] is also in widespread use. In some English-speaking countries, force is also measured in pounds (or pounds-force). Conversion formulas for units of force are to be found on p. XXIII.

If the mass of a body remains constant, force equals mass times acceleration:

(2.1:15) $$F = \frac{d}{dt}(m \cdot v) = m \cdot \frac{dv}{dt} = m \cdot a \,. \qquad [\mathrm{N}]$$

3. For every force, there is an opposing force of the same magnitude (action/reaction). Please see Table 2-4 for a list of typical forces encountered in ballistics.

As acceleration is a vector, it follows from Eqn 2.1:15 that force also behaves as a vector. In order to describe a force, we must therefore know three things: its *mag-*

Table 2-4. Typical forces acting on a bullet

Type of force	Force	
	[N]	[lbf]
Force of air acting on a rifle bullet	4	0.9
Mean drag acting on a bullet fired into water	5,000	1,125
Force on a bullet in a rifle barrel	12,000	2,700

nitude, its *direction* and its *point of application*. The best-known force is weight, G. Weight is the force that occurs when a body is subjected to the acceleration due to the earth's gravity, g. From Eqn 2.1:15:

$$G = m \cdot g \, . \qquad (2.1{:}16) \qquad [N]$$

In physics, it is often useful to express force as a function of the area on which it is acting. When a force acts perpendicular to a surface, it is termed either a *tensile* or a *compressive* force. If a force acts parallel to a surface, it is known as a *shear* force. Force per unit area is known as *stress*.

Stress (generally represented by the symbol σ) is often more important than the corresponding force. As a result, it has its own unit of measurement, the Pascal (Pa). 1 Pa ⇔ 1 N/m². Compressive stress may also be expressed in bar. This unit is widely used in ballistics:

$$1 \text{ bar} \Leftrightarrow 105 \text{ Pa} \, .$$

In some English-speaking countries, pressure is also measured in pounds per square inch (lbf/in^2). Conversion formulas for units of pressure are to be found on p. XXIII.

2.1.3.3 Work and energy

Work and energy, and their relationship to velocity, force, momentum and power, play a crucial role in ballistics. We shall therefore examine these terms in detail, the more so as they are not always used correctly in everyday speech.

Work W is defined as the force F acting in a given direction, multiplied by the distance covered under the influence of that force, s:

$$W = F \cdot s \, . \qquad (2.1{:}17) \qquad [J]$$

Any irregular movement, and any change in the structure of a material (deformation, destruction, etc.) requires work. The amount of work required depends not only on the force, but also on the distance covered. Even if the force is large, the work done will be small if the force acts over a short distance (which often equates to a short exposure time).

Given that work is defined as force times distance, the unit is the Newton-metre [N·m]. However, as work is such an important term it has its own unit, the Joule [J]:

$$1 \text{ J} \Leftrightarrow 1 \text{ N·m} \, .$$

Certain English-speaking countries still use the foot-pound-force [ft·lbf]. See p. XXIII for the conversion formula.

In order to move a body against the acceleration due to the earth's gravity, a force is required that corresponds to the weight of the body. From Eqn 2.1:17, we can calculate the work done W using the following formula:

$$W = m \cdot g \cdot y \, , \qquad [J] \tag{2.1:18}$$

where y is the distance that the body is moved against the force of gravity.

If, however, the body undergoes acceleration as a result of the force acting upon it, the work done results in movement. Substituting Equations 2.1:15, 2.1:7 and 2.1:6 in Eqn 2.1:17 (the latter two equations with $x_0 = 0$ and $v_0 = 0$) we obtain:

$$W = \tfrac{1}{2} \cdot m \cdot v^2 \, . \qquad [J] \tag{2.1:19}$$

If we ignore non-mechanical phenomena, the work done on a body therefore results either in an increase in distance within the gravitational field, or in movement. In the first instance, this can be expressed in terms of the weight of the body and the increase in distance (Eqn 2.1:18) and in the second case in terms of the mass of the body and the speed it attains (Eqn 2.1:19).

Both a body that has been raised from its initial position and a body in motion are capable of performing work. The ability to perform work is generally referred to as *energy*. In the case of a raised body we speak of *potential energy*, while in the case of a moving body the term is *kinetic energy* (see Table 2-5 for examples). In physics, therefore, work and energy are equivalent. Neither work or energy can be created or destroyed. All that can happen is that the one is converted into the other. From Eqn 2.1:19, we can therefore derive an analogous formula for the kinetic energy of a body in motion, with the same unit [J]:

$$E_{kin} = \tfrac{1}{2} \cdot m \cdot v^2 \, . \qquad [J] \tag{2.1:20}$$

Using Eqn 2.1:18, we can express potential energy thus:

$$E_{pot} = m \cdot g \cdot y \, . \qquad [J] \tag{2.1:21}$$

Wounding only occurs when energy is converted into work. However, this process is only partially mechanical, as the energy taken from the projectile is used primarily to deform and destroy tissue. In other words, this energy performs work

Table 2-5. Typical kinetic energy levels encountered in ballistics

Type of weapon		Muzzle energy	
		[J]	[ft·lbf]
Air rifle	over	10	7.4
Pistol		500	370
Modern military rifle (5.56 mm)		1,600	1,180
Older military rifle (7.62 mm)		3,000	2,215
Hunting rifle	up to	10,000	7,375

on the molecular structure of the material. Clearly, however, the severity of a wound (the quantity of tissue destroyed) can depend only on the energy taken from the bullet, and not on the total energy that the bullet possessed.

2.1.3.4 Rotation

If a force acts on a system capable of rotation, at a point away from its axis of rotation, the system will begin to rotate. The angular velocity will depend on the force and on the distance between the point of application and the axis of rotation (the lever arm). This phenomenon can be represented as:

(2.1:22) $$M = F \cdot r \ , \qquad [N{\cdot}m]$$

where M is the torque and r is the distance between the point of application and the centre of rotation (i.e. the length of the lever arm).

If we consider torque as a vector, it always acts perpendicular to both the direction of the force and the lever arm, and hence is always parallel to the axis of rotation.

From Equations 2.1:15, 2.1:10 and 2.1:12 we obtain:

(2.1:23) $$M = m \cdot \frac{dv}{dt} \cdot r = m \cdot \frac{r \cdot d\omega}{dt} \cdot r = m \cdot r^2 \cdot \alpha \ . \qquad [kg{\cdot}m^2/s^2]$$

From this equation, we can see that there is a linear relationship between the torque and the angular acceleration it produces, as is the case for force and acceleration. The factor that links the two is the moment of inertia J, a characteristic of the body related to the axis of rotation:

(2.1:24) $$M = J \cdot \alpha \ . \qquad [kg{\cdot}m^2/s^2]$$

The moment of inertia J therefore has the same relationship to rotation as does mass to linear motion.

The moment of inertia of a body can be calculated by analytical or numerical integration over the volume:

(2.1:25) $$J = \int r^2 \cdot dm \ . \qquad [kg{\cdot}m^2]$$

The moment of inertia can also be measured using a “moment of inertia” measuring instrument. The formulas for basic, regular bodies are in some cases quite simple.

Using the definition of force in Eqn 2.1:14, we can derive the following relationship from Eqn 2.1:24:

$$M = F \cdot r = \frac{dI}{dt} \cdot r = J \cdot \frac{d\omega}{dt} = J \cdot \alpha \ . \qquad [kg{\cdot}m^2/s^2]$$

Integrating the two corresponding terms, we obtain an equation with, on one side, the product of the torque and its distance from the axis of rotation. This product is the *angular momentum* or *spin*:

(2.1:26) $$L = I \cdot r = m \cdot v \cdot r = J \cdot \omega \ . \qquad [kg{\cdot}m^2/s]$$

Table 2-6. Comparison between linear and rotational motion

Linear motion (translation)		Rotational motion (rotation)	
Time	t	t	Time
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Mass	m	J	Moment of inertia
Force	$F = m \cdot a$	$M = J \cdot \alpha$	Torque
Momentum	$M = m \cdot v$	$L = J \cdot \omega$	Angular momentum
Kinetic energy	$E_{kin} = ½ \cdot m \cdot v^2$	$E_{rot} = ½ \cdot J \cdot \omega^2$	Rotational kinetic energy

In determining the kinetic energy of a rotating mass (its energy of rotation, or spin energy), we must remember that each mass particle has a velocity that depends on its distance from the axis of rotation. We must therefore integrate the differential energy over the entire volume:

$$E_{rot} = \tfrac{1}{2} \cdot \int_V v^2 \cdot dm = \tfrac{1}{2} \int_V \omega^2 \cdot r^2 \cdot dm = \tfrac{1}{2} \cdot \omega^2 \cdot \int_V r^2 \cdot dm \ ,$$

(2.1:27) $$E_{rot} = \tfrac{1}{2} \cdot J \cdot \omega^2 \ . \qquad [J]$$

The analogy between the formula for linear motion and that for rotation is so striking that it is worth placing them alongside one another (see Table 2-6).

It is often forgotten that yaw and partial rotation are also forms of rotation. In addition to rotating rapidly about its longitudinal axis, a moving bullet makes other rotational movements, such as those about a horizontal transverse axis. See Fig. 2-2.

2.1.3.5 Laws of conservation of mass, energy and momentum

All physical phenomena are controlled by a number of “laws of conservation.” This is particularly true of mechanics. The analysis of any physical process starts with the application of these fundamental laws, which generally make it possible to understand a phenomenon quite easily. However, it is important first to delimit the physical system in such a way that one can ignore any interaction with the

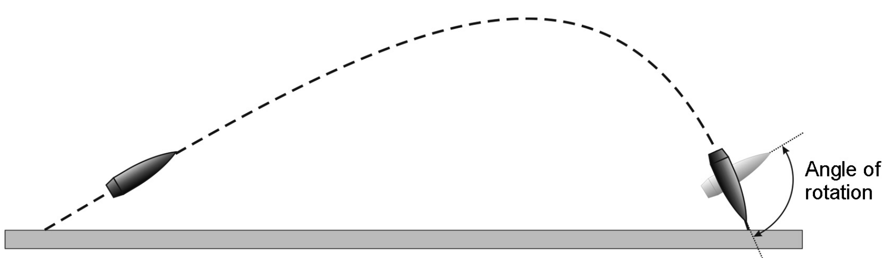

Fig. 2-2. Rotation of a bullet about a transverse axis along its trajectory.

outside world. Such a system is described as "closed." Laws of conservation apply only within closed systems.

1. The law of conservation of mass. The total mass within a closed system remains constant. Mass is neither created nor destroyed:

(2.1:28) $$m_{tot} = \sum m_i = \text{const} . \qquad [\text{kg}]$$

In a system consisting of several masses, the centre of gravity of the system also remains constant, regardless of the movements of the individual masses (such as in the case of a shot sheaf).

2. The law of conservation of momentum. The total momentum within a closed system (i.e. one on which no external forces are acting) remains constant:

(2.1:29) $$I_{tot} = \sum I_i = \text{const} . \qquad [\text{N·s}]$$

3. The law of conservation of angular momentum. The total momentum within a closed system (i.e. one on which no external moment is acting) remains constant:

(2.1:30) $$L_{tot} = \sum L_i = \text{const} . \qquad [\text{kg·m}^2/\text{s}]$$

4. The law of conservation of energy for mechanics. The sum of the potential, kinetic and rotation energy in a closed, frictionless system (with no energy entering or leaving the system) remains constant:

(2.1:31) $$E_{mech} = \sum E_{pot} + \sum E_{kin} + \sum E_{rot} = \text{const} . \qquad [\text{J}]$$

However, the law of conservation of energy goes beyond mechanics. It also applies to other, non-mechanical forms of energy.

5. The general law of conservation of energy. The total energy (i.e. the sum of all forms of energy) in a closed system remains constant.

The conversion of energy from one form to another within a system is not necessarily reversible.

2.1.3.6 Equations of motion

In physics, the motion of a body is described fully if its position, velocity and position in space are known at every point in time. The motion of a body is determined exclusively by the forces acting upon it and the accelerations those forces produce.

Linking the acceleration of a body to its position and velocity yields *equations of motion*. These form a system of differential equations that describe the time-dependent velocity and position functions of a body. In the case of a point mass (e.g. the centre of gravity of a body), the system consists of six equations – three for the spatial coordinates and three for velocity. A rigid body of non-zero volume requires six further equations, which describe the body's position in space and the rate at which its position changes. In many cases, however, the system of equations of movement can be considerably reduced. This is the case, for instance, when no force is acting in a particular direction.

D'Alembert's principle. Equations of movement can be derived from the law of conservation of energy or of momentum by differentiation. D'Alembert's principle allows us to take a more direct approach. This principle involves replacing the acceleration of a body by a fictitious force F_{fict} which, in accordance with Newton's Third Law of Motion, has to equal the sum of the forces applied:

(2.1:32) $$m \cdot a = F_{fict} = \sum F_i \, . \qquad [N]$$

The equations of motion utilize another basic principle of mechanics: the 'superposition principle'. This principle states that different movements of a body occurring simultaneously do not influence each other. Any movement can therefore be divided into separate components, one for each of the axes of the coordinate system. We can study each component independently of the others and hence can describe each component using simple equations.

Trajectory in a vacuum. One typical example of how equations of movement are developed and solved – and one of particular relevance to ballistics – is that of the trajectory of a body in a vacuum. No forces act along the x-axis, and the only force acting along the y-axis is the weight of the body. In accordance with d'Alembert's principle, we can write the following two equations:

(2.1:33a) $$m \cdot \dot{v}_x = 0 \qquad \text{(x-axis)}, \qquad [N]$$

(2.1:33b) $$m \cdot \dot{v}_y = -m \cdot g \qquad \text{(y-axis)}. \qquad [N]$$

These two differential equations form the system of equations of movement describing the *trajectory of a body in a vacuum*. We can readily integrate the two equations, using the initial values below, derived from the initial velocity v_0 and angle of departure θ_0:

$$v_{0x} = v_0 \cdot \cos\theta_0 \, , \qquad [m/s]$$

$$v_{0y} = v_0 \cdot \sin\theta_0 \, . \qquad [m/s]$$

Integration yields the following two equations:

(2.1:34a) $$x(t) = v_0 \cdot \cos\theta_0 \cdot t \, , \qquad [m]$$

(2.1:34b) $$y(t) = v_0 \cdot \sin\theta_0 \cdot t - \tfrac{1}{2} \cdot g \cdot t^2 \, . \qquad [m]$$

If we eliminate t from both equations, we obtain the equation for the (parabolic) trajectory of a body in a vacuum:

(2.1:35) $$y(x) = \tan\theta_0 \cdot x - \frac{g \cdot x^2}{2 \cdot v_0^2 \cdot \cos^2\theta_0} \, . \qquad [m]$$

Taking $y = 0$ (base of the trajectory), the maximum range x_e for a given angle of departure θ_0 is:

(2.1:36) $$x_e = \frac{v_0^2}{g} \cdot \sin(2 \cdot \theta_0) \ . \qquad [m]$$

Similarly, we can obtain the corresponding time of flight from Eqn 2.1:34b:

(2.1:37) $$t_e = \frac{2 \cdot v_0}{g} \cdot \sin\theta_0 \ . \qquad [s]$$

We can readily determine the highest point along the trajectory (the peak) if we bear in mind that the vertical component of the velocity must be zero at that point. This gives us the time of flight to the peak of the trajectory. Substituting this into Eqn 2.1:34b we obtain:

(2.1:38) $$y_s = \frac{v_0^2}{2 \cdot g} \cdot \sin^2\theta_0 \ . \qquad [m]$$

Combining Equations 2.1:37 and 2.1:38 gives us the principal equation for the vertex height. This equation is very useful in practice, and to a close degree of approximation is also valid for the trajectory of an object subject to air resistance:

(2.1:39) $$y_s = \tfrac{1}{8} \cdot g \cdot t_e^2 \ . \qquad [m]$$

Velocity as a function of distance. It is often clearer to show velocity as a function of distance instead of time. For these purposes, "distance" generally means the distance the projectile has travelled along its trajectory, although the term may refer to the distance travelled along the x-axis in some cases.

For example: we can say that a light, high-speed hunting bullet with a muzzle velocity of 1050 m/s undergoes a deceleration of 1700 m/s^2, but it is difficult to translate this information into anything practical. If, however, we say that the bullet loses 1.6 m/s per metre travelled, we can understand what is happening much more easily.

We can write the change in distance along the trajectory thus:

(2.1:40) $$ds = \sqrt{dx^2 + dy^2 + dz^2} \ , \qquad [m]$$

from which we can calculate velocity as follows:

$$v = \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \ , \qquad [m/s]$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \ . \qquad [m/s]$$

This yields the following formula for converting between velocity as a function of time and velocity as a function of distance (using the typographical convention specified in 2.1.2):

$$\dot{v} = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v' \cdot v \,. \qquad [m/s^2] \qquad (2.1{:}41)$$

2.1.4 Fluid dynamics

2.1.4.1 General

The materials involved in wound ballistics (certain types of tissue, together with the soap and gelatine used as simulants) act upon a bullet in a manner similar to that of a viscous liquid. It is considerably easier to understand such phenomena if we model them as such. On the other hand, the behaviour of a bullet in tissue or in a simulant depends to a large degree on the impact conditions. In turn, these conditions are directly linked to the movement of the bullet in air. The mechanics of fluids and gases – known as fluid dynamics – therefore play a major role in wound ballistics.

When we are studying the movement of a body in a medium it makes no difference whether we consider the medium to be stationary and the body in motion, or the body stationary and the medium in motion (in the opposition direction and at the same speed). In terms of physics, the two approaches are equivalent. We will therefore use whichever is clearer in a given context.

Motion in a liquid or gaseous medium always generates heat, whether through friction or as a result of subjecting a gas to increased pressure. It is therefore necessary to include thermodynamic processes in any consideration of fluid dynamics.

2.1.4.2 Basic concepts in thermodynamics

Temperature. Temperature is one of the basic parameters required to describe processes that involve heat. Temperature describes the thermal state of a body and serves as a measure of the total kinetic energy of the molecules that make up that body. It is independent of the mass and the material composition of the body.

If the thermal state of a material changes, so do a number of its physical characteristics, such as its dimensions (length, volume, etc.), its colour or its electrical conductivity. We can use this behaviour to measure temperature. However, it is only possible to conduct relative measurements. As a result, we are free to select the zero point and the units of a scale at will.

The SI unit for temperature is the Kelvin [K]. The only difference between the Kelvin and the older degree Celsius [°C] is the zero point used for the two systems. On the Celsius scale, 0 °C corresponds to the freezing point of water, whereas 0 K corresponds to “absolute zero,” the lowest temperature possible. Degrees Fahrenheit are in widespread use in some English-speaking countries. All scales are calibrated using internationally agreed reference points, corresponding

Table 2-7. Temperature reference points

		[K]	[°C]	[°F]
Water	Freezing point	273.15	0.00	32.00
	Boiling point	373.15	100.00	212.00
Oxygen	Boiling point	90.18	–182.97	–297.35
Gold	Melting point	1336.15	1063.00	1945.40

to the boiling and freezing points of various materials at normal pressure (1013.25 mb). See Table 2-7.

Conversion formulas for the various units appear after the table of symbols (p. XXIII).

Temperature and heat. If the temperature of a given quantity of a material rises during a physical process, then heat has been introduced. However, an increase in temperature means that the total kinetic energy of the molecules in the quantity of material under consideration has also increased. Heat is therefore a form of energy and is hence measured in Joule. The quantity of heat added is proportional to the mass and to the increase in temperature. The proportionality factor C is known as the *specific heat capacity* of the material:

$$(2.1{:}42) \qquad \Delta Q = C \cdot m \cdot \Delta T \,. \qquad [J]$$

States of matter. The state of a material can be *solid*, *liquid* or *gaseous*. The more precise terms for these three states are *crystalline*, *amorphous* and *gaseous*. In the solid state, the molecules are arranged in a crystal lattice, and are held in place not only by intermolecular forces of attraction but also by lattice linkage forces. Solid matter has a specific geometrical form and a fixed volume.

If so much heat is applied to a body that its molecules acquire enough kinetic energy to overcome the lattice forces, the material enters the amorphous (liquid) state. All that is keeping the material together at this point is the intermolecular force. Liquid matter has a fixed volume, but generally has no specific geometrical form.

If the intermolecular forces are sufficiently strong, amorphous materials may be shape stable. Examples of this include glass and wax.

If we add enough heat to raise the kinetic energy of the molecules to the point at which the intermolecular forces are overcome, the molecules become free to move. The material enters the gaseous state, in which its volume is limited not by the quantity of matter present, but by the total space available to it. From the above, we see that it is only possible to change the state of a material to the next higher level by adding energy in the form of heat. Similarly, the corresponding quantity of energy is released as the material returns to a lower level.

Equation of state for gases. Because of the kinetic energy of its free molecules, a gas exerts a pressure p on the surfaces that delimit its volume. The impacts of the particles against these surfaces result in a mean force per unit area, which can be explained by the law of conservation of momentum. Any change in temperature (i.e. in kinetic energy) or in volume (i.e. in surface area) will affect the pressure. Pressure, temperature and volume are the *thermodynamic state variables* of the gas in question. The equation that links these variables is known as an *equation of state*. The best-known equation of state is Boyle's law:

(2.1:43) $$p \cdot V = m \cdot R \cdot T \ . \qquad [J]$$

Where m is the mass of the gas and R is the special (material-dependent) gas constant. If a gas obeys Boyle's law (Eqn 2.1:43), it is known as an *ideal gas*. If the density of the gas is sufficiently low (i.e. if the pressure is low), many gases behave almost as ideal gases.

The special gas constant of air is 287.05 J/(kg·K).

In gas dynamics, we generally use density (mass per unit volume) rather than mass. Eqn 2.1:43 can then be written:

(2.1:44) $$\frac{p}{\rho} = R \cdot T \ . \qquad [J/kg]$$

If temperature remains constant, the right-hand side of the equation remains constant, and hence so does the left-hand side.

Heat, work and internal energy. As the volume of a material can only change considerably when it is in a gaseous state, it follows that materials can only convert heat energy into mechanical work (energy) when they are in that state. Devices that perform this conversion are known as *heat engines*.

Heat engines – which include firearms – always use materials in gaseous form. Those materials are usually created by heating or by combustion.

If the volume of a gas is increased by heating it, then the gas performs work. According to Eqn 2.1:17, the work done is proportional to the force exerted (pressure · area) and the distance moved (see Fig. 2-3):

(2.1:45) $$\Delta W = p \cdot A \cdot \Delta s = p \cdot \Delta V \ . \qquad [J]$$

As we know, the ability to do work is *potential energy*.

The potential energy of a gas of volume V at a pressure p is therefore:

(2.1:46) $$E_{dr} = p \cdot V \ . \qquad [J]$$

In this context, the potential energy of the gas is termed *pressure energy*.

When heat energy is added to a system, two things happen. The first is that the total kinetic energy of the molecules in that system (its internal energy U) increases, which means that its temperature also increases. The second is that the

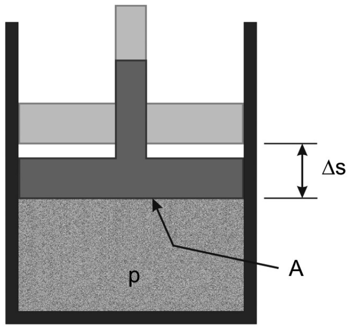

Fig. 2-3. Calculating the work done as a gas expands over a distance Δs.

system performs mechanical work. The following energy balance therefore applies:

(2.1:47) $$dQ = dU + dW = dU + p \cdot dV \,. \qquad [J]$$

If no heat is added to the system ($dQ = 0$) then, from Eqn 2.1:47:

$$dU + p \cdot dV = 0 \,,$$

and hence:

(2.1:48) $$U + p \cdot V = \text{const} \,. \qquad [J]$$

The sum of internal energy and pressure energy therefore remains constant.

This statement, taken with the corresponding law of mechanics (Eqn 2.1:31) constitutes the mechanical thermodynamic law of conservation of energy:

(2.1:49) $$E_{tot} = E_{mech} + E_{dr} + U = \text{const} \,. \qquad [J]$$

In order for the temperature of a gas to rise by a specific amount, the gas must absorb more heat at a constant pressure than at a constant volume. This is a direct consequence of Eqn 2.1:48.

If volume remains constant ($dV = 0$), all of the additional heat is used to raise the internal energy of the gas (i.e. its temperature), as no mechanical work can be done. At constant pressure, however, heat is “lost” to work, and the temperature increase is smaller.

This means that the specific heat capacity in Eqn 2.1:42 is greater at constant pressure than at constant volume:

(2.1:50) $$\gamma = \frac{c_p}{c_V} > 1 \,. \qquad [-]$$

The quotient γ is known as the *adiabatic coefficient*.

2.1.4.3 Material characteristics

Density and compressibility. When liquids and gases (fluids) are subjected to a force they change their form and, in many cases, their volume. In order to describe motion, we therefore need physical parameters that describe the main characteristics of the materials involved. In fluid dynamics, density ρ is more important than mass:

(2.1:51) $$\rho = \frac{m}{V}\ . \qquad [\mathrm{kg/m^3}]$$

The compressibility of a material is a measure of the relative change in the volume of that material in response to a change in pressure:

(2.1:52) $$\kappa = -\frac{1}{V}\cdot\frac{dV}{dp}\ . \qquad [\mathrm{1/Pa}]$$

The minus sign indicates that an increase in pressure brings about a decrease in volume.

Liquids are generally considered incompressible. At low speeds, we can make the same assumption for gases.

Viscosity. The absence of lattice force means that a fluid changes its form when subjected to an external force. However, the fluid reacts to that force with a certain inertia. This resistance, caused by the intermolecular forces of attraction, is termed *viscosity*. If a shear stress is applied to the fluid, with a shear force F_R, (see Fig. 2-4), a velocity gradient is created in the area under stress. In Newtonian fluids, this gradient is proportional to the stress:

(2.1:53) $$\sigma_s = \frac{F_R}{A} = \eta\cdot\frac{dv}{dy}\ . \qquad [\mathrm{N/m^2}]$$

The proportionality factor is the *dynamic viscosity*, η:

(2.1:54) $$\eta = \sigma_s\cdot\frac{dy}{dv}\ . \qquad [\mathrm{Pa\cdot s}]$$

Dynamic viscosity as a function of the density of the fluid is known as *kinematic viscosity*:

(2.1:55) $$\nu = \frac{\eta}{\rho}\ . \qquad [\mathrm{m^2/s}]$$

Viscosity depends very much on temperature, with the viscosity of a gas changing with temperature in exactly the opposite manner to that of a liquid.

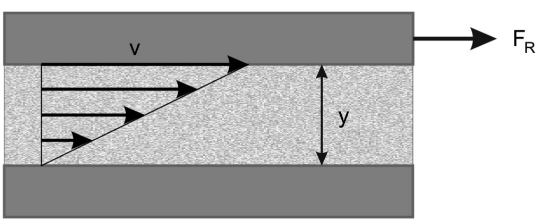

Fig. 2-4. Dynamic viscosity.

As the temperature of a liquid (and hence its internal energy) increases, the effect of the intermolecular forces decreases, and it becomes less viscous. In the case of a gas, internal friction is caused by the exchange of momentum between freely moving molecules. As temperature increases, the speed of those molecules increases, causing the gas to become more viscous.

If the mass remains constant, a fluid also resists any change in volume, as a change in volume can only occur if the form of the fluid also changes. This effect is known as *bulk viscosity*, which is to be distinguished from dynamic viscosity.

Speed of sound. Changes in pressure and density (disturbances) propagate through a fluid in the form of longitudinal waves.

In the case of longitudinal waves, the disturbances oscillate in the same direction as that in which the wave is moving.

If the disturbances remain small, they propagate at the speed of sound. In liquids, this speed depends on density and compressibility:

(2.1:56) $$c = \sqrt{\frac{1}{\kappa \cdot \rho}} \,. \qquad [\mathrm{m/s}]$$

In gases, it depends on density and pressure:

(2.1:57) $$c = \sqrt{\frac{\gamma \cdot p}{\rho}} = \sqrt{\gamma \cdot R \cdot T} \,. \qquad [\mathrm{m/s}]$$

The symbol γ represents the ratio of the specific heat capacities, c_p/c_v (Eqn 2.1:50). For air, its value is 1.4.

The general equation for the propagation speed of small disturbances in a gas is:

(2.1:57a) $$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_{\Delta S = 0} \,. \qquad [\mathrm{m^2/s^2}]$$

The significance of $\Delta S = 0$ is that changes in temperature have no effect on the heat (energy). This is known as *constant entropy*. In the case of an isentropic gas, this gives us the first part of Eqn 2.1:57.

In accordance with Eqn 2.1:43, the second part of Eqn 2.1:57 only applies to ideal (or nearly ideal) gases. In such cases, the speed of sound depends solely on temperature. As a result, this speed is a suitable reference speed for compressible media. This leads us to the definition of the Mach number, Ma:

(2.1:58) $$\mathrm{Ma} = \frac{v}{c} \,. \qquad [-]$$

The behaviour of a flow is closely related to the Mach number, which gives us a simple classification criterion.

If Ma < 1, the flow is described as *subsonic*. If Ma = 1, the flow is described as *transonic*. If Ma > 1, the flow is said to be *supersonic*.

2.1.4.4 Frictionless flow

Definition. Laws of conservation. The study of flow is much simplified if one ignores internal friction. This implies that no shear stresses occur, and hence that the fluid does not deform. This assumption also ignores the fact that every fluid adheres to a fixed surface. As a result, external friction is also ignored. Despite this limitation, a surprising number of phenomena can be described and presented in the form of laws.

This is because when a fluid flows over a body, the effect of friction is often limited to a thin layer at the surface of the body (known as the *boundary layer*). If the thickness of this boundary layer is very small compared to the other dimensions of the flow, we can safely ignore the effects of friction.

A fluid in which no internal friction forces occur is termed *frictionless*. If, in addition, the fluid is incompressible, it is said to be *ideal*. Flow is also described in terms of its relation to time. If the speed and thermodynamic characteristics of the flow depend only on position, and do not change over time, the flow is described as *stationary*.

The laws of conservation of mass, energy and momentum set out in 2.1.3.5 also apply to the movements of fluids. Here again, the conservation of mass plays a central role: the change in mass in any fixed volume element corresponds to the difference between the mass entering the system (subscript 1 in the equation below) and the mass leaving it (subscript 2):

(2.1:59) $$\frac{\Delta m}{\Delta t} = \rho_1 \cdot v_1 \cdot A_1 - \rho_2 \cdot v_2 \cdot A_2 \;. \qquad [\mathrm{kg/s}]$$

This relationship is known as a *continuity equation*. For an incompressible fluid (i.e. $\Delta m/\Delta t = 0$, $\rho = \text{const}$), Eqn 2.1:59 becomes:

(2.1:59a) $$v \cdot A = \text{const} \;. \qquad [\mathrm{m^3/s}]$$

Speed and flow cross-section are inversely proportional to one another. The law of conservation of energy applies. If no heat is added, the sum of pressure energy, kinetic energy, potential energy and internal energy remains constant (see Eqn 2.1:49):

$$E_{dr} + E_{kin} + E_{pot} + U = \text{const} \;, \qquad [\mathrm{J}]$$

(2.1:60) $$p \cdot V + \tfrac{1}{2} \cdot m \cdot v^2 + m \cdot g \cdot y + U = \text{const} \;. \qquad [\mathrm{J}]$$

Momentum and angular momentum are also conserved in a flow. As in the case of mechanics, we obtain the equations of motion for flow from the law of conservation of momentum, using d'Alembert's principle. In the case of flow, the formulas are known as Euler's formulas. The question of angular momentum arises only in turbulent flow, which we shall ignore in this context.

Bernoulli's equation. In theory, Bernoulli's equation is an equation of motion, and has to be derived from the equilibrium of forces acting on a liquid element. However, in the case of ideal flow (i.e. where ρ and U are both constant) it is easier to derive this equation from the law of conservation of energy (Eqn 2.1:60) by dividing by volume:

(2.1:61) $$p + \tfrac{1}{2} \cdot \rho \cdot v^2 + \rho \cdot g \cdot y = \text{const} ,$$ [N/m²]

or, if we also assume a constant flow height:

(2.1:61a) $$p_1 + \tfrac{1}{2} \cdot \rho_1 \cdot v_1^2 = p_0 + \tfrac{1}{2} \cdot \rho_0 \cdot v_0^2 = \text{const} .$$ [N/m²]

This means that pressure and flow velocity are directly related to one another. If pressure is high, fluid velocity is low. If pressure is low, fluid velocity is high. Acceleration occurs when there is a pressure gradient.

Flow forces. If a flow strikes a flat, stationary plate at right angles, the flow velocity drops to zero at the surface of the plate. If we use $_0$ to indicate the values for undisturbed flow and $_1$ for those at the plate surface, then from Eqn 2.1:61a, for $v_1 = 0$ we have:

$$p_1 = p_0 + \tfrac{1}{2} \cdot \rho_0 \cdot v_0^2 ,$$

(2.1:62) $$p_1 - p_0 = \tfrac{1}{2} \cdot \rho_0 \cdot v_0^2 .$$ [N/m²]

The left-hand side of Eqn 2.1:62 corresponds to the increase in pressure at the plate surface by comparison with the pressure in undisturbed flow. This is termed the *stagnation pressure* of the plane plate.

In the case of a real body, the streaming pressure depends not only on velocity but also on the form and dimensions of the body. The actual values will differ from those calculated using the equation above.

The streaming pressure on a specific body is described using the ratio of the actual increase in pressure to the plane stagnation pressure. This (dimensionless) quantity is known as the *pressure coefficient* of the body:

(2.1:63) $$C_p = \frac{p_{eff} - p_0}{\tfrac{1}{2} \cdot \rho \cdot v^2} .$$ [-]

The force that a flow exerts on a body (the flow resistance) is obtained by multiplying the force by the effective area. However, it is often difficult to determine this area. The effective area is therefore replaced by a fixed reference area A_0 which, together with the plane stagnation pressure from Eqn 2.1:62, yields a reference force that is independent of the shape of the body. Dividing the effective resistance F_D by this reference force gives another dimensionless quantity that takes account of all form-specific influences. This quantity is known as the *drag coefficient*:

(2.1:64) $$C_D = -\frac{F_D}{\frac{1}{2} \cdot \rho \cdot v^2 \cdot A_0},$$ [-]

(2.1:64a) $$F_D = -C_D \cdot \tfrac{1}{2} \cdot \rho \cdot v^2 \cdot A_0 \,.$$ [N]

The minus sign indicates that the force acts in the opposite direction to the movement.

A body subject to a flow experiences a lift force (F_D) – a force perpendicular to the direction of flow – when the pressure on its two sides is unequal as a result of the flow velocity on the two sides being unequal (Bernoulli's equation). The lift force is calculated in relation to the same reference force as the resistance/drag. The corresponding ratio is known as the *lift force coefficient*:

(2.1:65) $$C_L = \frac{F_L}{\frac{1}{2} \cdot \rho \cdot v^2 \cdot A_0} \,.$$ [-]

As a lift force generally acts at a point away from the centre of gravity, it will also induce a torque M, with its own reference distance d. The corresponding parameter is known as the *overturning moment coefficient*:

(2.1:66) $$C_M = \frac{M}{\frac{1}{2} \cdot \rho \cdot v^2 \cdot A_0 \cdot d_0} \,.$$ [-]

The reference area and distance chosen differ according to the area of study. In the aerodynamics of missiles, one often uses either the largest cross-section in the plane of the flow or the wing surface/wingspan. In ballistics, however, one generally uses the calibre cross-sectional area (which is generally smaller than the cross-sectional area of the bullet) and the calibre. It is therefore important to specify the reference value when stating a coefficient.

Furthermore, coefficients are always expressed as a function of the Mach number.

2.1.4.5 Flow of a viscous fluid

Forces and equations of motion. In accordance with d'Alembert's principle, equations of motion are derived by equating inertial force and the forces acting on a body. Because density and volume are subject to change, both are expressed as a function of mass.

The following discussion applies to a one-dimensional flow (a streamline). This makes the relationships considerably simpler and clearer, and facilitates comprehension of the subject. For complete versions of the general equations of motion for fluids, please consult the literature on the subject.

A particle in a flow can undergo a change in velocity under two circumstances:

- when the velocity field changes over time (unsteady flow);
- when, in a (stationary) velocity field, it is displaced to a position at which the flow velocity is higher or lower.

Total acceleration is derived from the combination of the two sources. This acceleration is known as the *substantial acceleration*:

(2.1:67) $$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial x} \,. \qquad [m/s^2]$$

The two terms on the right can be interpreted as time-dependent or position-dependent inertial forces, expressed as a function of mass.

In calculating the forces, we assume that the streamline under consideration is oriented along the x-axis and has a cross-section of dy × dz (see Fig. 2-5). The compressive force per unit mass on a flow element of length dx is then:

(2.1:68) $$\frac{(p + \partial p) \cdot dy \cdot dz \; - \; p \cdot dy \cdot dz}{\rho \cdot dx \cdot dy \cdot dz} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} \,, \qquad [N/kg]$$

and the frictional force is:

(2.1:69) $$\frac{(\sigma + \partial \sigma) \cdot dy \cdot dz \; - \; \sigma \cdot dx \cdot dz}{\rho \cdot dx \cdot dy \cdot dz} = \frac{1}{\rho} \cdot \frac{\partial \sigma}{\partial y} \,. \qquad [N/kg]$$

In the case of a Newtonian fluid, we can substitute the shear stress from Eqn 2.1:53:

(2.1:70) $$\frac{d\sigma}{dy} = \frac{\partial}{\partial y} \cdot \left(\eta \cdot \frac{\partial v}{\partial y} \right) = \eta \cdot \frac{\partial^2 v}{\partial y^2} = \nu \cdot \rho \cdot \frac{\partial^2 v}{\partial y^2} \,. \qquad [N/m^3]$$

Weight is omitted, as it does not exert a force along the x-axis, nor will we include any external forces, as these would only introduce an additional summand. From Equations 2.1:67, 2.1:68 and 2.1:69, and taking into account Eqn 2.1:70, we obtain:

(2.1:71) $$\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \nu \cdot \frac{\partial^2 v}{\partial y^2} \,. \qquad [m/s^2]$$

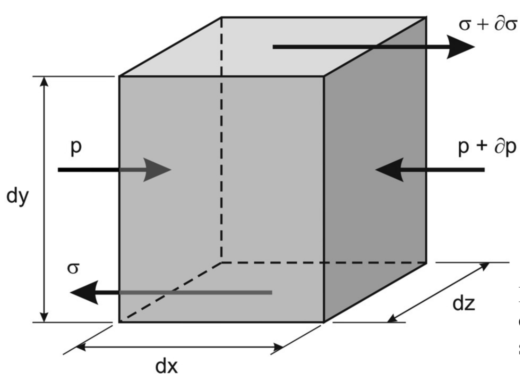

Fig. 2-5. Streamline element, with compressive stresses (p) and shear stresses (σ).

The three equations obtained by extending Eqn 2.1:71 to three dimensions are known as the *Navier-Stokes equations* for (incompressible) Newtonian fluids. They only produce unique solutions under very special conditions. To solve real problems, we must use numerical methods (e.g. finite elements).

For stationary ($\partial v/\partial t = 0$) and frictionless ($\nu = 0$) flow, given that

$$v \cdot \frac{\partial v}{\partial x} = \frac{\partial}{\partial x} \cdot (\tfrac{1}{2} \cdot v^2) ,$$

we obtain the following equation from Eqn 2.1:71:

$$\frac{\partial}{\partial x} \cdot (\tfrac{1}{2} \cdot \rho \cdot v^2 + p) = 0 .$$

This corresponds to Bernoulli's equation (Eqn 2.1:61a) which, as mentioned above, is derived from the equations of motion.

Taking account of viscosity. Reynolds number. From the parameters of a flow – length s, velocity v, time t, pressure p and density ρ – it is possible to derive a term for each of the mass-related forces in Eqn 2.1:71. By forming quotients, it is possible to derive a number of dimensionless values from these four terms, which can then be used to describe the forces in the flow.

In order to decide whether it is possible to ignore friction in a flowing fluid, we must know the ratio of inertia forces to friction forces. This (dimensionless) quantity is called the *Reynolds number*. After the Mach number, the Reynolds number is the most important parameter for describing a flow:

$$\text{(2.1:72)} \qquad Re = \frac{v^2 / s}{\nu \cdot v / s^2} = \frac{v \cdot s}{\nu} . \qquad [-]$$

If Re >> 1, e.g. if speed is high or viscosity is low, it is possible to ignore friction.

Since every flowing fluid adheres to a solid body, there is a layer at the surface of the body within which the velocity of the flow rises from zero to flow velocity, and within which friction occurs. For large Reynolds numbers this layer (the boundary layer) is very thin, and friction can be ignored.

If Re ≈ 1 or Re < 1, the friction forces must be taken into account in all parts of the flow. This is the case for low speeds, small dimensions or high levels of viscosity.

From the values in Table 2-8, we can see that the flight of a bullet in air can be treated as frictionless flow, whereas friction cannot be ignored when a bullet passes through a dense medium, such as glycerine soap or gelatine.

Table 2-8. Examples of Reynolds numbers for bullets

Bullet		Re
Rifle bullet, v = 800 m/s, s = 30 mm;	in air	$1.8{\cdot}10^6$
	in glycerine soap	0.05
Pistol bullet, v = 300 m/s, s = 15 mm;	in air	$3.4{\cdot}10^5$
	in gelatine	1.1

2.1.5 Fluid jets

2.1.5.1 General

A jet occurs when a fluid escapes from an opening in a container and penetrates another, stationary fluid, at a certain velocity. The density, state and composition of the two fluids can be the same or different. Such a jet always forms a mass flow. At high speeds or high densities, it can possess considerable kinetic energy. Under such circumstances, and up to a certain distance, a jet can be seen as a projectile, and can have a corresponding effect.

Exit flow phenomena occur when there is a pressure differential between the container and its surroundings. It is this differential that is responsible for the acceleration of the mass flow. The cross-sectional form of the opening plays a decisive role. If the cross-section of a tube varies, it is termed a nozzle. If the tube is at its narrowest at its exit, it is known as a muzzle.

2.1.5.2 Exhaust flow from a muzzle

In order to calculate the energy flux and the energy-flux density (which ultimately determines the wounding potential of the jet), we must first determine the exhaust velocity and the mass flow rate through the nozzle.

The process by which a gas leaves a nozzle is adiabatic, or nearly so. This means that heat exchange with the surroundings can be ignored. The equation of state for adiabatic processes is:

(2.1:73) $$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}, \qquad [-]$$

where γ is the adiabatic coefficient.

The exhaust velocity is calculated using Bernoulli's equation, with density as per Eqn 2.1:73. After some calculation, we obtain:

(2.1:74) $$v_m = \sqrt{\frac{2\cdot\gamma}{\gamma-1}\cdot\frac{p_i}{\rho_i}\cdot\left[1 - \left(\frac{p_m}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}, \qquad [m/s]$$

where v_m is the exhaust velocity, p_m the exit pressure and p_i and ρ_i the pressure and density of the fluid in the container.

Using Eqn 2.1:59, and taking Eqn 2.1:73 into account, the mass flow rate for an exhaust cross-sectional area A is:

(2.1:75) $$\dot{m} = A \cdot \sqrt{\frac{2 \cdot \gamma}{\gamma - 1} \cdot p_i \cdot \rho_i \cdot \left[\left(\frac{p_m}{p_i} \right)^{\frac{2}{\gamma}} - \left(\frac{p_m}{p_i} \right)^{\frac{\gamma+1}{\gamma}} \right]} \; . \qquad \text{[kg/s]}$$

The mass flow rate therefore depends upon the pressure and density of the fluid in the container, the cross-sectional area of the opening and the ratio of the pressure inside the container to the pressure at the muzzle. From Eqn 2.1:75, it is apparent that there exists a pressure ratio p_m/p_i at which the mass flow rate reaches a maximum.

Eqn 2.1:75 is a continuous function, with a value of zero both at $p_m/p_i = 1$ and at $p_m/p_i = 0$. A maximum must therefore occur somewhere between these two values.

That pressure ratio is known as the *critical pressure ratio* and the corresponding muzzle pressure is the *de Laval pressure*, p_L. Substituting extreme values into Eqn 2.1:75 we obtain:

(2.1:76) $$\frac{p_L}{p_i} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \; . \qquad \text{[-]}$$

The corresponding velocity is called the *de Laval velocity*. This is the maximum possible exhaust velocity for a gas leaving a muzzle:

(2.1:77) $$v_{max} = \sqrt{\frac{2 \cdot \gamma}{\gamma - 1} \cdot \frac{p_i}{\rho_i} \cdot \left[1 - \left(\frac{2}{\gamma + 1} \right) \right]} \; . \qquad \text{[m/s]}$$

The corresponding mass flow rate is given by the following equation:

(2.1:78) $$\dot{m} = A \cdot \sqrt{\gamma \cdot p_i \cdot \rho_i \cdot \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}} \; . \qquad \text{[kg/s]}$$

If the ambient pressure is greater than or equal to the de Laval pressure, the escaping gas can expand fully in the muzzle. The jet will be focused and will be capable of transferring kinetic energy. If the ambient pressure is lower than the de Laval pressure, the gas cannot expand fully in the muzzle. Expansion from muzzle pressure to ambient pressure will occur explosively, causing completely turbulent flow that is incapable of doing work.

2.1.5.3 De Laval nozzles (converging-diverging nozzles)

From the continuity equation and Bernoulli's equation, we can now show that there is a close relationship between the Mach number of the flow Ma and the form of a nozzle.

From the equation of motion, Eqn 2.1:71, we obtain Bernoulli's equation in differential form for a stationary, frictionless flow (the precondition for a jet):

$$\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + v \cdot \frac{\partial v}{\partial x} = 0 \,. \qquad (2.1{:}79) \qquad [m/s^2]$$

Multiplying the equation by ∂x and inserting ∂ρ in the first term and v in the second, we obtain:

$$\left(\frac{\partial p}{\partial \rho}\right) \cdot \frac{\partial \rho}{\rho} + v^2 \cdot \frac{\partial v}{v} = 0 \,. \qquad (2.1{:}80) \qquad [m^2/s^2]$$

However, from Eqn 2.1:57a, the first factor in the first summand is the speed of sound squared, which means that we can insert the Mach number Ma in Eqn 2.1:80:

$$\frac{\partial \rho}{\rho} + Ma^2 \cdot \frac{\partial v}{v} = 0 \,. \qquad (2.1{:}81) \qquad [-]$$

As we can assume that the flow is stationary, the mass flow rate over time is constant. We can therefore express Eqn 2.1:59 in differential form, thus:

$$\partial\left(\rho \cdot v \cdot A\right) = \frac{\partial \rho}{\rho} + \frac{\partial v}{v} + \frac{\partial A}{A} = 0 \,. \qquad (2.1{:}82) \qquad [-]$$

Replacing the first summand in Eqn 2.1:81 with the aid of Eqn 2.1:82, we obtain the following after calculation:

$$\frac{\partial v}{v} = -\frac{1}{1 - Ma^2} \cdot \frac{\partial A}{A} \,. \qquad (2.1{:}83) \qquad [-]$$

From this equation, it is apparent that acceleration only occurs at $Ma < 1$ (i.e. when $\partial v/v > 0$) with a convergent nozzle ($\partial A/A < 0$) and only occurs at $Ma > 1$ with a divergent nozzle ($\partial A/A > 0$).

In other words, subsonic flows will be accelerated by a convergent nozzle, supersonic flows by a divergent nozzle. With a combination of these two types of nozzle, it is therefore possible to achieve gas velocities well over the speed of sound. This type of nozzle is called a *convergent-divergent nozzle*, or *de Laval nozzle*. By choosing the length of the divergent section appropriately, it is possible to ensure that the pressure in the exit section is equal to ambient pressure. The nozzle is then said to be adjusted. It is under these conditions that the jet attains the maximum possible speed.

The exhaust gas velocity is calculated using Eqn 2.1:74, taking the pressure at the exhaust cross section for p. Eqn 2.1:78 gives the mass flow rate for the smallest cross section.

The exhaust gas velocity and the mass flow rate form the basis for calculating the jet. These two variables also determine the maximum energy-flux density (i.e. the energy hitting a given area per unit time) that can occur in the jet.

2.1.5.4 Jet velocity and energy

Except at very low velocities (which are unlikely to cause jet injuries), a free jet becomes turbulent very shortly after emerging from the nozzle.

In turbulent flow, irregular fluctuating motion is superposed upon the basic longitudinal motion. As a result, the fluid particles are able to move transverse to the direction of flow. In laminar flow, this does not occur.

The velocity of a turbulent flow must therefore be seen as a mean value over time. Because of the different directions in which the particles are moving, there is an apparent increase in internal friction by comparison with the mean movement (see 2.1.4.3). This *eddy kinematic viscosity* is an important descriptor of turbulent flow.

As a result of the turbulence, the jet partly mixes with the surrounding fluid, dragging particles of that fluid along with it. The mass of fluid picked up by the jet increases with distance, and the jet becomes broader. At the same time, its velocity decreases, as total momentum remains constant.

For the purposes of wound ballistics, we can limit ourselves to studying round jets, as both the gases that exit the muzzle of a weapon and the propulsion jet of a rocket belong to this category. One advantage of round jets is that the eddy kinematic viscosity is constant throughout the jet. This being so, the equation for the mean velocity distribution in the jet is as follows (only the component acting in the direction of the jet being relevant):

(2.1:84) $$v_x(x,r) = \frac{3}{8 \cdot \pi} \cdot \frac{I}{\rho \cdot \varepsilon \cdot x} \cdot \left(\frac{1}{1 + \frac{1}{4} \cdot \beta^2} \right)^2 , \qquad \text{[m/s]}$$

where β is calculated as follows:

(2.1:84a) $$\beta = \sqrt{\frac{3}{16 \cdot \pi}} \cdot \sqrt{\frac{I}{\rho}} \cdot \frac{r}{\varepsilon \cdot x} . \qquad \text{[-]}$$

Symbols for the formulas above:

x	distance from opening	r	distance from axis of jet
I	total momentum (per unit time)	ρ	density
ε	apparent kinematic viscosity		

Along the axis of the jet (r = 0), β is zero, which simplifies Eqn 2.1:84 considerably. If we know the velocity, we can use the mass flow rate from Eqn 2.1:59 to calculate the energy-flux density (energy density E′ per unit time) as follows:

(2.1:85) $$\frac{E'}{\Delta t} = \frac{1}{2} \cdot \rho \cdot v_x^3 .$$

The main difficulty in performing such calculations lies in determining the eddy kinematic viscosity. This can be accomplished by taking pressure measurements along the axis of the jet. From the pressure, we can determine the velocity, which in turn allows us to calculate ε from Eqn 2.1:84.

2.1.6 Measuring techniques for wound ballistics

2.1.6.1 General

Wound ballistics is very much an experimental science. The starting point is always the observation of phenomena, which are generally preceded by events of which the preconditions are unknown. In order to understand these phenomena, we perform experiments. That simply means repeating the events under known, defined conditions. This is the only way of understanding the sequence "*preconditions ⇒ event ⇒ phenomenon*" and, if possible, creating a physical model.

Scientific experiments must be reproducible. This means that the entire sequence of events must meet certain conditions. An experiment therefore involves measuring not only the events (the "results") but all the preceding processes and preconditions that could affect the results. Measurements are hence of vital importance to empirical science.

From a mathematical standpoint, measurements are a type of mapping. Numerical values are assigned in a physical "model space" to a time- and space-related situation observed in "real space." This implies that the observer conducting the measurements always has a model of the process in his head, even if he is unaware of this. As a result, there is the danger that he will attempt to use the results of measurements to confirm his preconceptions. It is important to adopt a critical attitude and to be prepared to revise one's assumptions, both in designing the experiment and when interpreting the results. *The aim of an experiment is not to confirm one's assumptions, but to discover "reality."*

2.1.6.2 Dynamic phenomena

Recording the motion of a bullet – especially in a dense medium – poses a number of problems. The most significant problem is that all the processes occur within a very short time. For instance, at an impact speed of 900 m/s it takes a bullet approximately 1 ms to penetrate to a depth of 30 cm. In the process, very high decelerations occur, the axis of the bullet rotates about a transverse axis and the bullet often deforms or breaks up into several pieces. Regardless of the type of measuring system, all measurements must be recorded at a frequency of a few μs.

Velocity and deceleration. In ballistics, the most relevant parameters are the velocity and deceleration of a bullet. If these two values are known for all points, the forces can be calculated from Eqn 2.1:14 and the energy present from Eqn 2.1:19. These are the physical parameters of greatest importance in describing the processes.

The most common approach to determining velocity is to measure distance travelled over time using photoelectric barriers, radar or high-speed cameras. A first differentiation gives speed over time, a second gives deceleration. In order to determine speed over time to a reasonable degree of accuracy, the measuring points need to be as close together as possible. A high sampling rate is therefore required.

High-speed video cameras operate at speeds of tens of thousands of frames per second, as long as there is sufficient light. Special cameras designed for very fast processes can record over 100 million frames per second, with up to 16 images being available for each process. It is possible to choose the interval between these images as required. In the case of opaque materials, x-ray flash photography is used. This generally produces a maximum of six images.

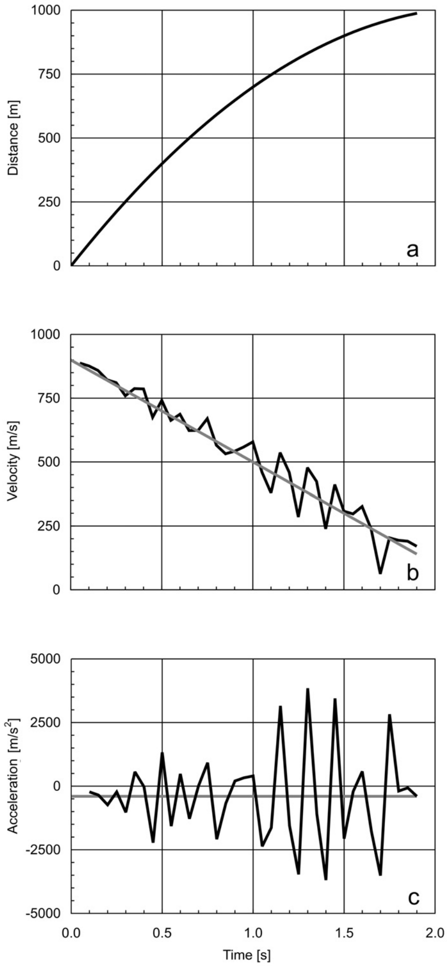

Fig. 2-6. Effect of measurement error on velocity and acceleration. The "correct" lines are shown in grey. See text for details.

Regardless of the method used, the double numerical differentiation required to calculate deceleration from time/distance measurements means that measurements must be extremely accurate. Even small errors in measuring distance and time will produce errors in the drag coefficient approximately two orders of magnitude greater.

Example: An object undergoes a constant deceleration of –600 m/s^2. A statistical error of just a few percent in the time/distance measurements will already cause visible discrepancies following differentiation over time (see Fig. 2-6b). The deceleration graph produced from a second numerical differentiation (see Fig. 2-6c) almost completely hides the fact that the object was undergoing constant deceleration.

Using pairs of photoelectric barriers and time measuring devices, it is possible to determine velocity directly. Two measuring points of this type make it possible to measure the change in velocity over a known distance, from which it is possible to derive deceleration by single differentiation (KNEUBUEHL 1982). This makes it possible to obtain a much higher degree of accuracy. Radar devices that utilize the Doppler principle can also be used to measure speed over time.

Doppler principle: A radar beam of a given frequency striking a moving body will be reflected at a different frequency. The change in frequency is proportional to the speed at which the body is moving.

However, neither method can be used for wound ballistics studies. Pairs of photoelectric barriers only allow us to measure the entry and exit speeds, and hence to calculate the drop in speed between the two points. They are not suitable for measuring a fast process that occurs over a short distance. Modern radar equipment can take measurements through simulants, which makes it possible to calculate the impact speed and residual speed of a bullet passing through a block of simulant. However, the sampling rate of such equipment is currently still too low

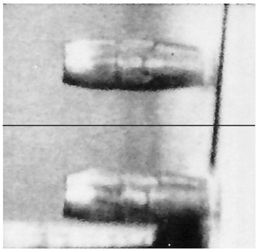

Fig. 2-7. Image of a bullet penetrating a block of soap, made using a mirror angled at 45°. *Upper picture:* plan. *Lower picture:* elevation. The angle of incidence can be determined from the two pictures (see also KNEUBUEHL 2004).

to record the change in speed over distance as the bullet passes through the material.

Proper motion of a bullet The proper motion of a bullet (the movement of the axis of the bullet about its centre of gravity) involves more than one plane. This motion can only be recorded using a measuring system that operates in two planes.

That involves taking a photograph of the bullet using two photographic plates at the same time, with the two plates arranged at right angles to each other. The photographic technology used (visible light or x-ray) is of secondary importance. From the pair of images produced, it is then possible to determine the orientation of the bullet's axis. This procedure is appropriate when one wishes to measure the angle of incidence of the bullet at a particular point (e.g. the point of impact) or to measure the change in angle of incidence along a section of the bullet's trajectory.

If the bullet is fired parallel to a mirror angled at 45° to the camera, it is possible to record both the side elevation and the plan view of the bullet on a single image. This method has the advantage that both images are created at exactly the same instant. See Fig. 2-7.

Another technique, which can only be used in the case of ferrous objects (e.g. bullets with a steel core or a thick steel jacket), utilizes the fact that a magnet rotating in a coil generates a potential difference. The change in potential difference (voltage) is proportional to the speed of rotation. If the bullet is magnetized and the target medium (soap or gelatine) placed inside a large coil, the rotating bullet will induce a voltage in the coil as it passes through. By recording this voltage, it is possible to determine the change in speed of rotation over distance (KNEUBUEHL 1990c; measuring method based on that of R. CATTIN).

2.1.6.3 Physical values

Centre of gravity. Moments of inertia. Determining the mass and the external dimensions of a bullet is both simple and commonplace. Determining the moments of inertia and the centre of gravity is less common, but is necessary in the field of ballistics, especially when one is studying stability.

The moment of inertia can be determined empirically using the principle of the physical (rotating) pendulum. The period of such a pendulum is directly related to the moment of inertia of the oscillating body relative to the centre of rotation. If a torsion wire is used to impart a retarding moment, measurements can be conducted through a bullet's centre of gravity, along its longitudinal or transverse axis. The moment of inertia of the measuring apparatus is measured beforehand using an object with known moments of inertia (see KNEUBUEHL 2004 for further details).

One way of finding the centre of gravity is to use a center of gravity measurement instrument. This involves sliding the bullet across a sharp edge until it is more or less in equilibrium. It is then held steady while the point in contact with the blade is marked.

If the structure of the bullet is not too complex, it is also possible to determine the centre of gravity and moments of inertia by calculation. This is the case for many handgun and rifle bullets. The process consists of dividing the bullet into thin slices, calculating the moment of inertia for each slice and calculating the sum of the moments.

Characteristics of materials. It is worth mentioning just a few specific measurement issues of relevance to wound ballistics. These include measuring viscosity and the speed of sound in dense media of the types used as simulants. Both parameters have a significant influence on shock waves.

Using Eqn 2.1:2, the speed of sound in a medium is determined by measuring the time a wave takes to pass through a layer of known thickness. Ultrasonic waves are generally used, although it is difficult to filter out the main signal.

Measuring the viscosity (and other mechanical parameters) of a simulant is important if fluid dynamics modelling is to be carried out. Measuring devices (cone viscometers) are available that allow standard measurements to be conducted even on very viscous materials.

2.2 Ammunition and weapons

2.2.1 Introduction

To understand wound ballistics, one needs a sound knowledge of ammunition in general and bullets in particular. We shall therefore be looking at the design of contemporary ammunition in some detail (2.2.2). Our focus will be on bullet design, as this has a major influence on wounding.

Ammunition and weapon are two halves of the same system. Sub-section 2.2.3 therefore gives an overview of key weapon characteristics and common types of weapon. This book is not the place for a detailed description of individual weapons, but exterior ballistics plays a major role in wound ballistics, as it tells us a great deal about the impact conditions. We shall be looking at exterior ballistics in sub-section 2.3.4.

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