Preface

Qui non se canta al modo de le rane,
qui non se canta al modo del poeta
che finge imaginando cose vane;
ma qui resplende e luce onne natura,
che a chi intende fa la mente leta;
qui non se gira per la selva oscura.

Dall'"Acerba", Cecco D'Ascoli

This book aims to introduce a systematic approach for solving mathematical problems arising in the study of hyperbolic-parabolic systems governing the motions of thermodynamic fluids. It appeals to a wide range of theoretical and applied mathematicians whose research interests may include compressible flows, capillarity theory, and control of motions. It aims to acquaint mathematicians and fluid dynamics researchers with recent results achieved in the investigation and of nonlinear asymptotic stability under no smallness assumption on initial data, and of loss of control from initial data for some steady flows of compressible fluids as well.

The main ideas are illustrated using three different model problems. The following classes of motions are studied:

(i) Barotropic viscous gases in rigid domains with compact, either impermeable or porous boundaries, and exterior domains.
(ii) Isothermal viscous gases with free boundaries.
(iii) Heat conducting viscous polytropic gases.

Equations governing non-steady flows of the fluids described in (i), (ii), (iii) are classified as follows:

(1) One vector parabolic equation for the velocity, and one scalar hyperbolic equation for the density.
(2) One vector parabolic equation for the velocity, and two scalar hyperbolic equations for the density and the free boundary.

(3) Two parabolic equations with one vectorial for the velocity, one scalar for the temperature, and one scalar hyperbolic equation for the density.

The main goal of this text is to reduce the study of stability, linear and nonlinear of basic motion S_b, to the sign of a suitable functional $\mathbb{E}$ called the ***modified energy functional****.* To this end, we employ the classical Lyapunov direct method[1] where the Lyapunov functional is identified with the *modified energy of perturbations* $\mathbb{E}(t)$. The modified energy $\mathbb{E}(t)$ consists of the difference between the total energies of unsteady $E(t)$ and basic E_b motions, plus an extra energy term given by the functional $\mathcal{I}(t)$, named *free work functional.* Namely $\mathbb{E}(t)$ is given by

$$\mathbb{E}(t) = E(t) - E_b + \gamma\mathcal{I},$$

where γ is an arbitrary parameter.

To apply the Lyapunov method using the modified energy as functional we are led to *formulate a differential equation governing the time evolution of the free work functional $\mathcal{I}(t)$, called the free work equation*, which is the key ingredient for the proof of uniqueness, asymptotic nonlinear stability, and loss of control from initial data, cf. [104]. Specifically, in coupled conservative-dissipative systems, the free work equation provides an artificial dissipative term for the variables satisfying conservative equations. As such, the free work equation, in combination with the energy equation for perturbations, results in a differential equation for the modified energy $\mathbb{E}(t)$, which then allows for a new a priori estimate that provides control of perturbations in particular norms that we call "natural norms"; cf. [4].

The two main tools that we will introduce in this book are:

(1) *A variant of Lyapunov's second method* (a generalization of Dirichlet method) to prove nonlinear stability;
(2) The *free work equation*, useful in proving asymptotic nonlinear stability, and loss of initial data control.

In our stability problems the control occurs on the employed "natural" norms for perturbations, that reduce only in particular cases to the L^2 norms of perturbations. In most cases, for regular flows, they will be equivalent to the

[1] Sometime the Lyapunov method is erroneously confused with the energy method. Actually in the energy method the Lyapunov functional is identified with the L^2 norm $K(t)$ of the difference $\mathbf{u} = \mathbf{v} - \mathbf{v}_b$ between the velocities $\mathbf{v}$, $\mathbf{v}_b$ of unsteady $S(t)$ and basic S_b motions. Of course $K(t) = \int_\Omega \mathbf{u}^2 dx$ doesn't coincide with the difference between the kinetic energies $E(t) = \int_\Omega \mathbf{v}^2 dx$, $E_b(t) = \int_\Omega \mathbf{v}_b^2 dx$ of the two motions,

$$K(t) \neq E(t) - E_b.$$

L^2 norm of perturbations. Furthermore, the basic flow S_b will represent either an equilibrium position or a steady motion.

In this text we will confine ourselves to the listed three cases (i), (ii) and (iii), as our aim is to explain a new algorithm, named the *free work identity.* It is introduced for the study of nonlinear stability of basic flows. We deem that our three examples of fluid motions, each containing an elastic behavior, cover a sufficiently wide mathematical set of partial differential equations (PDE), without being so many as to give rise to confusion.

Below is a summary of how the text will approach the subject matter.

(1) Chapter 1 is a prologue to thermo-fluid dynamics. In this chapter we introduce the equations for compressible fluids, with related boundary and initial-boundary value problems for the cases (i), (ii) and (iii) defined in the preface.
(2) In Chap. 2 we recall direct methods in the study of nonlinear stability, with the aid of three simple applications of the Dirichlet method. We end the chapter listing the main theorems proven in the paper.
(3) In Chap. 3 we consider barotropic fluids filling domains Ω with fixed, rigid, and compact closed boundaries $\partial\Omega$. The fluid may fill either the region Ω interior to $\partial\Omega$ where Ω is a bounded domain, or the region exterior to $\partial\Omega$, where Ω is an unbounded domain. For the exterior region we distinguish between cases when the fluid has finite mass, and when it has an infinite mass. We prove uniqueness theorems for several steady flows in a given regularity class of steady motions corresponding to the same data, thus we prove asymptotic decay of regular perturbed unsteady motions to these steady flows.
(4) In Chap. 4 we consider the rest state of an isothermal fluid in a section of horizontal layer with free boundaries. Periodicity in the horizontal direction is assumed. First we prove a uniqueness theorem of the rest state in the class of steady motions corresponding to the same data, thus we prove asymptotic decay to the rest. We also study the instability problem that occurs when the fluid is below the rigid plane of the layer. We extend this study by constructing rest states which are linearly stable, but not physically observable for large initial data. This result is achieved by introducing the concept of loss of initial data control.
(5) In Chap. 5 we consider the rest state of a polytropic viscous gas in a rigid, bounded domain, with perfectly heat conducting walls and periodicity in the horizontal direction. We then prove a uniqueness theorem for the rest state in the class of steady motions corresponding to the same data, and asymptotic decay to the rest.

Note that in Chaps. 2, 3 and 5:

(1) The initial perturbations may be large;
(2) The class of perturbations is quite large despite the fact that we will not be covering the problem of optimizing the regularity class of solutions.

This book deals with a single approach, in order not to obscure the main ideas.

To restrict the length of the book, the discussion of materials that require more extensive coverage, such as problems relating to fluid motions in pipes [57, 77, 111] and of heat conducting fluids with free boundaries [45], has been omitted.

The results described within the book are obtained using simple form, and the reader is required to know only the basic elements of classical and functional analysis.

Also note that each chapter is self-contained, and can be read independently.

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