

Introduction

Welcome to the second volume of the treatise “Mean fields models for spin glasses”. You certainly do *not* need to have read all of Volume 1 to enjoy the present work. For the low-temperature results of Part II, starting with Chapter 12, only (the beginning of) Chapter 1 is really needed. This is also true for Chapter 11.

In the first part of this volume we continue, at a deeper level, the study of four of the models that were introduced in Volume I. Chapter 8 continues the study of the Shcherbina-Tirozzi model of Chapter 3; Chapter 9 continues the study of the Perceptron model of Chapter 2. Both chapters culminate in the proof of the “Gardner formula” which computes the proportion of the sphere (respectively the discrete cube) that belongs to the intersection of many random half-spaces. Chapters 8 and 9 are somewhat connected. They could in principle be read with only the previous understanding of the corresponding chapter of Volume 1, although we feel that it should help to have also read at least a part of each of Chapters 2 to Chapter 4, where the basic techniques are presented.

Chapter 10 continues and deepens the study of the Hopfield model of Chapter 4. We achieve a good understanding for a larger region of parameters than in Chapter 4 and this understanding is better, as we reach the correct rates of convergence in $1/N$. This chapter can be read independently of Chapters 8 and 9, and in principle with only the knowledge of some of the material of Chapter 4.

Chapter 11 provides an in-depth study of the Sherrington-Kirkpatrick model at high temperature and without external field. As this is a somewhat simpler case than the other models considered in this work, we can look deeper into it. Only (the beginning of) Chapter 1 is a prerequisite from this point on.

In my lecture in the International Congress of Mathematicians in Berlin, 1998, I presented (an earlier form of) some of the results explained here. At the end of the lecture, while I was still panting under the effort, a man (whose name I have mercifully forgotten) came to me, and handed me one of his papers with the following comment “you should read this instead of doing this trivial replica-symmetric stuff”. To him I dedicate these four chapters.

The second part of this volume explores genuine low-temperature results. In Chapter 12 we describe the Ghirlanda-Guerra identities and some rather striking consequences. This chapter can be read without any detailed knowledge of any other material presented so far.

In Chapters 13 and 14 we learn how to prove a celebrated formula of G. Parisi which gives the value of the “limiting free energy” at any temperature for the Sherrington-Kirkpatrick model. A very special case of this formula determines that high-temperature region of this model. We present first this special case in Chapter 13. This seems to require all the important ideas, and these are better explained in this technically simpler setting. Parisi’s formula is believed to be only a small part of a very beautiful structure that we call the Parisi Solution. We attempt to describe this structure in Chapter 15 where we also prove as many parts of it as is currently possible. Chapter 15 can be read *without* having read the details of the (difficult) proof of Parisi formula in Chapter 14, and is probably the highlight of this entire work. We also explain what are the remaining (fundamental) questions to be answered before we reach a really satisfactory understanding.

In the final Chapter 16 we study the p -spin interaction model, in a case not covered by the theory of Chapter 14. The approach is based on a clear physical picture of what happens in the phase of “one step of replica-symmetry breaking” and new aspects of the cavity method.

I am very much grateful to Sourav Chatterjee and Albert Hanen who read this entire volume, sometimes in several versions in the most difficult parts, and also to Dmitry Panchenko and Marc Yor who read most of it. Each suggested countless many improvements, sometimes correcting serious errors. Special thanks are also due to Wei-Kuo Chen. I claim full responsibility for all the remaining mistakes.

<http://www.springer.com/978-3-642-22252-8>

Mean Field Models for Spin Glasses

Volume II: Advanced Replica-Symmetry and Low
Temperature

Talagrand, M.

2011, XII, 632 p., Hardcover

ISBN: 978-3-642-22252-8