

Chapter 2

Neoclassical Growth Theory and Standard Models

Thoughts and theories on economic growth can be traced back to the classical economists of the eighteenth and nineteenth century, whose works are briefly reviewed alongside the transition to neoclassical growth theory in Sect. 2.1. The basic outline of neoclassical growth models as first developed by Solow (1956) and Swan (1956) is presented in Sect. 2.2. The familiar but nonetheless special case of a Cobb-Douglas production function is examined in Sect. 2.3 in connection with the derivation of steady state levels of factors of production and output. Finally, Sect. 2.4 examines the inclusion of human capital as an additional factor of production and provides a note on endogenous growth theory.

2.1 From Classical to Neoclassical Growth Theory

Adam Smith's "An Inquiry into the Nature and Causes of the Wealth of Nations" includes some considerations on what is now referred to as economic growth. Although Smith (1776) does not develop a long run growth theory as such, conclusions on growth may be deduced, as he refers to the importance and effects of increasing labour productivity as well as saving. The *stationary state* is defined as a condition where capital accumulation and population size have reached their ceilings, and as a consequence the economy may not progress any more (Smith 1776, p. 82). In contrast to this rather pessimistic view, Smith also refers to technical progress, which raises aggregate output (Smith 1776, p. 75), but considers division of labour as an even more important potential for improving labour productivity (Smith 1776, p. 207).¹ However, division of labour may not be improved perpetually: whether long run growth of the aggregate economy is possible in Smith's model is open to interpretation. The crucial point in Smith's theorising is population growth – either it would grow to its maximum possible level, or it could be controlled. It follows implicitly that if the latter case were to be achieved, an increase of output per capita in the long run would be possible.

¹Smith famously exemplifies this with the production of pins.

When David Ricardo published his now best-known work “On the Principles of Political Economy and Taxation” in 1817, industrialisation was progressing so quickly that he already felt the need to rewrite those parts dealing with technology for the third edition in 1821. Like other economists of the industrial era, when the machine began to take precedence over the worker, it was impossible for him to foresee long run consequences. Ricardo was at first confident that productivity-augmentation due to machinery was to the benefit of all social classes, but revised his conclusions four years later: a scenario where profits rise while wages decline was possible (König 1997). The conclusion of Ricardo’s system concerning general economic growth is that technological progress may postpone, but not prevent the incidence of a stationary state (Pasinetti 1960). Ricardo assumes a two-sector economy that exhibits constant returns to scale in the manufacturing sector, but diminishing returns in the agricultural sector: as more land is cultivated, marginal quality of land worsens (Ricardo 1821). He considers capital owners to be the “productive class” of society, as they devote their profits to capital accumulation. This process of accumulation cannot go on infinitely, however, due to population growth. As a result, less fertile land will be cultivated, thus leading to aforementioned diminishing returns in agriculture (Ricardo 1821). Via feedback effects on employment growth and productivity, the profit rate will decrease until it has fallen to (almost) zero. This will prevent the capitalist class from accumulating, and the economy will have reached its stationary state, where all surplus is taken by the landlords.

A couple of decades later, economic development and growth are key themes in the studies of Karl Marx. He considers production to be interwoven with reproduction, distinguishes saving from consumption and accounts for depreciation and technological progress to develop a model of physical capital accumulation (Marx 1872). In this model, one part of the surplus value created in one period is consumed, while the other part becomes next period’s capital; thus concluding that after each turn, “capital has produced capital” (Marx 1872, p. 538).² Growth manifests itself in the ever-growing share of physical capital relative to labour and increasing labour productivity as a consequence of technological progress. This process is self-reinforcing: the working class, by producing physical capital, induces its own relative redundancy. Marx (1872) concludes that labour-demand growth is too low to compensate for decreases in employment following technological progress. In the long run, unemployment will rise until capitalism is abolished.

A further aspect of Marx’ studies is the interdependence of technical progress and the *falling tendency of the rate of profit*, a topic already featured prominently in Ricardo’s work as discussed above. Sustained (technological) progress requires an increasing ratio of fixed capital to output, which causes crises in the capitalist system and which could lead to a stationary state, although Marx (1894) considers several scenarios and countervailing forces as well. Apart from his visions of the demise of capitalism, Marx (1885) also developed a theory of medium run development

²“Das Kapital hat Kapital produziert.”

whereby the (capitalistic) economy grows at a constant rate (Krelle and Gabisch 1972). In this model of medium run growth, there exist a production goods sector and a consumption goods sector. Krelle (1988) later formalises this model for the two sectors and shows that both grow at identical rates, depending positively on the saving rate and the rate of surplus value. This bears the interesting result that wages may increase in absolute terms even in the case of a rising rate of surplus value (Krelle 1988, p. 55).³ Although Marx' model of medium run growth leaves some questions unanswered, it may be seen as an important forerunner of modern growth theory: it addresses to the same questions asked by economists today, and it anticipates the characteristic feature of neoclassical growth theory, namely the steady state.

Until the turn of the twentieth century and long after, economists remained remarkably silent on the issue of growth, with just a few exceptions. Joseph A. Schumpeter greatly stresses the role of innovations and therefore technical and technological progress, but less so the role of capital accumulation. He consciously distinguishes between economic *growth* and *development*, the latter being caused by endogenous factors that lead to groundbreaking innovation, thereby changing technique and productive organisation (Schumpeter 1926). The entrepreneur is greatly acknowledged for his pioneering in the field of new technologies, and therefore embodying the driving force of economic development (Schumpeter 1926). It is a crucial characteristic of Schumpeter's work that development does not happen gradually, but rather cyclical in the wake of innovations. He also used (and popularised) the term *creative destruction* to describe the process of transformation that accompanies innovations (e.g. Schumpeter 1942, p. 138). In the wake of new conceptions and theories of economic growth, the concept of creative destruction has been taken up and strongly emphasised again in the 1990s by endogenous growth theorists (e.g. Aghion and Howitt 1992).

One of the first economists to focus on the *rate of growth* was Roy F. Harrod (1939). Building on the works of John Maynard Keynes, he develops a theory that sets the base for conditions required for long run equilibrium growth. Keynes himself did not develop a growth theory as such, as he was primarily interested in short term developments, especially in the interplay of aggregate income and investments. Shortly afterwards, Evsey D. Domar (1946) stresses the importance of a dynamic view in order to gain insights into long run growth. Although Harrod and Domar developed their theories independently from each other, they display a number of similarities, which is why the resulting growth model is usually referred to as the *Harrod-Domar model*. Due to "implausible assumptions" and "undesirable outcomes" (Barro and Sala-i-Martin 1995, p. 49) as a result of building on Keynes'

³It is also interesting to note that political arguments in defence of declining wage shares (i.e. rising rates of surplus value) nowadays often go like this: "The wage share may decline, but workers are still better off because wages and salaries continue to grow at real terms" (see, for instance, *Der Standard*, July 21, 2007, p. 32: "...die Verdienste der unselbständig Beschäftigten [schrumpfen] – statistisch gesehen – nicht. Ihre Einkommen wachsen nur langsamer als Kapital- und Unternehmensgewinne.").

short term analysis to study long run problems, the Harrod-Domar model is now of primarily historical interest, but nevertheless remains appreciated as an intermediate step between classical and neoclassical theory.

2.2 The Basic Outline of Neoclassical Growth Models

In the introduction to his paper that forms the foundation of neoclassical growth theory, Robert Solow (1956) criticises the Harrod-Domar model by identifying its assumption of fixed proportions of labour and capital as the cause of an equilibrium growth that in fact balances on a knife's edge (Solow 1956, p. 65). As a tendency toward instability is particularly dissatisfying for any approach dealing with long run problems, Solow (1956) and Swan (1956) turn to neoclassical production functions with varying shares of labour and capital inputs. These two approaches provide the first neoclassical model of long run economic growth and mark the starting point for most studies on economic growth up to the present day, including the model presented in Part II.

The Solow model focuses on a closed economy where output Q is produced by the factors labour, L , and capital, K . The production function takes the form

$$Q_t = f(K_t, L_t) \quad (2.1)$$

where t denotes time. The critical assumption of the production function is that it shows constant returns to scale; Solow (1956) departs here from the classical assumption of scarce land or any non-augmentable resources. Romer (1996) interprets the assumption of constant returns to imply that the economy under consideration is big enough that the gains from specialisation have been exhausted.⁴

Technically speaking, the neoclassical production function is homogenous of degree one and implies that both factors must be available, or else output would equal zero (i.e. the economy would not exist). The function allows for an unlimited substitutability between capital and labour, which means that to produce any given output, any amount of capital can be efficiently used with the appropriate amount of labour. As a consequence of this assumption, the capital-output ratio can take on any nonnegative value. Furthermore, Eq. (2.1) exhibits positive and diminishing marginal products (quasi-concave) with $f(0) = 0$, $f'(\cdot) > 0$, $f''(\cdot) < 0$. The factors of production grow at constant rates:

$$\dot{L}_t = nL_t \quad (2.2)$$

$$\dot{K}_t = s_K f(K_t, L_t) \quad (2.3)$$

where a dot over a variable denotes a derivative with respect to time, and the labour force growth rate n as well as the saving rate s_K are exogenous parameters.

⁴The issue of what is actually “big enough” becomes relevant again when dealing with regions, and will be explored in Sect. 6.1.

Equation (2.2) implies that $L_t = L_0 e^{nt}$, and can be looked at as a supply curve of labour, with the labour force growing at an exponential and completely inelastic rate (Solow 1957, p. 67). Since L_t denotes both labour supply in Eq. (2.2) as well as total employment in Eq. (2.3), the model implies that full employment is perpetually maintained. Under the conditions of full employment and inelastic supply of both labour and capital at any point in time, both factors earn their marginal product, where the real wage v and the real interest rate r are given by

$$v_t = \frac{\partial Q_t}{\partial L_t} \quad (2.4)$$

$$r_t = \frac{\partial Q_t}{\partial K_t} \quad (2.5)$$

and where the price level is conveniently assumed to be constant. These two assumptions, of market-clearing conditions at any point and of constant price levels, are seen by some as weak points of neoclassical theory and should be briefly commented upon at this point of the study, as both are seldom found in the real world: as for market-clearing conditions, Krelle (1988, p. 89) notes that a constant degree of monopolisation of markets shifts both distribution of income and the propensity to save, thus changing numerical results, but has no effect on the main results. As for price levels, the underlying assumption may be understood as being not important enough to matter for *long run* growth or, alternatively, its fluctuations are being brought under control. The latter argument also applies to fiscal and monetary policies.

As long as closed economies are considered, output equals income by definition, therefore, it follows from Eqs. (2.1), (2.4) and (2.5) that

$$Q_t = v_t L_t + r_t K_t \quad (2.6)$$

which is the sum of wages and profits. In the terminology of macroeconomic accounting, Eqs. (2.1) and (2.6) thus equal gross value added (GVA) at t . Since indirect taxes are assumed to equal zero, gross value added is equal to gross domestic product (GDP). Furthermore, if there is no physical capital migration, gross domestic product is equal to gross national income (GNI).⁵

The capital-labour ratio k_t is defined as

$$k_t = \frac{K_t}{L_t} \quad (2.7)$$

The derivative of Eq. (2.7) (with no depreciation) with respect to time is

$$\dot{k}_t = \frac{\dot{K}_t L_t - K_t \dot{L}_t}{L_t^2} = \frac{\dot{K}_t}{L_t} - n k_t \quad (2.8)$$

⁵Gross national income is conceptually identical to gross national product.

Since the production function is homogenous of degree one, output per labour unit q_t can be expressed as

$$q_t = f(k_t) \quad (2.9)$$

where $f(k_t) \geq 0$ for $k_t \geq 0$. Like Eq. (2.1), (2.9) also exhibits positive and diminishing marginal products with $f(0) = 0, f'(\cdot) > 0, f''(\cdot) < 0$.

From Eqs. (2.3), (2.8) and (2.9), it follows that physical capital equipment per labour unit grows at

$$\dot{k}_t = \frac{s_K f(K_t, L_t)}{L_t} - nk_t = s_K f(k_t) - nk_t \quad (2.10)$$

The differential equation (2.10) is the fundamental equation of the Solow model: it states that the rate of change of the capital stock per labour unit is the difference between two terms. The first term, $s_K f(k_t)$, displays the increment of capital and represents actual investment per labour unit; the second term, nk_t , accounts for the increase of labour and as such represents the break-even investment necessary to keep k at its existing level. When $\dot{k}_t = 0$, the capital-labour ratio is constant, and consequently (with no depreciation) the aggregate capital stock K must be expanding at the same rate as the labour force n . If this is the case, the system is in equilibrium and is defined as being in a state of *balanced growth* (Solow 1956, p. 70).

The stability of the balanced growth path is shown in Fig. 2.1: if the current capital-labour ratio k_t is smaller than the equilibrium capital-labour ratio k^* , then actual investment per labour unit is greater than break-even investment and the capital-labour ratio rises with $\dot{k}_t > 0$. If $k_t > k^*$, then $\dot{k}_t < 0$, and hence k will decrease toward k^* . Whatever the starting point of the system,⁶ it converges to the

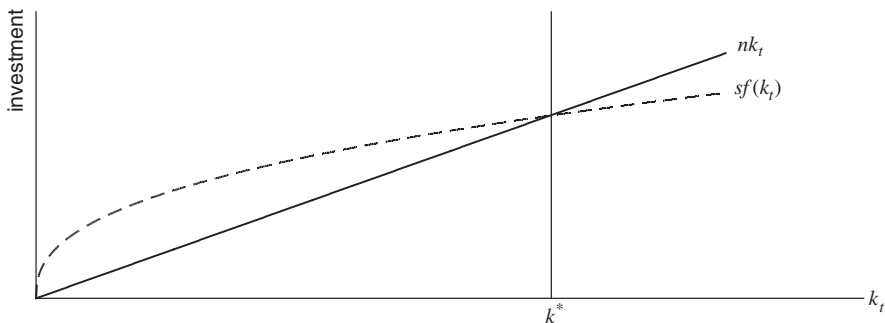


Fig. 2.1 Capital stock per labour unit in equilibrium

⁶Except for the possibility that there is no physical capital at all, which corresponds to the second intersection of the two lines at the point of origin in Fig. 2.1. As this mathematically possible equilibrium represents a non-existing economy, it is of no further interest.

equilibrium value of $k_t = k^*$, with $\dot{k}_t = 0$. Once this point has been reached, represented by the intersection of the two lines in Fig. 2.1 at $k = k^*$, the system stays there – thus the equilibrium is stable with $\dot{k}_t = 0$.⁷ If for some reason the economy moves temporarily away from equilibrium, it will be forced to return to the balanced growth path. In general, a situation in which the various quantities grow at constant rates is defined as a *steady state*.⁸

2.3 The Solution with Technological Progress and a Cobb-Douglas Production Function

Both Solow (1956) and Swan (1956) were primarily interested in economic growth as a consequence of capital accumulation, and studied the case of technological progress only briefly in their original papers. In a successive paper, Solow (1957) estimates technological progress for a 1909–1949 time series as a residual of capital and labour for explaining output growth of the US-American economy.⁹ Here the main conclusion is that technological progress appears to be *neutral* when it comes to scale effects: shifts in the production function therefore have no effect on marginal rates of substitution at given capital-labour ratios (Solow 1957, p. 316).

In general, a production function that takes the form as given in Eq. (2.1), where technological progress enters as labour-augmenting, is defined as *Harrod-neutral*:

$$Q_t = f(K_t, A_t L_t) \quad (2.11)$$

where A_t represents the level of technology at t . If technological progress is capital-augmenting of the form

$$Q_t = f(A_t K_t, L_t) \quad (2.12)$$

it is defined as *Solow-neutral*. Finally, if technological progress simply multiplies the production function by an increasing scale factor as

$$Q_t = A_t f(K_t, L_t) \quad (2.13)$$

it is defined as *Hicks-neutral*.¹⁰ Barro and Sala-i-Martin (1995) show that technological progress can always be expressed as labour-augmenting. In this context

⁷A formal proof for the stability of the equilibrium can be found in Krelle (1988).

⁸The term steady state reminds one of the stationary state as discussed in Sect. 2.1. The difference is that at steady state, an economy continues to grow at constant rate along the steady state growth path. Note that growth at constant rates also includes the possibilities of zero or even negative growth.

⁹This residual has become known as the *Solow residual*.

¹⁰Definitions and discussions of the various ways of implementing technological progress in a neoclassical production function can be found in Beckmann and Künzi (1984).

they prove mathematically that only labour-augmenting technological change is consistent with the existence of a steady state.

A Cobb-Douglas production function with Harrod-neutral technological progress therefore takes the form

$$Q_t = K_t^a (A_t L_t)^{1-a}, \quad 0 < a < 1 \quad (2.14)$$

where the exponents a and $1 - a$ denote the output elasticities of capital and labour, respectively. Marginal product of each factor is very large when its amount is sufficiently small, and becomes very small when the amount becomes large. This satisfies the *Inada-conditions* (Barro and Sala-i-Martin 1995, following Inada 1963) of the production function, in particular that the limit of the derivative towards zero is positive infinity, and that the limit of the derivative towards positive infinity is zero. These conditions are fulfilled by a Cobb-Douglas production function, whose intensive form is found by dividing Eq. (2.14) by technology-augmented labour $A_t L_t$:

$$\hat{q}_t = \left(\frac{K_t}{A_t L_t} \right)^a = \hat{k}_t^a \quad (2.15)$$

where output per unit of effective labour \hat{q}_t is a function of capital per unit of effective labour \hat{k}_t . It follows from Eq. (2.14) that output per labour unit at t equals

$$q_t = \left(\frac{K_t}{L_t} \right)^a = A_t^{1-a} k_t^a \quad (2.16)$$

which corresponds to GVA per worker at t .

If capital stock depreciation is considered in the equation that determines the evolution of the capital stock, Eq. (2.3) takes the form

$$\dot{K}_t = s_K Q_t - d K_t \quad (2.17)$$

where d is the rate of depreciation. Therefore, the aggregate capital stock grows as long as the left-hand term on the right side of Eq. (2.17) is greater than the right-hand term. In steady state, the amount of capital per unit of effective labour is constant; therefore break-even investment has to take technological progress into account:

$$\dot{\hat{k}}_t = s_K \hat{k}_t^a - (n + g + d) \hat{k}_t \quad (2.18)$$

where g is the rate of technological progress:

$$\dot{A}_t = g A_t \quad (2.19)$$

Thus it can be seen that if steady state growth is defined as an equilibrium where the growth rate of output per unit of effective labour \hat{q} is nil, while output per

unit of labour (i.e. per worker = per capita) q grows with technological progress g , then aggregate output Q grows with population *and* technological progress. In steady state, the ratio of capital to effective labour is constant, therefore growth of the physical capital stock per effective labour in steady state also equals zero. In other words, Eq. (2.15) expresses output per labour unit corrected for technological progress, which serves two illustrative purposes: firstly, growth per unit of effective labour in the steady state is a straight line, as can be seen from Fig. 2.2; secondly, if the question of interest is growth per capita that is *not* due to technological progress, correcting for technological progress makes comparisons easier.

The steady state level of \hat{k} is calculated from the right-hand side of Eq. (2.18) by setting both terms as equal:

$$\hat{k}^* = \left(\frac{s_K}{n + g + d} \right)^{\frac{1}{1-a}} \quad (2.20)$$

From Eqs. (2.15) and (2.20) it follows that the steady state output per unit of effective labour equals

$$\hat{q}^* = \left(\frac{s_K}{n + g + d} \right)^{\frac{a}{1-a}} \quad (2.21)$$

With A and L growing at constant rates, the model at steady state can be solved at any t for output per labour unit

$$q_t^* = A_t \hat{q}^* = A_0 e^{gt} \left(\frac{s_K}{n + g + d} \right)^{\frac{a}{1-a}} \quad (2.22)$$

and aggregate output in steady state at any t equals

$$Q_t^* = A_t L_t \hat{q}^* = A_0 L_0 e^{(g+n)t} \left(\frac{s_K}{n + g + d} \right)^{\frac{a}{1-a}} \quad (2.23)$$

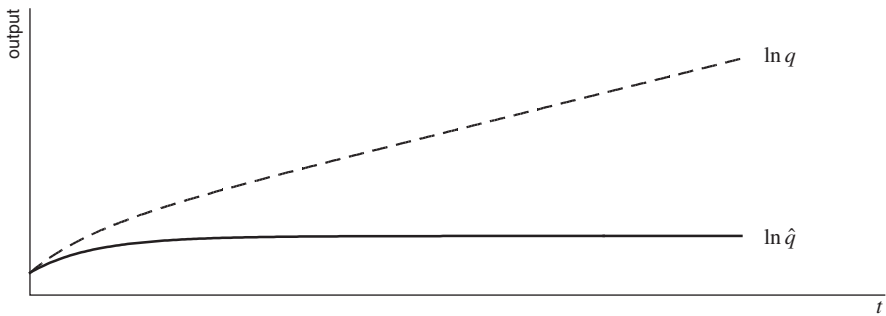


Fig. 2.2 Steady state per capita growth of output

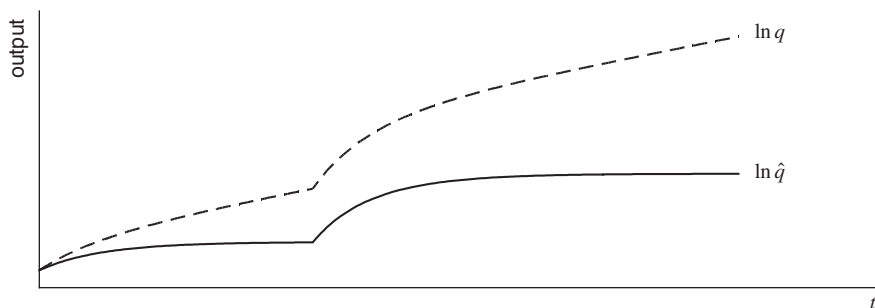


Fig. 2.3 An increase in the saving rate

From Eqs. (2.21), (2.22) and (2.23) it follows that an increase in the saving rate s_K raises output and therefore income in the long run: as can be seen from Fig. 2.3, output growth will be higher until the economy has reached its new steady state growth path. In other words, a change in the saving rate has a level effect in the long run: if the saving rate increases, growth will temporarily be higher. Long run growth, however, remains unaffected: growth rates of output per labour unit (= per capita), of the capital stock per labour unit and of the wage rate will equal the rate of technological progress as soon as the economy returns to its balanced growth path.

2.4 Human Capital as an Additional Factor of Production

One of the most controversial conclusions drawn from the Solow model and its successors is the implication of immense incentives to invest in economies where the marginal product of capital is highest, that is if rates of return across economies differ according to the model. Generally speaking, according to the Solow model as presented above, one would expect capital flows from wealthy countries (those with currently high stocks of physical capital, which for this reason should exhibit a relatively low marginal productivity of physical capital) to less wealthy countries (where conditions are the other way round). In addition, within the model these capital flows are supposed to happen incidentally, that is unhindered by time or space.

Capital of course does flow from wealthy countries to less wealthy countries; it is a process which is commonly understood to be one of the crucial characteristics of today's world economy. In fact, these flows have increased significantly since 1990 (The World Bank 2004a), although large discrepancies exist (The World Bank 2004b): the vast majority of investments takes place within the capital's origin country, and the vast majority of foreign direct investments (FDI) takes place within the developed world (The World Bank 2008). This phenomenon is often referred to as the Lucas paradox, as discussed by Lucas (1990). He identifies differing human

capital endowments and capital market imperfections as candidate answers for the question of why standard neoclassical predictions on capital flows fail at least in part when confronted with reality.

Mankiw, Romer and Weil (1992) explore this issue by augmenting the original Solow model to include the accumulation of human capital in addition to physical capital. The Mankiw-Romer-Weil model focuses on economies that converge to their steady states as long as the levels of both physical and human capital per worker rise, where human capital is supposed to be embodied in skilled workers. In analogy to Sects. 2.2 and 2.3, the rates of saving, population growth and technological progress are taken as exogenously given. With a Harrod-neutral Cobb-Douglas production function assumed, production at any point in time t is given by

$$Q_t = K_t^a H_t^b (A_t L_t)^{1-a-b}, \quad a > 0, \quad b > 0, \quad a + b < 1 \quad (2.24)$$

where K captures exclusively the stock of physical capital, H is the stock of human capital, b denotes the output elasticity of human capital, and the other variables are defined as in the Solow model. As can be seen from Eq. (2.24), there is a clear distinction between human capital H , and abstract knowledge A .¹¹ Human capital is defined as consisting of the abilities, skills and knowledge of particular workers, and is thus rival and excludable (Romer 1996, p. 126). Furthermore, although human capital is embodied in workers and hence represents in fact a specific kind of labour, it is treated as a second type of capital in analogy to physical capital. This definition of human capital has two major effects within the modelling framework: firstly, introducing human capital implies that the sum of shares of output paid to capital of both kinds is raised. Secondly, devoting more resources to the accumulation of either type of capital increases the amount of output that can be produced in the future.¹²

Dividing the production function of Eq. (2.24) by technology-augmented labour $A_t L_t$ yields output per unit of effective labour

$$\hat{q}_t = \hat{k}_t^a \hat{h}_t^b \quad (2.25)$$

where output per unit of effective labour is now a function of physical capital per unit of effective labour and human capital per unit of effective labour.

The evolution of the economy is determined by the key equations

$$\dot{\hat{k}}_t = s_K \hat{q}_t - (n + g + d) \hat{k}_t \quad (2.26)$$

¹¹The terms “abstract knowledge”, “state of technology” and, if appropriate, “effectiveness of labour” are usually used as synonyms here as well as in the bulk of growth literature.

¹²The term *human capital* (“Humankapital”) has been voted *Ghastly Neologism of the Year 2004* (“Unwort des Jahres”) in Germany, which provoked some controversy. It should be stressed that the jury was appalled by *non-technical* applications of the expression (*Spiegel Online*, 18th January 2005 (<http://www.spiegel.de/kultur/gesellschaft/0,1518,337259,00.html>), queried on 30-July-2007)).

and

$$\dot{\hat{h}}_t = s_H \hat{q}_t - (n + g + d) \hat{h}_t \quad (2.27)$$

where s_H is the fraction of output invested in human capital, so that $s_H Q_t$ represents the part of current aggregate output devoted to education expenditures. Note that human capital underlies the same assumptions as physical capital, in particular that human capital depreciates at the same rate, d , as physical capital. With the right-hand sides of Eqs. (2.26) and (2.27) set to zero, this system of two equations is solved for the steady state values of physical capital:

$$\hat{k}^* = \left(\frac{s_K^{1-b} s_H^b}{n + g + d} \right)^{\frac{1}{1-a-b}} \quad (2.28)$$

... and the steady state of human capital:

$$\hat{h}^* = \left(\frac{s_K^a s_H^{1-a}}{n + g + d} \right)^{\frac{1}{1-a-b}} \quad (2.29)$$

... and, as it follows from Eq. (2.25), steady state output equals:

$$\hat{q}^* = \hat{k}^{*a} \hat{h}^{*b} = \left(\frac{s_K s_H}{(n + g + d)^{a+b}} \right)^{\frac{1}{1-a-b}} \quad (2.30)$$

The economy is on a balanced growth path if $\dot{\hat{k}} = 0$ and $\dot{\hat{h}} = 0$, in which case output per effective labour unit is constant. The long run growth rate of output per worker thus equals the rate of technological progress g ; it applies to the growth rates of both production factors and is identical to the original Solow model. Accordingly, changes in either s_K or s_H will have a shift-effect on output (equivalent to those sketched in Fig. 2.3), but will leave long run growth unchanged.

The phase-diagram (for the method see Chiang and Wainwright 2005) in Fig. 2.4 displays the stability of the two differential equations given in (2.26) and (2.27): whatever the initial values of \hat{k} and \hat{h} , the system converges to the intersection point of the two curves $\dot{\hat{k}} = 0$ and $\dot{\hat{h}} = 0$.¹³ Once the intersection point is reached, the economy stays there. Although the system becomes more complex with the inclusion of a third factor, it remains stable: if the economy has moved away from equilibrium, it will return to the intersection point and hence to the balanced growth path.¹⁴

¹³In analogy to Sect. 2.2, the possibility of initial values of either type of capital equalling zero is ignored.

¹⁴A formal proof of the stability of the Mankiw-Romer-Weil model can be found in Gandolfo (1997).

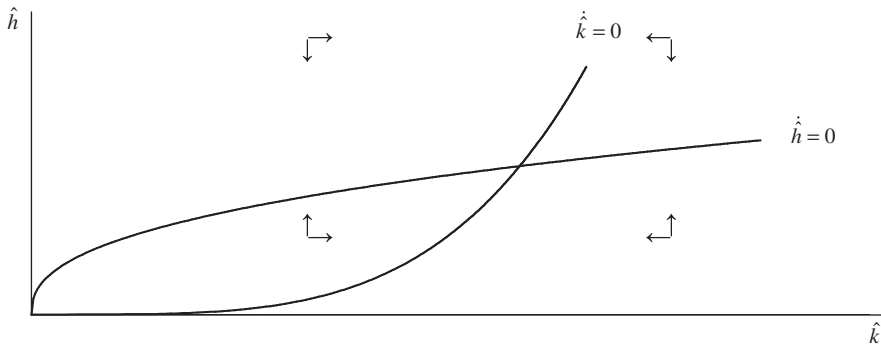


Fig. 2.4 The dynamic stability of the Mankiw-Romer-Weil model

While the similarity of the Mankiw-Romer-Weil model's qualitative conclusions to those of the Solow model follows from the model's structure, the introduction of human capital has a considerable impact on quantitative analysis. In particular, raising either saving rate, or both of them, will have shift-effects on long-term output levels. Depending on b , output elasticities with respect to s_K , s_H , g and n will differ significantly from those of the Solow model. The influence of b can be observed most easily by taking the natural logarithm of steady state output per unit of effective labour, $\hat{q}^* = \hat{k}^* a \hat{h}^{*b}$:

$$\ln \hat{q}^* = \frac{a}{1-a-b} \ln s_K + \frac{b}{1-a-b} \ln s_H - \frac{a+b}{1-a-b} \ln(n+g+d) \quad (2.31)$$

Note that Eq. (2.31) is identical to the natural logarithm of Eq. (2.21) if b is set to zero. In an empirical test, Mankiw, Romer and Weil (1992) show that their model is able to explain considerable differences of cross-country income levels, and also serves to explain why it may be not that attractive to invest somewhere just because the current level of physical capital is low. In this sense, the Mankiw-Romer-Weil model gives one of Lucas' (1990) candidate answers in detail.¹⁵ Furthermore, this model provides a seamless expansion of the Solow model, with long run growth still viewed as identical to technological progress – an aspect now seen as a characteristic feature of neoclassical models of (long run) growth.

The 1980s marked not only a deepening interest in the role and importance of human capital, but also in the origins of technological progress. Models of long

¹⁵Economic history is somewhat reflected in these two most important neoclassical growth models: the Solow model adjusts the focus on physical capital accumulation, and provides an explanation of worldwide economic growth after the Second World War; the Mankiw-Romer-Weil model adds a focus on human capital accumulation to the former, and provides an explanation of worldwide economic growth since the decline of industrial employment in Western Europe and North America. Seen from a meta-level, these two models perfectly mirror their respective ages of economic development.

run growth were developed that explained technological progress within the model (sometimes called “endogenised”) by relating it to other economic forces. From these models *endogenous growth theory* (new growth theory) has evolved, which tries to model technological progress *and* growth – that is, it postulates that long run growth rate can in fact be influenced by economic factors. In this spirit, P. M. Romer (1986) develops a model in which the creation of new knowledge by one firm is assumed to have a positive external effect on the production possibilities of other firms. This non-rivalry of knowledge is further developed by Lucas (1988), who assumes that human capital releases spillovers whereby each producer in an economy benefits from the average level of human capital in the economy. Endogenous growth theory has spawned further noteworthy approaches, such as those by P. M. Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), whose models focus mainly on the interdependence and feedback effects of various producers and sectors of the economy. These models are able to explain interdependence and developments of factors within *one* economy, but become highly complex when mutual influence of various economies is considered; thus they are problematic if unambiguous results are needed for empirical tests.

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