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Linear Algebra for Everyone

by Lorenzo Robbiano

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It is rare to find a new book, on a subject you love and have taught for years, that brings a smile on your face each time you read it. Lorenzo Robbiano's is such a book. It starts with a superb foreword, where we are introduced to a quest for a linear algebra's book that will appeal *everyone*. Starting from here, you are permanently under the impression that a witty Italian professor is talking directly to you. And you are most certainly enjoying listening to him.

The book is originally written in Italian. Under the masterful translation of Anthony Geramita you still can hear the beautiful Italian accent that permeates the book.

How do you write a book, for everyone, and still explain why your results are true ? Why your algorithms work ? What is their cost ? The author does so without introducing the formal language of mathematics. Very few proofs are to be found. Still the necessity of proving your results is emphasized (and then left to the proverbial mathematician).

Robbiano starts his book analyzing the simplest form of linear equations, $ax = b$, with a and b integers. We learn that there are different ways to approach this problem. What sort of solutions can we expect ? In what sort of trouble we can get into ? How do we deal with fractions ? What does a computer ? What happens if we are working over the real numbers, where there is no way of avoiding the need of approximate solutions ? All of this is elegantly explored in a few examples. At the end of the chapter, the reader knows what to look for. The quest for answers starts.

Chapter 1 consists of examples from "real life" that motivate the need of solving systems of linear equations. The examples are also used to introduce the formalism of matrices and vectors. In Chapter 2, we peek at the algebra of matrices, and learn that matrix multiplication is not commutative. The reader discovers that, when it exists, the inverse of a matrix can be used to solve the system of linear equations $AX = b$. A few questions leave the reader wondering if there exists a better way of solving systems of linear equations.

Chapter 3 is the heart of the book. It is here that Gauss's algorithm is finally introduced, together with the LU factorization. The question of when a matrix is invertible is tackled. The cost of Gauss's algorithm is analyzed. The reader finally knows a fast and efficient way of dealing with the system of linear equations $AX = b$. Determinants appear as an oracle that tells us if a matrix is invertible. Using the method of Gauss and analyzing the pivots, the determinant of 2 by 2 and 3 by 3 matrices are obtained.

Chapter 4 ends the first part of the book. Here the author introduces coordinates, scalar products and the modulus of a vector. Care is used to explain the idea of orthonormality. We learn how to perform changes of bases. The determinant of a general square matrix is defined as an alternating sum of monomials, and the reader is invited to verify the main properties of the determinant from this definition. This last discussion on the determinant of a general square matrix may be too terse to be understood by *everyone*, confirming the current tendency of decreasing the role of determinants in a first linear algebra course.

Chapter 5 starts the second part of the book. It deals with the study of real quadratic forms. Symmetric matrices are introduced, and we learn how to perform elementary operations to reduce them into their canonical form. The concepts of positive definite (and semidefinite) forms are studied, and Sylvester's theorem (a quadratic form represented by a symmetric matrix A is positive definite if and only if all of its principal minors are positive) is stated. The chapter ends with a description of the Cholesky decomposition of a matrix.

Chapter 6 looks deeper into the concepts of orthogonality. It starts by asking the reader : why are orthogonal coordinate systems particularly useful. For example, we learn that the inverse of an orthogonal matrix is its transpose. The rank of a matrix is introduced, and its properties studied. The chapter ends with the introduction of the Gram-Schmidt procedure as a method of constructing orthonormal bases, and the QR decomposition.

In Chapter 7 we finally learn what a linear transformation is. We see that linear transformations are represented by matrices, and learn how to compute their kernel and image. We compute the matrix of a projection and use it to solve the problem of least squares, and to compute pseudoinverses.

In chapter 8, the last chapter of the book, we come back to the idea of a matrix as a description of a linear transformation. When is a matrix similar to a diagonal matrix ? What does this mean ? The concepts of eigenvalue and eigenvector are introduced. We learn how to diagonalize a matrix, and analyze when this can be done. As applications, we compute powers of matrices, play with Fibonacci numbers, and study some families of differential equations.

An appendix teaches the reader how to solve linear algebra problems using the computer algebra system CoCoA.

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