Testing Basic Laws of Gravitation – Are Our Postulates on Dynamics and Gravitation Supported by Experimental Evidence?

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Abstract Gravity is the most fundamental interaction; it not only describes a particular interaction between matter, but also encompasses issues such as the notion of space and time, the role of the observer, and the relativistic measurement process. Gravity is geometry and, in consequence, allows the existence of horizons and black holes, nontrivial topologies, a cosmological big bang, time-travel, warp drive, and other phenomena unknown in nonrelativistic physics. Here we present the experimental basis of General Relativity, addressing its foundations encoded in the Einstein Equivalence Principle and its predictions in the weak and strong gravity regimes. We discuss several approaches in the search to reveal an influence of the much sought-after quantum theory of gravity. We emphasize assumptions underlying the dynamics – for example, Newton's axioms and conservation laws – and the current extent to which they are supported by experiment. We discuss conditions under which gravity can be transformed away locally, and examine higher order time derivatives in the equations of motion.

1 Introduction – Why Gravity Is So Exceptional

Gravity is the most fundamental interaction in physics: it is not only a very particular interaction between particles, but also it is related to the notion of space and time, the description of the observer, and the relativistic measurement process. Thus, *any issue related to gravity is also of concern for the description of all other interactions*.

Even by itself, General Relativity (GR), the relativistic theory of gravity, is highly interesting. Since GR is related to the space–time geometry, the gravitational interaction modifies the structure of space–time and leads to surprising phenomena, such as black holes. It is remarkable that we have a theory capable of predicting that a

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region of space–time can “disappear” and no longer be accessible to the observer. Other unexpected effects, like lensing or cosmological implications such as the big bang, have had a big impact on science, and even on the philosophy of science; in particular, they have attracted very much the attention of the general public.

It is fascinating to follow the present observational exploration of black holes, for example, in the center of our Milky Way [146]. In parallel, there are mathematical studies of known black hole solutions of GR, and the search for new solutions of the Einstein field equations, such as the solution for a disk of dust [127]. There are also numerical studies of the merging of binary black holes which, when spinning, may exhibit an unexpected acceleration [56].

GR in general, and solutions with black holes in particular, have lead to very beautiful, highly interesting, and exceedingly stimulating mathematics studies. In particular, these studies include questions about the geometry and the topology of black holes and our universe. These issues have stimulated a veritable laboratory for *gedanken* experiments, which have lead to consideration of the information paradox [68], time travel [123] (for a recent discussion see, e.g., [88]), warp drive [5] (for a more recent discussion see, e.g., [89]), etc.

In recent years, increasing effort has been spent on developing a quantum theory of gravity. A large number of people have attempted to develop a unification of quantum theory along the lines of string theory [111], loop quantum gravity [81, 140] or noncommutative geometry [126] (see also references therein). While string theory lays emphasis on the particle content of our physical world and neglects somewhat the geometrical nature of gravity, loop quantum gravity starts from gravity as space–time geometry and neglects the particle content. Within string theory, higher dimensional theories are experiencing a renaissance and, for example, black holes display even more unusual features than are known from four dimensions [51, 82].

Gravity is one area in physics where something new is expected, which will undoubtedly lead to another revolution in the physics paradigm. Very unusual effects are expected to arise in quantum gravity and there are both theoretical and experimental efforts under way in the search for the new phenomena it should entail. Until now all experiments are in agreement with standard GR. However, substantial efforts are being made to find experimental signatures of quantum gravity. Any experimental result in this direction will guide the development of the theory itself. New experiments have been designed and new technologies have been developed to improve available accuracy in the search for possible quantum gravity effects. It is speculated that perhaps the LHC has the potential to see related phenomena.

Since gravity is such a fundamental interaction – it covers the notion of space–time, the space–time geometry, the observer, the measurement process, etc. – it is clear that thinking about gravity and questioning its underlying principles can open up many unusual possibilities that should be tested by experiment. These range from questioning Newton’s axioms, conservation laws, the time dependence of constants, etc. One may also speculate whether under extreme situations, like extremely weak gravity, small accelerations, large accelerations, highest energies, ultralow temperatures etc., some of the principles underlying today’s physics lose their meaning.

Similarly exciting is quantum theory. The experimental realization of the strange behavior of quantum systems is always truly astonishing, as Bohr said: "If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet." However, since quantum theory is based on a scheme that is not directly related to experiments, that is, there is no real operational approach to quantum theory, it is much more difficult to systematically question various assumptions underlying quantum theory. For a survey of experiments testing quantum theory see [102].

In this chapter, we first describe the remarkable features of GR and then present its experimental basis. This basis consists in the principles underlying the fact that today gravity is described by a metric tensor representing the space–time geometry. This metric theory then predicts certain effects which, for Einstein's GR, acquire particular values. Then we give reasons why it is important to improve these experiments and to perform new ones, and we also present a strategy for such new tests, where emphasis is placed on tests of gravity and relativity in extreme situations. Finally we focus on unusual questions related to possible effects rarely discussed in the literature, like tests of Newton's axioms and of conservation laws, etc. In fact, all tests of gravity can be regarded as searches for "new physics". This is a considerably enlarged version of an earlier article [92].

2 Key Features of Gravity

Gravity is singled out and characterized by a set of universality principles that are shared by no other interaction.

1. Universal presence of gravity
   - Gravity is *everywhere*
   - Gravity *always can be transformed away* locally
2. Universal action on masses
   - Gravity acts on *all bodies*
   - Gravity acts on all bodies *in the same way*
3. Universal action on clocks
   - Gravity acts on *all clocks*
   - Gravity acts on all clocks *in the same way*
4. Universal creation of gravitational field
   - *Each mass* creates a gravitational field
   - Each mass creates a gravitational field *in the same way*

The last of these features means that all, say, spherically symmetric masses of the *same weight* create the same gravitational field. That means that a measurement of a gravitational field of a spherically symmetric body only gives the mass of the gravitating body and not its composition.

3 Standard Tests of the Foundations of Special and General Relativity

The basic structure of GR, and of all other physics, is encoded in the Einstein Equivalence Principle (EEP). This principle states that (i) if all nongravitational interactions are switched off, all pointlike particles move in a gravitational field in the same way, (ii) all nongravitational clocks[1] are influenced by the gravitational field in the same way, and (iii) locally, Special Relativity is valid, in that all physical laws are Lorentz covariant.

These principles are so important because they imply the following:

- The gravitational interaction is described by means of a metrical tensor. The mathematical frame for that is a Riemannian geometry.
- The equations of motion for a point particle, for a spin-$\frac{1}{2}$-particle, of the electromagnetic field, etc. have to be the geodesic equation, the Dirac equation, the Maxwell equations in Riemannian space–times with a certain space–time metric.
- All these Riemannian metrics have to be the same.

Owing to their importance it is clear that these principles have to be confirmed with the highest possible accuracy. We describe appropriate experiments below.

3.1 Tests of Special Relativity

Lorentz invariance, the symmetry of SR which also holds locally in GR, is based on the constancy of the speed of light and the relativity principle. For recent reviews see, e.g., [9, 116].

3.1.1 The Constancy of the Speed of Light

The constancy of the speed of light has many aspects:

1. The speed of light should not depend on the velocity of the source. Otherwise, it would be possible to measure in one space–time event in one direction two light rays with different velocities. This independence from the velocity of the source has been confirmed in various experiments in the laboratory as well as by astrophysical observations. If the velocity of light depends on the velocity of the source, then this can be written as $c' = c + \kappa v$, where v is the velocity of the source (in some frame) and κ some parameter. Within this model, it is possible that the light of a star in a binary system may overtake light that was emitted earlier. Such a reversal of the chronological order has never been observed, allowing the estimate $\kappa \leq 10^{-11}$ [27]. Laboratory experiments performed at CERN used protons hitting a Beryllium target to create π^0 mesons with a velocity of

[1] Pendula and hourglasses are not allowed.

$v = 0.99975\,c$. These moving mesons decay into photons whose velocity has been measured and compared with the velocity of photons emitted from a source at rest. No difference in the speed of the photons was found giving [7] $\kappa \leq 10^{-6}$ though, from a nonrelativistic point of view, one would expect almost $2c$. The constancy of c for photons appears to hold for all velocities of the source.

2. The speed of light does not depend on frequency or polarization. The best results for this are from astrophysics. From radiation at frequencies $7.1 \cdot 10^{18}$ Hz and $4.8 \cdot 10^{19}$ Hz of Gamma Ray Burst GRB930229 one obtains $\Delta c/c \leq 6.3 \cdot 10^{-21}$ [143]. In a theoretical model of a hypothetical photon rest mass the best restriction is $m_\gamma \leq 10^{-47}$ kg from radiation from GRB980703 [143]. Analysis of the polarization of light from distant galaxies yields an estimate of $\Delta c/c \leq 10^{-32}$ [84].
3. The speed of light is universal. This means that the velocity of all other massless particles, as well as the limiting maximum velocity of all massive particles, coincides with c. The maximum speed of electrons, neutrinos, and muons in vacuum has been shown in various laboratory experiments to coincide with the velocity of light at a level $|c - c_{\text{particle}}|/c \leq 10^{-6}$ [6,29,58,80]. Astrophysical observations of radiation from the supernova SN1987A yield an estimate for the comparison of photons and neutrinos, which is two orders of magnitude better [109, 157].
4. The speed of light does not depend on the velocity of the laboratory. This can be tested, for example, by comparing the frequency of an optical resonator that depends on the speed of light and the frequencies of an atomic clock, in a modern version of the corresponding Kennedy–Thorndike experiment. The best estimate today yields $\Delta c/c \leq 10^{-16}$ [71].
5. The speed of light does not depend on the direction of propagation. This isotropy of the speed of light has been confirmed, by modern Michelson–Morley experiments using optical resonators, to a relative accuracy of $\Delta c/c \leq 10^{-17}$ [71].

These results mean that the speed of light is universal, so it can be interpreted as part of the space–time geometry. The implied causal structure is an essential part of the operational description of space–time proposed by Ehlers, Pirani, and Schild [50].

3.1.2 The Relativity Principle

The relativity principle states that the outcome of all experiments when performed identically within a laboratory, that is, without reference to the external world, is independent of the orientation and the velocity of the laboratory. This applies to the photon sector as well as to the matter sector. For the photon sector this can be tested with the Michelson–Morley and Kennedy–Thorndike type experiments already discussed above.

Regarding the matter sector, the corresponding tests are Hughes–Drever type experiments. In general, these are nuclear or electronic spectroscopy experiments. Such effects can be modeled by an anomalous inertial mass tensor [67] of the corresponding particle. For nuclei, one then gets estimates of the order

$\delta m/m \leq 10^{-30}$ [35, 103, 135]. Modeling with an anisotropic speed of light, as in the $TH\varepsilon\mu$-formalism [168], yields $\Delta c/c \leq 10^{-21}$. In addition to the possibility of an anisotropic mass tensor, there is also the possibility of an anomalous coupling of the spin to some given cosmological vector or tensor fields, which would destroy Lorentz invariance. Recent tests have given no evidence for any anomalous spin coupling either to the neutron [19, 20], to the proton [74], or to the electron [69, 72]. All anomalous spin couplings are absent to the order of 10^{-31} GeV (see also [165] for a review). Similarly, higher order derivatives in the Dirac and Maxwell equations generally lead to anisotropy effects [110].

A further consideration is that there could be intrinsic anisotropies in the Coulomb or Newtonian potentials [83, 84]. Anisotropies in the Coulomb potential should affect the lengths of optical cavities which, in turn, might influence the frequency of light in the cavity, It has been shown that the influence of the anisotropies of the Coulomb potential are smaller than the corresponding anisotropies in the velocity of light [124]. Anisotropies in the Newtonian potential of the Earth have recently been searched for using atomic interferometry [125], which has constrained the anisotropies at the 10^{-8} level.

Future spectroscopy of anti-hydrogen may yield further information about the validity of the PCT symmetry.

3.1.3 The Consequence

The consequence of the above experiments is that within the accuracy given by these experiments, vectors transform with the Lorentz–transformations. The best adapted mathematical framework thus introduces a four-dimensional space–time, which, locally, is equipped with a Minkowski metric $\eta_{ab} = \mathrm{diag}(+1, -1, -1, -1)$. More can be found in standard textbooks; for example, see [139, 149].

3.2 Tests of the Universality of Free Fall

The Universality of Free Fall (UFF) states that all neutral point-like particles move in a gravitational field in the same way, that is, that the path of these bodies is independent of the composition of the body. The corresponding tests are described in terms of the acceleration of these particles in the reference frame of the gravitating body: the Eötvös factor compares the normalized accelerations of two bodies $\eta = (a_2 - a_1)/[\frac{1}{2}(a_2 + a_1)]$ in the same gravitational field. In the frame of Newton's theory this can be expressed as $\eta = (\mu_2 - \mu_1)/[\frac{1}{2}(\mu_2 + \mu_1)]$, where $\mu = m_{\mathrm{g}}/m_{\mathrm{i}}$ is the ratio of the gravitational to inertial mass. Though there are no point particles, it is possible experimentally to manufacture macroscopic bodies such that their higher gravitational multipoles are either very small or very well controlled. In other cases, a numerical integration yields the effective gravitational force on the extended body. Both these methods are used in the various tests of the UFF.

There are two principal schemes in which to perform tests of the UFF. The first scheme uses the free fall of bodies. In this case the full gravitational attraction toward the Earth can be exploited. However, these experiments suffer from the fact that the time-of-flight is limited to roughly 1 s and that a repetition needs new adjustment. The other scheme uses a restricted motion confined to one dimension only, namely a pendulum or a torsion balance. The big advantage is the periodicity of the motion, which by far beats the disadvantage that only a fraction of the gravitational attraction is used. In fact, the best test today of the UFF uses a torsion pendulum and confirms it at the level of $2 \cdot 10^{-13}$ [145]. Newly proposed tests in space, the approved mission MICROSCOPE [160], and the proposal STEP [108] will combine the full advantages of free fall and periodicity.

Quantum gravity inspired scenarios hint that the UFF might be violated below the 10^{-13} level [39, 40]. From cosmology with a dynamical vacuum energy (quintessence), a violation at the 10^{-14} level can also be derived [167]. If the validity of the UFF holds, we can impose bounds on the time variability of various constants, such as the fine structure constant and the electron-to-proton mass ratio [42].

According to GR, spinning particles couple to the space–time curvature [15, 70] and, thus, violate the UFF. However, the effect is far beyond any current experimental reach. Testing the UFF for spinning matter amounts to a search for an anomalous coupling of spin to gravity. Motivation for anomalous spin couplings came from the search for the axion, a candidate for the dark matter in the universe initially introduced to resolve the strong PC puzzle in QCD [122]. In these models, spin may couple to the gradient of the gravitational potential or to gravitational fields generated by the spin of the gravitating body. Tests of the first case by weighing polarized bodies show that, for polarized matter, the UFF is valid at a level of order 10^{-8} [73].

Charged particles, too, must couple to space–time curvature [44], again at a level that is too small to be detectable. It is possible to introduce a charge-dependent violation of the UFF by proposing a charge-dependent anomalous inertial and/or gravitational mass. It is also possible to choose the model such that, for a neutral atom, the UFF is fulfilled exactly while it is violated for isolated charges [45]. It has been suggested that a corresponding experiment be carried out in space [45].

3.3 Tests of the Universality of the Gravitational Redshift

A test of the universal influence of the gravitational field on clocks based on different physical principles requires clock comparison during their common transport through different gravitational potentials. There is a large variety of clocks that can be compared:

1. Light clocks (optical resonators)
2. Atomic clocks based on

   (a) Hyperfine transitions
   (b) Fine structure transitions
   (c) Principal transitions

3. Molecular clocks based on
   (a) Rotational transitions
   (b) Vibrational transitions
4. Gravitational clocks based on revolution of planets or binary systems
5. The rotation of the Earth
6. Pulsar clocks based on the spin of stars
7. Clocks based on particle decay

At a phenomenological level, the comparison of two collocated clocks is given by

$$\frac{\nu_{\mathrm{clock}\,1}(x_1)}{\nu_{\mathrm{clock}\,2}(x_1)} = \left(1 - (\alpha_{\mathrm{clock}\,2} - \alpha_{\mathrm{clock}\,1})\,\frac{U(x_1) - U(x_0)}{c^2}\right)\frac{\nu_{\mathrm{clock}\,1}(x_0)}{\nu_{\mathrm{clock}\,2}(x_0)} \tag{1}$$

where $\alpha_{\mathrm{clock}\,i}$ are phenomenologically given clock-dependent parameters, U is the Newtonian potential, and x_0 and x_1 are two positions. If this frequency ratio does not depend on the gravitational potential then the gravitational redshift is universal. This is a null-test of the quantity $\alpha_{\mathrm{clock}\,2} - \alpha_{\mathrm{clock}\,1}$. It is obviously preferable to employ a large difference in the gravitational potential, which clearly shows the need for space experiments. In experiments today, the variation of the gravitational field is induced by the motion of the Earth around the Sun and thus requires that the clocks used have very good long-term stability.

The best test to date has been performed by comparing the frequency ratio of the 282 nm $^{199}\mathrm{Hg}^+$ optical clock transition to the ground state hyperfine splitting in $^{133}\mathrm{Cs}$ over 6 years. The result is $|\alpha_{\mathrm{Hg}} - \alpha_{\mathrm{Cs}}| \leq 5 \cdot 10^{-6}$ [14, 54]. Other tests compare Cs clocks with the hydrogen maser, and Cs or electronic transitions in I_2 with optical resonators. We are looking forward to ultrastable clocks on the ISS and on satellites in Earth orbit, or even in deep space as proposed by SPACETIME, OPTIS, and SAGAS [94, 113, 169], which should considerably improve the quality of the scientific results.

So far there are no tests using anti-clocks, that is, clocks made of antimatter. However, since the production of anti-hydrogen is a well established technique today, attempts to perform high-precision spectroscopy of anti-hydrogen have been proposed. These measurements should first test special relativistic CPT invariance but, as a long-term measurement, could also be used to test the Universality of the Gravitational Redshift for a clock based on anti-hydrogen.

3.4 The Consequence

A consequence of the validity of the EEP is that gravity can be described by a Riemannian metric, $g_{\mu\nu}$, a symmetric second rank tensor defined on a differentiable manifold that is identified as the collection of all possible physical events. The purpose of this metric is twofold: First, it governs the rate of clocks, that is,

$$s = \int ds, \qquad ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu} \tag{2}$$

is the time shown by clocks where the integration is along the world-line of those clocks. Second, the metric gives the equation of motion for massive point particles as well as for light rays,

$$0 = D_v v \qquad \Leftrightarrow \qquad 0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} \tag{3}$$

where D_v is the covariant derivative along v and

$$\left\{ {\mu \atop \rho\sigma} \right\} = \frac{1}{2} g^{\mu\nu} \left(\partial_\rho g_{\nu\sigma} + \partial_\sigma g_{\nu\rho} - \partial_\nu g_{\rho\sigma} \right)$$

is the Christoffel symbol. Here $x = x(s)$ is the world-line of the particle parametrized by its proper time and $v = dx/ds$ the tangent vector along this world-line. While $g(v, v) = 1$ for particles, we have $g(v, v) = 0$ for light, so that we must use some affine parameter to parametrize the world-line of a light ray. More on that can be found in many textbooks on gravity; see, for example, [66, 121, 166]. It can be shown that this notion also describes the propagation of, for example, the spin vector, $D_v S = 0$, where S is a particle spin. (This is valid at first order in the spin vector; in the case of spin–spin interactions as they appear for spinning binary systems, terms of $\mathcal{O}(S^2)$ have to be added, see, e.g., [53].) In generalized theories of gravity there might be additional terms in the equations of motion for v and for S.

For a general, static, spherically symmetric space–time metric, which we take to have the form:

$$ds^2 = g_{tt} dt^2 - g_{rr} dr^2 - r^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2), \tag{4}$$

we obtain an effective equation of motion

$$\frac{1}{2} \left(\frac{dr}{ds} \right)^2 = \frac{1}{2} \left(\frac{E^2}{g_{tt} g_{rr}} - \frac{1}{g_{rr}} \left(1 + \frac{L^2}{r^2} \right) \right), \tag{5}$$

where E and L are the conserved (specific) energy and angular momentum, respectively. In the case of asymptotic flatness it is possible to uniquely define an effective potential [79]

$$U_{\text{eff}} = \frac{1}{2} \left(E^2 - 1 - \frac{E^2}{g_{tt} g_{rr}} + \frac{1}{g_{rr}} \left(1 + \frac{L^2}{r^2} \right) \right), \tag{6}$$

which completely governs the motion of the particle.

In order to solve the equations of motion one has to know the metric. The metric is given by independent field equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{7}$$

where G is a prescribed differential operator acting on the metric and T is the energy–momentum tensor of the matter creating the gravitational field.

4 Tests of Predictions

Gravity can be explored only through its action on test particles (or test fields). Accordingly, the gravitational interaction has been studied through the motion of stars, planets, satellites, and light. There are only very few experiments that demonstrate the effects of gravity on quantum fields.

Any metric theory of gravity leads to effects like the gravitational redshift, the deflection of light, the perihelion precession, the Lense–Thirring effect, the Schiff effect, etc. GR is singled out through certain values for these effects. In the case of weak gravitational fields, such as occur in the Solar system, and of asymptotic flatness, any deviation of a gravitational theory from GR can be parametrized by a few constants, namely the PPN parameters [168]. Many astrophysical observations and space experiments that probe fundamental physics are designed to make precise measurement of these effects and, thus, to better ascertain the PPN parameters.

For Einsteins GR we have, in the left hand side of the field Eq. 7,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{8}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and Ricci scalar, respectively. For a spherically symmetric gravitating body we obtain the Schwarzschild metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{1}{1 - \frac{2M}{r}} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2\vartheta d\varphi^2. \tag{9}$$

Use of this metric in the equation of motion yields an ordinary differential equation

$$\left(\frac{dr}{d\varphi}\right)^2 = \frac{r^4}{L^2}\left(E^2 - \varepsilon + \varepsilon \frac{2M}{r} - \frac{L^2}{r^2} + 2\frac{ML^2}{r^3}\right), \tag{10}$$

($\varepsilon = 1$ for massive particle, $\varepsilon = 0$ for light), which can be solved in terms of the Weierstrass $\wp$-function [65]

$$r(\varphi) = \frac{2M}{\wp(\frac{1}{2}\varphi; g_2, g_3) + \frac{1}{3}}, \tag{11}$$

where the Weierstrass invariants given by

$$g_2 = 4\left(\frac{1}{3} - \varepsilon\left(\frac{2M}{L}\right)^2\right) \tag{12}$$

$$g_3 = 4\left(\frac{2}{27} + \frac{2}{3}\varepsilon\left(\frac{2M}{L}\right)^2 - E^2\left(\frac{2M}{L}\right)^2\right) \tag{13}$$

depend on M, E, and L. This solution can be used to calculate most of the Solar system effects.

The Kerr metric is a vacuum solution of Einstein's field equation that describes a rotating black hole. This metric contains the product of $d\varphi\, dt$, which appears also in the metric of a rotating observer in Minkowski space–time. The gravitational field of a rotating star is not given by the Kerr solution but, for weak fields, the Kerr solution is a very good approximation to the solution for a rotating star (for which no exact solution exists) so one can, in practice, use the Kerr solution when describing effects related to the addition of rotation. In a weak field limit, the rotation of a star adds to the Schwarzschild metric (9) a term proportional to $J_i\, dt\, dx^i$, where J_i is the angular momentum of the rotating star. The solutions of the geodesic equation in the Kerr solution are quite complicated but are still given by elliptic integrals [34].

The situation in space–times with cosmological constant is much more complicated. A spherically symmetric mass in a universe with a cosmological constant is described by the Schwarzschild-de Sitter solution (see, e.g., [139]), and the corresponding geodesic equation can be solved by means of hyperelliptic integrals [62, 63]. Also in Kerr–de Sitter space–times the geodesic equation can now be solved analytically [61] (see also [64]), and even more generally in all Plebański–Demiański space–times without acceleration [60].

4.1 The Gravitational Redshift

The gravitational redshift compares the frequencies of a light ray measured by two different observers. The general situation is shown in Fig. 1. A light ray intersects the world-lines O_1 and O_2 of two observers at the space–time events x_1 and x_2. The measured frequency is given by $\omega = k(u) = k_\mu u^\mu$, where k is the 4-wave

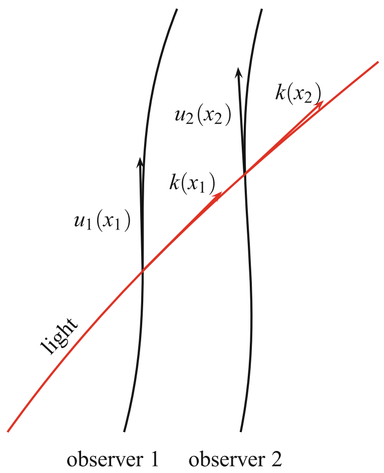

Fig. 1 The geometry of the gravitational redshift: a light ray crosses the world-lines of two observers that both measure the frequency of the light ray

vector of the light ray and u the 4-velocity of an observer. Accordingly, the gravitational redshift is given by the ratio

$$\frac{\nu_2}{\nu_1} = \frac{k(u_2)}{k(u_1)}, \tag{14}$$

($\omega = 2\pi\nu$). This relation gives the total redshift, consisting of the gravitational redshift and the Doppler effect.

In a stationary gravitational field this ratio can be presented in a very simple form. For a stationary gravitational field there exists a timelike Killing vector ξ, so that $k(\xi) = \omega_0 = \text{const}$ along the light ray. It then follows that

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{g_{tt}(x_1)}{g_{tt}(x_2)}} \approx 1 - \frac{GM}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{15}$$

where r_1 and r_2 are the radial positions of the two observers. The right part of the equation follows if we assume the validity of the Einstein theory of gravity.

This effect was observed first by Pound and Rebka [134] who confirmed the predictions to within 1%. Later, in a space experiment where the time of a hydrogen maser in a rocket was compared with the time of an identical hydrogen maser on Earth, the confirmation has been improved to 1 part in 10^4 [164]. The gravitational redshift also plays an important role in satellite navigation and positioning systems. In the passage of one day the redshift will account for a distance of several km on Earth.

A further aspect of the gravitational redshift is the coupling of gravity to the Maxwell field. Assuming a stationary situation, that is, assuming a Killing vector field ξ and an electromagnetic field strength F that is stationary, $\mathcal{L}_\xi F = 0$, it can be shown [78] that there exists a generalized scalar electrostatic potential ϕ so that $i_\xi F = d\phi$ (i being the inner product). With the observer's 4-velocity given by $u = e^{-\varphi}\xi$, where φ is a gravitational potential (in a Newtonian approximation it is mgz), we then have $d\phi = e^{\varphi} i_u F = e^{\varphi} E$ where E is the electric field measured by the observer u. Since ϕ is constant along the paths of charged particles, we have const. $=\Delta\phi \approx E(1+\varphi)$. As a consequence, the voltage between two identical batteries depends on their position in the gravitational field. This has been experimentally verified at the percent level [77]. This also confirms the universality of the coupling of gravity to all forms of matter.

4.2 Light Deflection

The deflection of light was the first prediction of Einstein's GR to be confirmed by observation, which occurred only four years after the complete formulation of the theory. With the exact solution of the geodesic equation for light given in Eq. 11, the deflection angle is defined as the difference between the angles φ_1 and φ_2 for which $\wp(\frac{\varphi}{2}; g_2, g_3) + \frac{1}{3} = 0$. Explicitly,

$$\delta\varphi = \frac{4}{\sqrt{e_1 - e_2}} F(\alpha, k), \quad \sin\alpha = \sqrt{-\frac{e_3 + \frac{1}{3}}{e_2 - e_3}}, \quad k^2 = \frac{e_2 - e_3}{e_1 - e_3} \tag{16}$$

where

$$F(\alpha, k) = \int_0^\alpha \frac{dx}{1 - k^2 \sin^2 x} \tag{17}$$

is the elliptic integral of the first kind [2] and $e_1 > e_2 > e_3$ are the three real zeros of the polynomial $4x^3 - g_2 x - g_3$ (in our light deflection scenario $e_3 < -\frac{1}{3}$). Here, $e_2 = \frac{2M}{r_2} - \frac{1}{3}$ where r_2 is the radial coordinate of closest approach of the deflected light ray. In an approximation for weak gravitational fields or small mass M this is $\delta\varphi = M/b$, where b is the impact parameter. In the frame of the PPN formalism we obtain $\Delta\varphi = \frac{1}{2}(1 + \gamma)M/b$.

Today's observations use Very Long Baseline Interferometry (VLBI); this has lead to a confirmation of Einstein's theory at the 10^{-4} level [151].

4.3 Perihelion/Periastron Shift

The exact value of the perihelion shift is

$$\delta\varphi = \frac{2}{\sqrt{e_1 - e_3}} F\left(\frac{\pi}{2}, k\right) - 2\pi, \tag{18}$$

where again $k^2 = \frac{e_2 - e_3}{e_1 - e_3}$ and $e_1 > e_2 > e_3$ are the real zeros of the corresponding polynomial (the values of k, e_1, e_2, and e_3 are here different from the corresponding values in the previous subsection). Here $e_2 = \frac{2M}{r_2} - \frac{1}{3}$ and $e_3 = \frac{2M}{r_3} - \frac{1}{3}$ so that we can relate e_2 and e_3 to the minimum and maximum radial distances, r_2 and r_3, of the orbit. In a post-Newtonian approximation one obtains $\delta\varphi = \frac{6\pi M}{a(1-e^2)}$, where a is the semimajor axis and e the eccentricity of the orbit. In the PPN framework this has to be multiplied with $(2 + 2\gamma - \beta)/3$.

It was first observed by Le Verrier in the nineteenth century that the perihelion shift of Mercury was larger than that calculated on a Newtonian basis from the influence of other planets. Today this post-Newtonian perihelion shift has been determined as 42″98 per century, with an error of the order 10^{-3} [133].

Recently, a huge periastron shift of a candidate binary black hole system in the quasar OJ287 has been observed, where one black hole is small compared to the other [161]. The observed perihelion shift is approximately 39° per revolution, which takes 12 years.

4.4 Gravitational Time Delay

In the vicinity of masses, electromagnetic signals move slower than in empty space. This effect is referred to as the gravitational time delay, see Fig. 2, which has been confirmed by observations and experiments. There are two ways to detect this effect: (i) direct observation, that is, by comparing the time of flight of light signals in two situations for fixed sender and receiver, and (ii) by observing the change in the frequency induced by this gravitational time delay.

4.4.1 Direct Measurement

The gravitational time delay for signals that pass in the vicinity of a body of mass M is given by [168]

$$\delta t = 2(1+\gamma)\frac{GM}{c^3}\ln\frac{4x_{\mathrm{Sat}}x_{\mathrm{Earth}}}{b^2}, \tag{19}$$

where x_{Sat} and x_{Earth} are the distances of the satellite and the Earth, respectively, from the gravitating mass and b is the closest distance of the signal to the gravitating mass. If the gravitating body is the Sun and if we the take b to be the radius of the Sun, then the effect would be of the order 10^{-4} s, which is clearly measurable. Reflection of radar signals from the surface of Venus has confirmed this effect [150]. An improved result is obtained by using Mars ranging data from the Viking Mars mission [136]. GR, characterized by $\gamma = 1$, has thus been confirmed by $|\gamma - 1| \leq 10^{-4}$.

4.4.2 Measurement of Frequency Change

Though the time delay is comparatively small, the induced modification of the received frequency can indeed be measured with higher precision, the reason being that clocks are very precise and can thus resolve frequencies very precisely.

The corresponding change in the frequency is easily derived. The emission time of the first wave crest is t_{s1}. This first wave crest will be received at

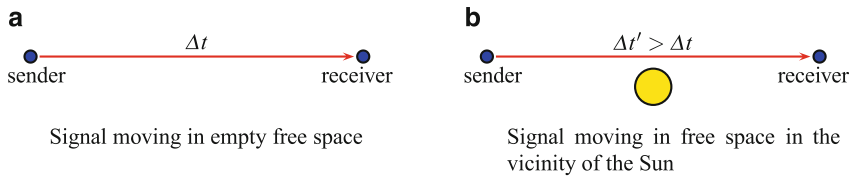

Fig. 2 Gravitational time delay. A signal from the sender to the receiver passing the Sun (**b**) needs a longer time than a signal in empty free space (**a**)

$t_{r1} = t_{s1} + \Delta t(t_{s1})$. Now, the second wave crest will be emitted at $t_{s2} = t_{s1} + \frac{1}{\nu_0}$ and received at $t_{r2} = t_{s1} + \Delta t(t_{s2})$. The measured frequency then is given by

$$\nu = \frac{1}{t_{r2} - t_{r1}}. \tag{20}$$

With the result (19) one can easily derive the relative frequency shift

$$y(t) = \frac{\nu - \nu_0}{\nu_0} = 2(\alpha + \gamma)\frac{GM}{c^3}\frac{1}{b(t)}\frac{db(t)}{dt}, \tag{21}$$

where ν_0 is the emitted frequency. It should be noted that, in this formula, it is the time dependence of the impact parameter that is responsible for the effect, which has been measured by the Cassini mission. The associated mission scenario is shown in Fig. 3. The calculated time dilation and frequency shifts are shown in Fig. 4. One

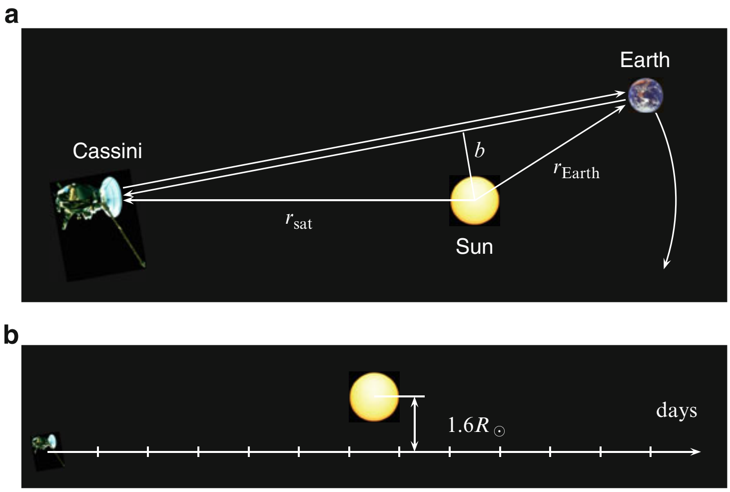

Fig. 3 Cassini mission scenario: (**a**) top view, (**b**) sight-of-line view form Earth

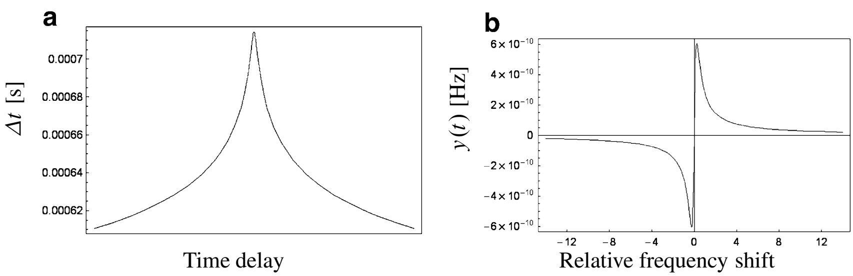

Fig. 4 (**a**) Calculated time delay, (**b**) relative frequency shift

important feature of the actual measurement was that three different wavelengths for the signals were used, which made it possible to eliminate dispersion effects near the Sun and to verify this time delay with an accuracy of 10^{-5} [23].

4.4.3 Remarks

The theoretical description of the gravitational time delay requires some additional remarks. In the above treatment – and this is the standard description of this effect – we compared a measurement in the presence of a gravitational field with a measurement without a gravitational field. However, within an exact framework for gravitational effects there is no definition for the unique identification of points with and without a gravitational field. Therefore, there is no definition of a gravitational time delay; there is no situation that can be taken as reference with respect to which the signal can be delayed.

Within an exact treatment there is only a combined effect due to the gravitational time delay, redshift, kinematical time delay (Doppler effect), and light bending. There is no way to isolate a gravitational time delay; this is only possible asymptotically, in the weak field approximation.

4.5 Lense–Thirring Effect

The metric component $J_i\, dt\, dx^i$ that reflects the rotation of a gravitating body can be regarded as representing a gravitomagnetic vector potential, the curl of which gives a Lorentz type gravitational force acting on bodies. The influence of this field on the trajectory of satellites results in a motion of the nodes (mathematically this is related to a period of the analytical solution of the geodesic equation), which has been measured by observing the LAGEOS satellites via laser ranging. Together with new data of the Earth's gravitational field obtained from the CHAMP and GRACE satellites, the confirmation recently reached the 10% level [36].

The gravitomagnetic field also influences the rate of clocks. It is easily shown that the geodesic equation for *circular* orbits in the equatorial plane reduces to

$$\frac{d\varphi}{dt} = \pm\Omega_0 + \Omega_{\text{Lense–Thirring}}, \tag{22}$$

where $\Omega_{\text{Lense–Thirring}}$ is the frame-dragging angular velocity that is proportional to the angular momentum of the gravitating source. The $\pm$ is related to the two different directions of the circular orbit. From this we obtain the difference of the proper time of two counterpropagating clocks, see Fig. 5,

$$s_+ - s_- = 4\pi \frac{J}{M}. \tag{23}$$

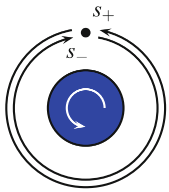

Fig. 5 Clocks tick differently when orbiting a rotating mass in opposite directions along the same orbit

It should be remarked that this quantity does not depend on G and r. In principle, this effect can be calculated for arbitrary orbits. It decreases with increasing inclination and vanishes for polar orbits. For clocks in satellites orbiting the Earth, this effect can be as large as 10^{-7} s [115].

4.6 Schiff Effect

The gravitational field of a rotating gravitating body also influences the rotation of gyroscopes. This effect is currently being considered by the data analysis group of the GP-B mission that flew in 2004. Analysis is expected to be complete in 2010. Though the mission met all design requirements, a huge technological success, it turned out after the mission that contrary to all expectations and requirements the gyroscopes lost more energy than anticipated [57]. For updates of the data analysis one may contact GP-B's Web site [57]. Full analysis of the experiment requires the determination of further constants characterizing this spinning down effect, which affects the overall accuracy of the measurement of the Schiff effect that was expected to be of the order of 0.5%. Nevertheless, recent reports of the GP-B data analysis group give at the moment an error of about 10% [52, 57].

It should be noted that although both effects within GR are related to the gravitomagnetic field of a rotating gravitational source, the Lense–Thirring effect and the Schiff effect differ conceptually, even measuring different quantities, so they may be regarded as independent tests of GR. In a generalized theory of gravity, spinning objects may couple to different gravitational fields (like torsion) than the trajectory of orbiting satellites. Moreover, the Lense–Thirring effect is a global effect related to the whole orbit while the Schiff effect observes the Fermi-propagation of the spin of a gyroscope.

4.7 The Strong Equivalence Principle

The gravitational field of a body contains energy that adds to the rest mass of the gravitating body. The strong equivalence principle now states that EEP is also valid for self gravitating systems, that is, that the UFF is valid for the gravitational energy, too. This has been confirmed by Lunar Laser Ranging with an accuracy of 10^{-3}

[168] where the validity of the UFF had to be assumed. However, the latter has been tested separately for bodies of the same composition as the Earth and Moon and confirmed with an accuracy of $1.4 \cdot 10^{-13}$ [16].

5 Why New Tests?

It is evident that the number of high precision tests relating to gravity has increased considerably in the last decade. This is certainly not due to some impact from the official Einstein year 2005, but is the consequence of (i) improved technology, (ii) the quest for a quantum theory of gravity, and (iii) problems in the understanding of observational data within standard GR.

5.1 Dark Clouds – Problems with GR

Despite all the confirmation catalogued above, some serious problems with GR may exist. In most cases there is no doubt concerning the data. The main problem is the interpretation of the observations and measurements. Each phenomenon that cannot be explained within standard GR is, inevitably, motivation to propose new theories. One should, nevertheless, spend considerable effort in searching for conventional explanations. Below, besides the "standard" interpretation of the phenomena we also mention activities regarding more conventional explanations.

5.1.1 Dark Matter

It was first observed by Zwicky in 1933 that in the Coma cluster of about 1,000 galaxies, the galaxies move with a velocity that is much higher than what is expected from the standard laws of gravity. This feature has since been confirmed for many other galaxy clusters, and even for stars within galaxies; it has also been confirmed with gravitational lensing. The apparent gravitational field is too strong. In order to keep the Einstein equations one introduces dark matter that accounts for the observed strength of gravity [158]. Structure formation also appears to need this dark matter. However, so far there is no single observational hint at which particles might constitute this dark matter. Consequently, there are alternative attempts to describe the same effects by a modification [141] of the gravitational field equations, for example, by a term of Yukawa form, or by a modification of the dynamics of particles, as in the MOND ansatz [120, 142], which has recently been formulated in a relativistic framework [21]. With the current lack of direct detection of Dark Matter particles, all these attempts remain on an equal footing.

Another attempt to solve the dark matter problem involves taking into account the full nonlinear Einstein equation. There are suggestions that many of the observations that are usually "explained" by dark matter could be explained by a stronger gravitational field which emerges from more fully taking the Einstein equations into account [17, 37].

5.1.2 Dark Energy

Observations of type Ia supernovae indicate an accelerating expansion of the universe and that 75% of the total energy density consists of a dark energy component with negative pressure [131, 137]. Furthermore, WMAP measurements of the cosmic microwave background [152], the galaxy power spectrum, and the Lyman-alpha forest data lines [129,159,162] all support the existence of Dark Energy, rather than a modification of the basic laws of gravitation [130]. However, in this case too, there are attempts to give an explanation in terms of modified field equations; see, for example, [128]. Recently it has been claimed that dark energy or, equivalently, the observed acceleration of the universe can be explained by inhomogeneous cosmological models, such as the spherically-symmetric Lemaitre–Tolman–Bondi model, see, for example, [13, 33, 163].

Buchert and Ehlers [31] have shown, first in a Newtonian framework, that with a spatial averaging of matter and the gravitational field, rotation, and shear of matter can influence the properties of the averaged gravitational field as would be described in effective Friedman equations. Their observation also holds in the relativistic case [30]. Therefore, it is still an open question whether or not the need for dark energy is just the result of an incorrect averaging procedure. An influence of the averaging has certainly been found in the interpretation of existing data [106, 107].

5.1.3 Pioneer Anomaly

The Pioneer anomaly is an anomalous, unexplained acceleration of the Pioneer 10 and 11 spacecraft of

$$a_{\text{Pioneer}} = (8.74 \pm 1.33) \cdot 10^{-10}\ \text{m/s}^2 \tag{24}$$

toward the Sun [11,12]. This acceleration seems to have been turned on after the last flyby of Jupiter and Saturn, and has stayed constant within a 3% range. Until now, no convincing explanation has been found. An anisotropy of the thermal radiation might explain the acceleration. In particular, while the power provided by the plutonium decays exponentially with a half life of 87.5 y (which would mean a decrease of more than 10% during 10 years), the acceleration has stayed constant within a margin of 3%. Presently, much further work is being done on a good thermal modeling of the spacecraft [138], and a reanalysis of the early tracking data is still underway. Improvements in ephemerides are also helping to eliminate various proposed explanations and theories [154].

5.1.4 Flyby Anomaly

It has been observed on several occasions that satellites after an Earth swing-by possess a significant unexplained velocity increase of a few mm/s. This unexpected and unexplained velocity increase is called the *flyby anomaly*. For a summary of recent analyses, see [100]. In a recent article [10] a heuristic formula has been found, which describes all flybys

$$\Delta v = v\frac{\omega R}{c^2}\left(\cos\delta_{\mathrm{in}} - \cos\delta_{\mathrm{out}}\right) \tag{25}$$

where R and ω are the radius and the angular velocity, respectively, of the Earth, and δ_{in} and δ_{out} are the inclinations of the incoming and outgoing trajectory.

Although no explanation has been found so far, it is expected that the effect is either (i) a mismodeling of the thermal influence of the Earth's and the Sun's radiation on the satellite, (ii) a mismodeling of reference systems (this is supported by the fact that all the flybys can be modeled by Eq. 25 containing geometrical terms only), or (iii) a mismodeling of the satellite's body by a point mass. There are also more hypothetical considerations: in [118, 119] a model was introduced in which the inertial mass experiences a modification that depends on the Hubble scale and the acceleration of a body. Within this model, the additional term accounts for the Pioneer anomaly and also gives a modification of the velocities of spacecraft during a flyby. Another proposal [32] relates the flyby anomaly to an anisotropic speed of light, which, however, only resorts to a non-understood early measurement reported by D.C. Miller 75 years ago and neglects all new confirmations of the isotropy of light at the level of 10^{-17}. In [3], S. Adler discusses the possibility that the flyby anomaly may be related to dark matter around the Earth. This proposal would lead to severe restrictions on the dark matter model (e.g., a two component dark matter model around the Earth is needed), which are unlikely to be consistent with other observations. In [132] a modification of Special Relativity, based again on a violation of the relativity principle, has been used in a scheme for obtaining a modified velocity. Within a certain five-dimensional theory of gravity [55] an additional acceleration occurs, which may be account for the flyby as well as the Pioneer anomaly. An attempt to understand the flyby anomaly on a conventional level has been carried forward by J.P. Mbelek [117], who claims that the observation was due to a mismodeling of Special Relativity in the orbit determination.

5.1.5 Increase of Astronomical Unit

The analysis of radiometric distances measured between the Earth and the major planets, and observations from Martian orbiters and landers from 1961 to 2003, both lead to reports of a secular increase of the Astronomical Unit of approximately 10 m/cy [87] (see also the article [153] and the discussion therein). This increase cannot be explained by a time-dependent gravitational constant G because the $\dot{G}/G$ needed is larger than the restrictions obtained from LLR. Such an increase might be mimicked, though, by a long-term increase in the density of the solar plasma.

5.1.6 Quadrupole and Octupole Anomaly

Recently, an anomalous behavior of the low-l contributions to the cosmic microwave background has been reported. It has been shown that (i) there exists an alignment between the quadrupole and octupole with $>99.87\%$ C.L. [43], and (ii) that the quadrupole and octupole are aligned to the Solar system ecliptic to $>99\%$ C.L. [148]. No correlation with the galactic plane has been found.

The reason for this anomaly is totally unclear. One may speculate that an unknown gravitational field within the Solar system slightly redirects the incoming cosmic microwave radiation (in a similar way that motion with a certain velocity with respect to the rest frame of the cosmological background redirects the cosmic background radiation and leads to modifications of the dipole and quadrupole parts). Such a redirection should be more pronounced for low-l components of the radiation. It should be possible to calculate the gravitational field needed for such a redirection and then to compare that with the observational data of the Solar system and the other observed anomalies.

5.2 The Search for Quantum Gravity

There are many experiments proving that matter must be quantized and, indeed, all experiments in the quantum domain are in full agreement with quantum theory, with all its seemingly strange postulates and consequences. Consistency of the theory also requires that the fields to which quantized matter couples also have to be quantized. Therefore, the gravitational interaction has to be quantized too. However, though gravity is an interaction between particles, it also deforms the underlying geometry. This double role of gravity seems to prevent all quantization schemes from being successful in the gravitational domain.

The incompatibility of quantum mechanics and GR is not only due to the fact that it is not possible to quantize gravity according to known schemes, but also because time plays a different role in quantum mechanics and in GR. Moreover, it is expected that a quantum theory of gravity will solve the problem of the singularities appearing within GR. It is also hoped that such a theory would lead to a true unification of all interactions and, thus, to a better understanding of the physical world.

Any theory is characterized by its own set of constants. It is believed that the Planck energy $E_{\mathrm{Pl}} \approx 10^{28}$ eV sets the scale for quantum gravity effects. All expected effects scale with this energy or the corresponding Planck length, Planck time, etc.

5.3 Possible New Effects

The low energy limit of string theory, as well as some semiclassical limit of loop quantum gravity and results from noncommutative geometry, suggest that many of the standard laws of physics will suffer some modifications. At a basic level these modifications show up in the equations of the standard model (Dirac equation, Maxwell equations, etc.) and in Einstein's field equations. These modifications then result in the following (see, e.g., [9, 38, 116]):

- Violation of Lorentz invariance
  - Different limiting velocities of different particles
  - Modified dispersion relations leading to birefringence in vacuum
  - Modified dispersion relations leading to frequency-dependent velocity of light in vacuum
  - Orientation- and velocity-dependent effects
- Time and position dependence of constants (varying α, G, etc.)
- Modified Newtonian law at short and large distances

In recent years there have been increased efforts to search for these possible effects, so far without success.

Besides these effects expected to result from quantum gravity, there are some more "exotic" issues that are usually taken for granted but are also worth testing experimentally. Such issues include:

- Violations of Newton's inertial law $\boldsymbol{F} = m\ddot{\boldsymbol{x}}$.
- Violation of *actio* = *reactio*.
- Violation of charge conservation.
- Violation of mass or energy conservation.
- Questioning that gravity can be transformed away even if UFF is fulfilled.

In most cases there is no basic theory from which these effects can be derived, due in part to the fact that equations of motions cannot normally be derived without an action principle. Nevertheless, since these issues are at the very basis of our description of physical dynamics, they should be tested to the highest accuracy possible.

6 How to Search for "New Physics"

If one looks for "new physics" then one has to measure effects that have never previously been measured. Strategies by which it might be possible to find new things include (i) using more precise devices, (ii) exploring new parameter regions, and (iii) testing "exotic" ideas.

6.1 Better Accuracy and Sensitivity

It is clear that in searching for tiny effects, better accuracy is always a good strategy. It is amazing how the accuracy for testing Lorentz invariance, for example, has increased over the years. It took more than 20 years to improve the results of the

Brillet and Hall experiment of 1979 [28]; within another few years the accuracy improved by two orders of magnitude and it is still improving further.

It would be of interest to find examples where present-day technologies have, at least in principle, sensitivity to quantum gravity effects. One such example arises with gravitational wave interferometers [8], which currently have a strain sensitivity of 10^{-21}. With Advanced LIGO the sensitivity will become 10^{-24}. Thus, for a continuous gravitational wave with a frequency in the maximum sensitivity range between 10 and 1,000 Hz a continuous observation over one year would reach a sensitivity of slightly less than 10^{-28}. This is the sensitivity needed for observing Planck scale effects (10^{28} eV) by optical laboratory devices (which have an energy scale of $\sim$1 eV). It is, thus, the level of sensitivity required to detect Planck-scale modifications in the dispersion relation for photons [8].

6.2 Extreme Situations

Often, "new physics" is discovered when new situations are explored. We discuss various scenarios of this kind.

6.2.1 Extreme High Energy

One possibility for exploring new physics is to probe physical processes at very high energies. With the LHC, where energies of the order 10^{13} eV should be achievable, it is hoped that signals of the Higgs particle and of supersymmetry will be found. This energy range is still far away from the quantum gravity scale. The best that one can do is to observe high energy cosmic rays that have energies of up to 10^{21} eV. It has, in fact, been speculated that the observations of high energy cosmic rays – which according to standard theories are forbidden owing to the GZK-cutoff – could indicate a modified dispersion relation [9, 116].

6.2.2 Extreme Low Energy

The other extreme, very low temperatures, might also provide a tool for investigating possible signals of quantum gravity. One may speculate that the influence of expected space–time fluctuations on the dynamics of quantum systems is more pronounced at very low temperatures. One may even speculate that such space–time fluctuations may give rise to a temperature threshold above absolute zero.

Very low temperatures may be achievable in BECs for which a long period of free evolution is possible. Recently a free evolution time of more than 1 s has been sustained at the Bremen drop tower where a BEC has been created during a period of 4.7 s of free fall [171]. These BECs may be used for novel investigations, including a search for deviations from standard physics predictions.

6.2.3 Large Distances

The unexplained phenomena, dark matter, dark energy, and the Pioneer anomaly are related to large distances. It is questionable whether the ordinary laws of gravity should be modified at large distances. Recently, some suggestions have been made:

- It has been examined whether a Yukawa modification of the Newtonian potential may account for galactic rotation curves [141].
- In the context of higher dimensional braneworld theories, deviations from Newton's potential arise [48]. At large distances the potential behaves like $1/r^2$, as one would expect from the Poisson equation in five dimensions. A comparison with cosmological and astrophysical observations has been reviewed in [112].
- From considering a running coupling constant, it has been suggested that the spatial parts of the space–time metric possess a part that grows linearly with distance [75]. This approach is in agreement with present solar system tests and also describes the Pioneer anomaly [76].

6.2.4 Small Accelerations

An acceleration, a, being of physical dimension $\mathrm{m\,s^{-2}}$ can be related to a length scale $l_0 = c^2/a$. Now, the largest length scale in our universe is the Hubble length $L_{\mathrm{H}} = c/H$, where H is the Hubble constant. The corresponding acceleration is cH, at an order of magnitude that remarkably coincides with the Pioneer acceleration and the MOND acceleration scales. As a consequence, it really seems mandatory to perform experiments that explore physics for such small accelerations (see below).

6.2.5 Large Accelerations

Analogously, since the smallest length scale is the Planck length l_{Pl}, the corresponding acceleration is $a = 2 \cdot 10^{51}\ \mathrm{m\,s^{-2}}$, which, however, is far outside any experimental reach. For the smaller accelerations that might be reached by electrons in the fields of strong lasers, one might be able to detect Unruh radiation or to probe the physics near black holes [144, 147].

6.2.6 Strong Gravitational Fields

Most observations and tests of gravity are being performed in weak fields: the solar system, galaxies, galaxy clusters. Recently, it became possible to observe phenomena in strong gravitational fields: in binary systems and in the vicinity of black holes.

The observation of stars in the vicinity of black holes [146] may, in one or two decades, give improved measurements of the perihelion shift and of the Lense–Thirring effect. Binary systems present an even better laboratory for observing strong field effects.

The inspiral of binary systems, which has been observed with very high precision, can be completely explained by the loss of energy through the radiation of gravitational waves as calculated within GR [24]. The various data from such systems can be used to constrain hypothetical deviations from GR. As an example, such data can be used for a test of the strong equivalence principle [41] and of preferred frame effects and conservation laws [22] in the strong field regime.

Double pulsars have recently been detected and studied. These binary systems offer possibilities for analyzing spin effects, thus, opening up an entirely new domain for exploration of gravity in the strong field regime [85, 86]. Accordingly, the dynamics of spinning binary objects has been intensively analyzed [25, 53, 156].

6.3 Investigation of "Exotic" Issues

We describe several "unusual" questions which are rarely posed but that are worth investigating both experimentally and theoretically. A class of these peculiarities addresses Newton's axioms, particularly their dynamical part related to forces:

1. Test of *actio* = *reactio*. Tests of this axiom can be encoded in a difference between active and passive charges (electric charge, masses, magnetic moments, etc., generally, any quantity that creates a corresponding field).
2. Test of the inertial law $m\ddot{x} = F$ where F is the force acting on a body. What is being measured here? The measured acceleration together with the knowledge of the mass (which can be determined, e.g., through elastic scattering) leads to the exploration of the force. This can be illustrated with the Lorentz force. If one sends charged particles through a condenser, their trajectory will be deflected in response to the voltage applied to the condenser. The deflection gives the force and the force defines the electric field E.
   Therefore, the question of testing the inertial law may have at least two meanings:
   (a) Why are there no higher time derivatives in the inertial law? (In fact, owing to back reaction all equations of motion are of higher than second order. For charged particles, for example, we have the third order Abraham–Lorentz equation. This back reaction force can be calculated from the basic equations of motion which are of second order only. Therefore, the question is why the underlying basic equations of motion are of second order.)
   (b) Does the inertial law hold for all forces, no matter how large or small? (in our example, do we have $m\ddot{x} = qE$ even if E becomes extremely large or small?)
3. Test of the superposition of forces.

7 Testing "Exotic" but Fundamental Issues

7.1 Active and Passive Mass

The notion of active and passive masses and their possible non-equality was first introduced and discussed by Bondi [26]. The *active mass* m_a is the source of the gravitational field (here we restrict to the Newtonian case with the gravitational potential U) $\Delta U = 4\pi m_\mathrm{a}\delta(x)$, whereas the *passive mass* m_p reacts to it

$$m_\mathrm{i}\ddot{x} = m_\mathrm{p}\nabla U(x). \tag{26}$$

Here, m_i is the inertial mass and x the position of the particle. The equations of motion for a gravitationally bound two-body system then are

$$m_{1\mathrm{i}}\ddot{x}_1 = G m_{1\mathrm{p}} m_{2\mathrm{a}} \frac{x_2 - x_1}{|x_2 - x_1|^3}, \qquad m_{2\mathrm{i}}\ddot{x}_2 = G m_{2\mathrm{p}} m_{1\mathrm{a}} \frac{x_1 - x_2}{|x_1 - x_2|^3}, \tag{27}$$

where $1, 2$ refer to the two particles and G is the gravitational constant.

For the equation of motion of the center of mass, $X = (m_{1\mathrm{i}}x_1 + m_{2\mathrm{i}}x_2)/M_\mathrm{i}$, we find

$$\ddot{X} = \frac{m_{1\mathrm{p}}m_{2\mathrm{p}}}{M_\mathrm{i}} C_{21} \frac{x}{|x|^3} \qquad \text{with} \qquad C_{21} = \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} - \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} \tag{28}$$

where $M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}}$ and x is the relative coordinate. Thus, if $C_{21} \neq 0$ then active and passive masses are different and the center of mass shows a self-acceleration along the direction of x. This is a violation of Newton's *actio* equals *reactio*. A limit has been derived by Lunar Laser Ranging (LLR): no self-acceleration of the moon has been observed yielding a limit of $|C_{\mathrm{Al-Fe}}| \leq 7 \cdot 10^{-13}$ [18].

The dynamics of the relative coordinate

$$\ddot{x} = -G \frac{m_{1\mathrm{p}}m_{2\mathrm{p}}}{m_{1\mathrm{i}}m_{2\mathrm{i}}} \left(m_{1\mathrm{i}} \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} + m_{2\mathrm{i}} \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} \right) \frac{x}{|x|^3}. \tag{29}$$

have been probed in a laboratory experiment by Kreuzer [90] with the result $|C_{21}| \leq 5 \cdot 10^{-5}$.

The issue of the equality of active and passive gravitational mass is of the same quality as the issue of the equality of inertial and passive gravitational mass. While the UFF is an equivalence of all bodies *reacting* to the gravitational field, here we have an equivalence of all masses *creating* a gravitional field: all (spherically symmetric) masses of the same weight create the same gravitational field, independent of their internal composition. The equality of active and passive masses constitutes a universality principle that we may call the *Universality of the Gravitational Field*.

It is interesting to note that there is no Lagrange function from which the equations of motion (27) can be directly derived. As a consequence there is no Hamiltonian, which means that there is no quantum version of this system. Only the equation of motion for the relative distance can be quantized.

7.2 Active and Passive Charge

Similarly, one can think of active and passive charges, which have been discussed recently [98]. Though electric charges have no direct link to gravity, a discussion of the similarities and differences to the gravitational case will underline the universality of this question and can lead to a better understanding of the gravitational case. As an example, we will see that on the one hand the weakness of the gravitational interaction helps in a search for a difference of active and passive masses, while on the other hand the fact that negative charges are possible may help in circumventing the short timescales present in the electromagnetic interaction, which at first sight are a big obstacle in searching for a difference in active and passive electric charges. Furthermore, since in the weak field approximation there are many similarities between gravity and electromagnetism, a different active and passive charge would give a strong indication of a possible difference of active and passive masses. Moreover, as charged bodies also gravitate, a difference in active and passive charges would probably lead to a modified behavior for interacting charged black holes. This realization has not yet been fully developed.

The resulting equations of an electrically bound system with different active and passive charges are similar to the equations for a gravitationally bound system with different active and passive masses. The only difficulty that arises here is that the self acceleration of the center of mass cannot be observed, since within atoms the timescale is too short so that, as a result, this effect averages out.

However, there is one substantial difference between this and the massive case: there are positive and negative charges. This opens up the possibility of defining active as well as passive neutrality. In order to exploit this possibility one has to consider a bound system in an external electric field E

$$m_{1\mathrm{i}}\ddot{x}_1 = q_{1\mathrm{p}}q_{2\mathrm{a}}\frac{x_2 - x_1}{|x_2 - x_1|^3} + q_{1\mathrm{p}}E(x_1), \quad m_{2\mathrm{i}}\ddot{x}_2 = q_{2\mathrm{p}}q_{1\mathrm{a}}\frac{x_1 - x_2}{|x_1 - x_2|^3} + q_{2\mathrm{p}}E(x_2), \tag{30}$$

where $q_{1\mathrm{p}}$, $q_{1\mathrm{a}}$, $q_{2\mathrm{p}}$, and $q_{2\mathrm{a}}$ are the passive and active charges. The equations of motion of the center of mass and the relative coordinate are

$$\ddot{X} = \frac{q_{1\mathrm{p}}q_{2\mathrm{p}}}{M_\mathrm{i}}\bar{C}_{21}\frac{x}{|x|^3} + \frac{1}{M_\mathrm{i}}\left(q_{1\mathrm{p}} + q_{2\mathrm{p}}\right)E, \qquad \ddot{x} = -\frac{1}{m_\mathrm{red}}q_{1\mathrm{p}}q_{2\mathrm{p}}\bar{D}_{21}\frac{x}{|x|^3}, \tag{31}$$

where

$$\bar{C}_{21} = \frac{q_{2\mathrm{a}}}{q_{2\mathrm{p}}} - \frac{q_{1\mathrm{a}}}{q_{1\mathrm{p}}}, \qquad \bar{D}_{21} = \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\frac{q_{1\mathrm{a}}}{q_{1\mathrm{p}}} + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\frac{q_{2\mathrm{a}}}{q_{2\mathrm{p}}}. \tag{32}$$

Thus, if active and passive charges are different, the center of mass shows a self-acceleration along the direction of x, in addition to the acceleration caused by the external field E. Due to fast internal motion the self-acceleration of the center of mass is not observable.

However, it is now possible to define active neutrality through $0 = q_{\mathrm{a}1} + q_{\mathrm{a}2}$ as well as passive neutrality $0 = q_{\mathrm{p}1} + q_{\mathrm{p}2}$. We may now prepare an actively neutral system by the condition that it creates no electric field (which may be explored by other test charges). This actively neutral system might be passively non-neutral and may react on an external electric field. Also, a passively neutral field may actively create an electric field. If actively neutral systems are also passively neutral, then the active and passive charge are proportional. These procedures can be carried out with high precision resulting in $\bar{C}_{12} \leq 10^{-21}$ [98]. Atomic spectra represent a cleaner test but yield only an estimate of the order $\bar{C}_{12} \leq 10^{-9}$ [98].

7.3 Active and Passive Magnetic Moment

A similar analysis can be carried out for magnetic fields created by magnetic moments. If active and passive magnetic moments are different, then again we would observe a self-acceleration of the center of mass. In this case atomic spectroscopy is more useful and yields an (unsurpassed) estimate $\widetilde{C}_{12} \leq 10^{-5}$ [98].

7.4 Charge Conservation

Charge conservation is a very important feature of the ordinary Maxwell theory:

- It is basic for an interpretation of Maxwell-theory as a $U(1)$ gauge theory.
- It is necessary for the compatibility with standard quantum theory insofar as it relates to the conservation of probability.

Recently, some models that allow for a violation of charge conservation have been discussed. Within higher dimensional brane theories it has been argued that charge may escape into other dimensions [46,47], leading to charge nonconservation in four-dimensional space–time. Charge nonconservation may also occur in connection with variable-speed-of-light theories [104]. A very important aspect of charge nonconservation is its relation to the EEP, which is at the basis of GR [105]. Charge nonconservation necessarily appears if, phenomenologically, one introduces into the Maxwell equations, in a gauge-independent way, a mass for the photon [95, 97].

The more important a particular feature of physics is, the more firmly this feature should be based on experimental facts. There seem to be only three classes of experiments related to charge conservation:

1. *Electron disappearing:* Charge is not conserved if electrons spontaneously disappear through $e \rightarrow \nu_e + \gamma$ or, more generally, through $e \rightarrow$ any neutral particles. Decays of this kind have been searched for using high-energy storage rings but they have not been observed [4, 155]. For the general process, the probability for such a process has been estimated to be $2 \cdot 10^{-22}$ year^{-1} [155]; for two specific processes the probability is as low as $3 \cdot 10^{-26}$ year^{-1} [4]. Even for a strict non-disappearance of electrons, the charge of an electron may vary in time and

thus may give rise to charge nonconservation. Thus, while charge-conservation implies the non-disappearance of electrons, electron non-disappearance does not imply charge conservation.

2. *Equality of electron and proton charge:* Another aspect of charge conservation is the equality of the absolute value of the charge of elementary particles like electrons and protons. Tests of the equality of q_e and q_p through the neutrality of atoms [49] yield very precise estimates because a macroscopic number of atoms can be observed. The result is $|(q_e - q_p)/q_e| \leq 10^{-19}$.
3. *Time-variation of α:* The most direct test of charge conservation is implied by the search for a time-dependence of the fine structure constant $\alpha = q_e q_p/\hbar c$. Since different hyperfine transitions depend in a different way on the fine structure constant, a comparison of various transitions is sensitive to a variation of α. Recent comparisons of different hyperfine transitions [114] lead to $|\dot{\alpha}/\alpha| \leq 7.2 \cdot 10^{-16}\ \mathrm{s}^{-1}$. This may be translated into an estimate for charge conservation $|\dot{q}_e/q_e| \leq 3.6 \cdot 10^{-16}\ \mathrm{s}^{-1}$, provided $\hbar$ and c are constant and $q_p = q_e$. However, this direct translation does not hold within the framework of varying c theories. An estimate that is more than one order of magnitude better comes from an analysis of the natural OKLO reactor [38], but it requires some additional assumptions on the α-dependence of various nuclear quantities.

Apparently, we have *no dedicated direct experiment to test charge conservation.*

7.5 Small Accelerations

Since the effect of gravity is observed by its influence on orbits of satellites and stars, a modification of Newton's first law, $F = ma$, will dramatically change the interpretation of the orbits and, therefore, the relation between the observation and the deduced gravitational field. This is, for example, the basis of the MOND (MOdified Newtonian Dynamics) ansatz proposed by Milgrom [120] and put into a relativistic formulation by Bekenstein [21].

The MOND ansatz replaces $m\ddot{x} = F$ by

$$m\ddot{x}\mu(|\ddot{x}|/a_0) = F, \tag{33}$$

where $\mu(x)$ is a function that behaves as

$$\mu(x) = \begin{cases} 1 & \text{for} \quad |x| \gg 1 \\ x & \text{for} \quad |x| \ll 1. \end{cases} \tag{34}$$

For Newtonian gravity this means that from the equation $F = m\nabla U$ we obtain the special cases

- For large accelerations: $\ddot{x} = \nabla U$.
- For small accelerations: $\ddot{x}|\ddot{x}| = a_0 \nabla U \to |\ddot{x}| = \sqrt{a_0 |\nabla U|}$.

This result for small accelerations, such as are present in the outer regions of galaxies, describes many galactic rotation curves very well, and may also reproduce dynamics of galactic clusters. The acceleration scale a_0 is of the order 10^{-10} m s^{-2}.

A recent laboratory experiment using a torsion balance tests the relation between the force acting on a body and the resulting acceleration [59]. No deviation from Newton's inertial law has been found for accelerations down to $5 \cdot 10^{-14}$ m s^{-2}. However, this does not mean that the MOND hypothesis is ruled out. Within MOND it is required that the full acceleration should be smaller than approximately 10^{-10} m s^{-2}, while in the above experiment only two components of the acceleration were small while the acceleration due to the Earth's attraction was still present. This means that better tests must be performed in space. An earlier test [1] went down to accelerations of $3 \cdot 10^{-11}$ m s^{-2}, though the applied force was nongravitational. It might be questioned whether the MOND ansatz applies to all forces or to the gravitational force only. There exists a short time and space window (of the order 1 s and 10 cm) for performing tests capable of such a distinction on Earth [170].

It has also been questioned whether the MOND ansatz can describe the Pioneer anomaly [12, 120] but positive confirmation has not been convincingly demonstrated. In any case, it is a very remarkable coincidence that the Pioneer acceleration, the MOND characteristic acceleration, and the cosmological acceleration are all of the same order of magnitude, $a_{\text{Pioneer}} \approx a_0 \approx cH$, where H is the Hubble constant.

What is the principal meaning of such tests? When we are testing $m\ddot{x} = F$ for small F, this at first sight means nothing. The only measured quantity in this equation is x as function of time from which we can derive $\ddot{x}$. Such measurements of $\ddot{x}$ are used to *define* the force F and to explore the charge-to-mass ratio. Therefore, this kind of measurement does not provide any kind of test.

The only way to give these experiments a meaning is if one has a model for the force. If the force is given by, for example, a gravitating mass, $F = m\nabla U$ with $U = G\int \rho(x')/|x-x'|dV'$, then one may ask whether the acceleration decreases linearly with decreasing gravitating mass. If the gravitating mass is spherically symmetric, $U = GM/r$, then the question is whether $\ddot{x} \to \alpha\ddot{x}$ for $M \to \alpha M$, particularly in the case of small M. This is an operationally well-defined question.

Since all components of the acceleration should be extremely small, it is necessary to perform such tests in space. It has been suggested that such a test should be carried out in a satellite located at a Lagrange point of the Earth–Sun system.

7.6 Test of the Inertial Law

The question we ask here is how one can test experimentally whether equations of motion possess second or higher order time derivatives. If the equation of motion is of nth order, then the solution for the path depends on n initial conditions. To enable a theoretical description of such tests we set up equations of motion of higher order where the higher order terms are characterized by some parameters which vanish in the standard equations of motion. This means that, besides their mass, particles are characterized by further parameters related to the additional higher order time derivatives. We solve these equations of motion and try to exploit already completed experiments, or propose new ones in order to obtain estimates on the extra parameters. So as not to be too general, we use the Lagrange formalism, which, for our purposes, is of higher order with a Lagrangian depending on higher derivatives. A complete description of a particle's dynamics requires the introduction of an interaction with, for example, the electromagnetic field. The structure of this coupling may differ from what we know in a more familiar, first order Lagrangian.

7.6.1 Higher Order Equation of Motion for Classical Particles

In order to get a feeling of what might happen we take for simplicity a (nonrelativistic) second order Lagrangian $L = L(t, x, \dot{x}, \ddot{x})$, see [101] for more details. The Euler–Lagrange equations read

$$0 = \frac{\partial L}{\partial x^i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^i} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{x}^i}. \tag{35}$$

It can be shown that these equations of motion remain the same if we add to the Lagrangian a total time derivative of a function $f(t, x, \dot{x})$,

$$\frac{d}{dt} f(t, x, \dot{x}) = \partial_t f(t, x, \dot{x}) + \dot{x}^i \frac{\partial}{\partial x^i} f(t, x, \dot{x}) + \ddot{x}^i \frac{\partial}{\partial \dot{x}^i} f(t, x, \dot{x}). \tag{36}$$

According to the gauge principle, one should replace the derivatives $\partial_t f(t, x, \dot{x})$, $\nabla f(t, x, \dot{x})$, and $\nabla_{\dot{x}} f(t, x, \dot{x})$ by gauge fields, which then yield gauge field strengths. However, it makes no sense to have velocity-dependent gauge fields. Therefore we assume that f is a polynomial in the velocities, $f(t, x, \dot{x}) = \sum_{k=0}^{N} f_{i_1,\ldots i_k}(x)\dot{x}^{i_1} \cdots \dot{x}^{i_k}$.

In the simplest case, $N = 0$ and $L = \frac{1}{2}\varepsilon\ddot{x}^2 + \frac{m}{2}\dot{x}^2$. In this case the gauged Lagrange function reads $L = \frac{1}{2}\varepsilon\ddot{x}^2 + \frac{m}{2}\dot{x}^2 + q\phi + q\dot{x}^i A_i$ that yields as an equation of motion

$$\varepsilon \ddddot{x} + m\ddot{x} = qE(x) + q\dot{x} \times B(x) = F(x), \tag{37}$$

where E and B are the electric and magnetic field derived as usual from the scalar and vector potentials ϕ and A. More general cases are discussed in [101].

This equation of motion may be solved in a first approximation by using, to begin with, the substitution $x = \varepsilon\bar{x} + x_0$ where x_0 is assumed to solve the equation of motion without the fourth order term. If we assume that the force is very smooth and that the deviation $\varepsilon\bar{x}$ is very small, that is, if $\bar{x} \cdot \nabla F(x_0) \ll m\ddot{\bar{x}}$ and can be neglected, then we obtain

$$\ddddot{x}_0 + \varepsilon \ddddot{\bar{x}} + m\ddot{\bar{x}} = 0. \tag{38}$$

This equation can be integrated twice

$$\ddot{x}_0 + \varepsilon\ddot{\bar{x}} + m\bar{x} = at + b, \tag{39}$$

where a and b are two integration constants. Inserting the equation for $\ddot{x}_0$ yields

$$\ddot{\bar{x}} + \frac{m}{\varepsilon}\bar{x} = -\frac{1}{m\varepsilon}F(x_0) + \frac{1}{\varepsilon}at + \frac{1}{\varepsilon}b. \tag{40}$$

With a new variable $\hat{x} = \bar{x} - \frac{1}{m}at + \frac{1}{m}b$ we have

$$\ddot{\hat{x}} + \frac{m}{\varepsilon}\hat{x} = -\frac{1}{m\varepsilon}F(x_0). \tag{41}$$

If ε is small (and m large),[2] then m/ε becomes large. Then the term $\frac{m}{\varepsilon}\hat{x}$ is dominant compared with the term on the right-hand side. If, furthermore, we take ε to be positive, then $\hat{x}$ is a fast oscillating term (for negative ε we have runaway solutions). The total solution then is

$$x(t) = x_0(t) + \varepsilon\left(\hat{x}(t) + \frac{1}{m}at - \frac{1}{m}b\right). \tag{42}$$

This solution consists of the standard solution $x_0(t)$, which is the main motion, a small displacement, a small linearly growing term, and a small fast oscillating term, a kind of *zitterbewegung*. From ordinary observations, a and b should be very small. Neglecting these particular contributions, the standard solution of the standard second order equation of motion seems to be rather robust against the addition of a higher order term.

The question now is how to search for the deviations from the standard solution. One way might be to look for the linearly growing term, which, however, requires a long observation time. Another way might be to search for a fundamental variation in the final position resulting from well-defined initial conditions. Some corresponding proposals have been worked out in [101].

7.6.2 Higher Order Equation of Motion for Quantum Particles

It is easier to consider the question of the order of the time derivative at the quantum level. If one adds, for example, a second time derivative to the Schrödinger equation, then this will change the spacing between the energy levels. A comparison with measurements yields an estimate on the strength of such a term [93]. A higher order time derivative in the Maxwell equations would, for example, modify the dispersion relation by adding cubic or higher order energy terms. Such additional terms could, in principle, be observed in high energy cosmic radiation or in experiments with gravitational wave interferometers, as described above in Section 6.1.

[2] We assume that ε is independent of m.

7.7 Can Gravity Be Transformed Away?

It might be thought that, with the validity of the UFF, it would be possible to eliminate gravity from the equations of motion of a neutral point particle. This is not the case. The UFF merely implies that the equation of motion should have the general form $\ddot{x}^\mu + \Gamma^\mu(x, \dot{x}) = 0$, where it is essential that no particle parameters enter this equation. If gravity can be transformed away (Einstein elevator), then the second term has to be bilinear in the velocity $\Gamma^\mu(x, \dot{x}) = \Gamma^\mu_{\rho\sigma}\dot{x}^\rho\dot{x}^\sigma$. This is not the case, for example, in Finsler geometries or in the model presented in [100]. These are examples where the UFF is valid but Einstein's elevator fails to hold; they constitute a gravity-induced violation of Lorentz invariance.

7.7.1 Finsler Geometry

An indefinite Finslerian geometry is given by

$$ds^2 = F(x, dx) \qquad \text{with} \qquad F(x, \lambda dx) = \lambda^2 F(x, dx), \tag{43}$$

so that

$$ds^2 = g_{\mu\nu}(x, dx)dx^\mu dx^\nu \qquad \text{with} \qquad g_{\mu\nu}(x, y) = \frac{1}{2}\frac{\partial^2 F(x, y)}{\partial y^\mu \partial y^\nu}, \tag{44}$$

where $g_{\mu\nu}(x, dx)$ is a kind of metric, which, however, depends on the vector it is acting on. The motion of light rays and point particles is to be described by the action principle $0 = \delta \int ds^2$.

There are two main consequences of such a Finslerian framework. (i) Since the Christoffel connection depends on the 4-velocity, it cannot be transformed away, so the equation of motion will not reduce to $\ddot{x}^\mu = 0$ for all possible particle 4-velocities. Therefore, gravity cannot be transformed away in the whole tangent space as it can be in GR. (ii) There is no coordinate transformation by which the Finslerian metric could acquire a Minkowskian form. Therefore, a Finslerian metric violates Lorentz invariance.

A very simple example of a Finslerian metric is given by

$$ds^2 = F(dx^\mu) = dt^2 - D(dx^i), \qquad D(\lambda dx^i) = \lambda^2 D(dx^i), \tag{45}$$

with

$$(D(dx^i))^r = D_{i_1 \ldots i_{2r}} dx^{i_1} \cdots dx^{i_{2r}} = (\delta_{ij} dx^i dx^j)^r + \phi_{i_1 \ldots i_{2r}} dx^{i_1} \cdots dx^{i_{2r}}, \tag{46}$$

where $i, j, \ldots = 1, 2, 3$. The anisotropy is encoded in the tensor field $\phi_{i_1 \ldots i_{2r}}$, which, by comparison with many experiments, can be assumed to be very small: $\phi_{i_1 \ldots i_{2r}} \ll 1$.

7.7.2 Testing Finslerian Anisotropy in Tangent Space

In [96] this ansatz was used for describing tests of Finslerian models, in the photon sector given by $ds^2 = 0$, using Michelson–Morley experiments. From a comparison with the best available optical data, see page 29 in Section 3.1.1, one deduces that $\phi_{i_1 \ldots i_{2r}} \leq 10^{-16}$.

In the matter sector, within the nonrelativistic realm, one may start with a Hamiltonian of the form

$$H = H(p) \qquad \text{with} \qquad H(\lambda p) = \lambda^2 H(p), \tag{47}$$

where $p_i = -i\hbar\partial_i$. For a "power-law" ansatz we have

$$H = \frac{1}{2m} \left(g^{i_1 \ldots i_{2r}} p_{i_1} \cdots p_{i_{2r}} \right)^{\frac{1}{r}} . \tag{48}$$

The deviation from the standard case may again be parametrized as

$$H = \frac{1}{2m} \left(\Delta^p + \phi^{i_1 \ldots i_{2r}} p_{i_1} \cdots \partial_{i_{2r}} \right)^{\frac{1}{r}} \approx \frac{1}{2m} p^2 \left(1 + \frac{1}{r} \frac{\phi^{i_1 \ldots i_{2r}} p_{i_1} \cdots p_{i_{2p}}}{p^{2r}} \right) . \tag{49}$$

The second term is a nonlocal operator that has influence on, for example,

- The degeneracy of Zeeman levels given by $H_{\text{tot}} = H + \sigma \cdot B$. If H_0 deviates from p^2 then the Zeeman levels split, as can be explored in Hughes–Drever type experiments, which lead to estimates $\phi^{i_1 \ldots i_{2r}} \leq 10^{-30}$, see Section 3.1.2.
- On the phase shift in atomic interferometry. The atom–photon interaction leads to a phase shift

$$\delta\phi \sim H(p+k) - H(p) \approx \frac{k^2}{2m} + \frac{1}{m} \left(\delta^{il} + \frac{1}{r} \frac{\phi^{ili_3 \ldots i_{2r}} p_{i_3} \cdots p_{i_{2r}}}{p^{2(r-1)}} \right) p_i k_l , \tag{50}$$

  where we have used $k \ll p$. This is a modified Doppler term: while rotating the whole apparatus we get different Doppler terms.

7.7.3 Finslerian Geodesic Equation

In Finslerian space–time gravity cannot, in general, be transformed away. In [99] we discuss a Finslerian model of gravity by appropriately modifying the ansatz (45) for a Finslerian metric function

$$ds^2 = h_{00} dt^2 - \left((h_{i_1 i_2} \cdots h_{i_{2r-1} i_{2r}} + \phi_{i_1 \ldots i_{2r}}) dx^{i_1} \cdots dx^{i_{2r}} \right)^{\frac{1}{r}} , \tag{51}$$

which reduces to a Riemannian space–time for $\phi_{i_1 \ldots i_{2r}} = 0$. For the case of a spherically symmetric Finsler space–time, it is possible to calculate the geodesic equation to first order in the Finslerian deviation $\phi_{i_1 \ldots i_{2r}}$. We assumed for $h_{\mu\nu}$ the Schwarzschild form and found, for circular orbits, a modified Kepler law

$$\frac{r^3}{T^2} = \left(1 - \frac{A(r)}{r^4}\right) \frac{GM}{4\pi^2}, \tag{52}$$

where $A(r)$ is an arbitrary function, related to one component of the spherically symmetric tensor $\phi_{i_1 \ldots i_{2r}}$.

For a radial free fall we obtain

$$\frac{d^2 r}{d\tau^2} = -\left(1 - B(r)\left(1 - \frac{2GM}{r}\right)^2\right) \frac{GM}{r^2}, \tag{53}$$

where τ is the proper time and $B(r)$ another function related to another component of the spherically symmetric tensor $\phi_{i_1 \ldots i_{2r}}$. In the Newtonian approximation this gives

$$\frac{d^2 r}{dt^2} = -\left(1 - B(r)\right) \frac{GM}{r^2}. \tag{54}$$

Comparison of (52) with (54) reveals that radial motion and circular motion "feel" different gravitational constants, which, in general, may depend on the radial distance [99],

$$\frac{r^3}{T^2} = \frac{G_1 M}{4\pi^2}, \qquad \frac{d^2 r}{dt^2} = -\frac{G_2 M}{r^2}. \tag{55}$$

The geodesic equation in Finsler space–time thus implies that the gravitational attraction of a body falling vertically towards the center of the Earth is different from the gravitational attraction that keeps a satellite on its bound orbit, see Fig. 6. From the orbit of the Earth around the Sun one can determine GM of the Sun with a relative accuracy of approximately 10^{-9}. This mass can be taken to determine the gravitational field of the Sun and the acceleration that bodies experience within standard theory. The acceleration of a satellite on a radial escape orbit can be measured with an accuracy of the order 10^{-10} m/s^2, which would allow a determination of GM of the Sun with an accuracy of the order 10^{-8} (at a distance of approximately 1 AU). As for the Earth, the gravitational acceleration of a body falling on Earth can be measured with an accuracy of 10^{-8} m/s^2 [91] leading to a relative accuracy of the determination of GM of the Earth of the order 10^{-9}. So, if all observations and measurements are compatible within standard theory, then the equality of the acceleration of horizontally moving satellites and planets and vertically falling

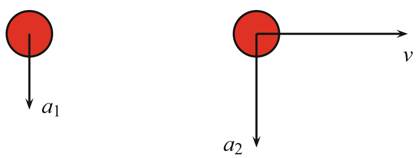

Fig. 6 A body falling toward the center of the Earth may feel a gravitation acceleration toward the center of the Earth different from that of a body moving horizontally

bodies is confirmed to within the order of 10^{-8}. As a consequence, the functions G_1 and G_2, or A/r^4 and B, should differ by less than 10^{-8}.

It is clear from the given formulae that Finsler geometry offers the possibility of having different properties for escape and bound orbits (the gravitational attraction depends on the orbit) and, thus, is in the position to describe effects like the Pioneer anomaly; for example, a very simple choice in this case might be $A = 0$ and $B = B_0 r^2$ (assuming that the observed anomalous acceleration is of gravitational origin and not a systematic error). Further studies on experimental and observational consequences of Finsler gravity are in progress [99].

8 Summary

In this chapter, we have described the underlying principles of GR encoded in the EEP, and their corresponding experimental verification. We have also described observations relating to the predictions of GR, ranging from the weak field Solar system to strong field effects in compact binary systems. Besides the standard principles, we also focussed some attention on assumptions that are usually taken for granted, even though their experimental basis is sometimes not strong, or the interpretation of related experiments is not unique. These assumptions include charge conservation, equality of active and passive mass, charge, and magnetic moment, the order of the time derivative in classical and quantum equations of motion, and the issue of whether gravity can be transformed away locally.

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