Periodic and Chaotic Motions in a Gear-pair Transmission System with Impacts

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Abstract Nonlinear dynamical behaviors of a gear transmission system with impacts are investigated. The transmission system is described through an impact model with possible stick between the two gears. Based on the mapping structures, periodic motions of such a system are predicted analytically. To understand the global dynamical behaviors of the gear transmission system, system parameter maps are developed. Numerical simulations for periodic and chaotic motions are performed from the parameter maps.

Keywords Gear-pair transmission systems · Impact chatter · Stick motion

1 Introduction

Gear transmission systems are extensively used in mechanical engineering and an efficient gear transmission is necessary to save energy in mechanical transmission as discussed in Changenet et al. [1]. From the current principles and theories, impacting chatter is a source to dissipate energy, and the released energy will cause vibration and noise in the system. On the other hand, the reduction of vibration and noise in transmission systems will enhance the corresponding transmission efficiency.

The early investigations of gear transmission systems focused on the mesh geometries, kinematics and strength of teeth as in Buckingham [2, 3]. For low-speed gear systems, the linear model was developed, which gave a reasonable prediction of gear-tooth vibrations. With increasing rotation speed in gear transmission systems, vibrations and noise become serious. Hartog and Mikina [4] used a piecewise linear system without damping to model gear transmission systems, and the symmetric periodic motion in such a system was investigated. Ozguven and Houser [5] gave

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a survey on the mathematical models of gear transmission systems. The piecewise linear model and the impact model were the two main mechanical models to investigate the origin of vibration and noise. In 1984, Pfeiffer [6] presented an impact model of gear transmissions, and the theoretical and experimental investigations on regular and chaotic motions in the gear box were later carried out in Karagiannis and Pfeiffer [7]. One also used a piecewise linear model to investigate the dynamics of gear transmission systems (e.g., Theodossiades and Natsiavas [8]).

To model vibrations in gear transmission systems, Luo and Chen [9] gave an analytical prediction of the simplest, periodic motion through a piecewise linear, impacting system. In addition, the local singularity theory in Luo [10] was used to obtain the corresponding grazing of periodic motions, and chaotic motions were simulated numerically through such a piecewise linear system. the grazing mechanism of the strange fragmentation of such a piecewise linear system was discussed in Luo and Chen [11]. Luo and Chen [12] used the mapping structure technique to analytically predict arbitrary periodic motions of such a piecewise linear system. In this piecewise linear model, it was assumed that impact locations were fixed, and the perfectly plastic impact was considered. Separation of the two gears occurred at the same location as the gear impact. Compared with the existing models, this model can give a better prediction of periodic motions in gear transmission systems, but the related assumptions may not be realistic to practical transmission systems. In this paper, the two gears will be considered to be independent, and impacts between the two gears occur at different locations. This gear transmission system with impact will be modeled by a piecewise linear system with impacts. Luo and O'Connor [13, 14] discussed the mechanism of impacting chatter with stick, and analytical prediction of periodic chatter with/without stick. In this paper, the global nonlinear behaviors of such a gear transmission system will be discussed and parameter maps will be developed. Numerical illustrations will be presented for parameter characteristics of impacting chatter with/without stick.

2 Equations of Motion

To model the gear transmission system, consider a periodically forced oscillator confined between the teeth of a second oscillator, as shown in Fig. 1. Interaction between the two gears causes impacting and sticking together. Since the gears are supported by shafts, each gear m_i $(i = 1, 2)$ is connected to a spring and a damper. The spring stiffness k_i is from the twisting shafts of a gear transmission system, and the damper damping r_i is from lubricating fluids. The free-flying gap between two teeth of the driven gear is d. The external force $B_0 + A_0 \cos \Omega t$ acts on the driving gear m_1 where A_0 and Ω are the amplitude and frequency of the oscillation torque, respectively. B_0 is from the constant torque. The displacements of each mass measured from their equilibriums are expressed by $x^{(1)}$ and $x^{(2)}$. Impacts between two gears are described through the impact law with restitution coefficient e. The equilibrium of the first gear is set at the center of the two teeth of the second gear at

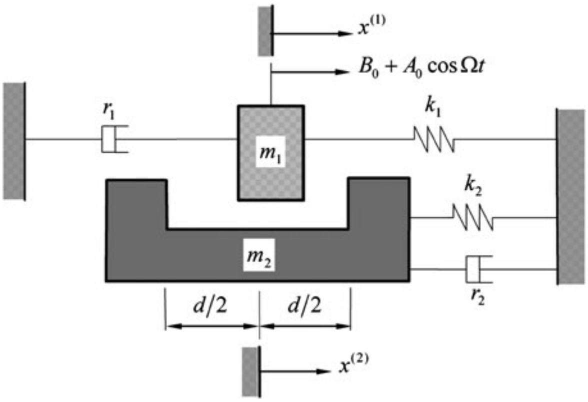

Fig. 1 A mechanical model for a gear transmission

equilibrium. Without any interaction between two gear oscillators, the equations of motion are for $i = 1, 2$

$$\ddot{x}_2^{(i)} + 2\zeta_2^{(i)} \dot{x}_2^{(i)} + (\omega_2^{(i)})^2 x_2^{(i)} = b_2^{(i)} + Q_2^{(i)} \cos \Omega t \tag{1}$$

where

$$\left.\begin{array}{llll} \zeta_2^{(i)} = \dfrac{r_i}{2m_i}, & \omega_2^{(i)} = \sqrt{\dfrac{k_i}{m_i}} & (i = 1, 2); & \\ b_2^{(1)} = \dfrac{B_0}{m_1}, & Q_2^{(1)} = \dfrac{A_0}{m_1}, & b_2^{(2)} = 0, & Q_2^{(2)} = 0 \end{array}\right\} \tag{2}$$

for the mechanical model in Fig. 1. Once $|x_2^{(i)} - x_2^{(\bar{i})}| = \frac{d}{2} (i, \bar{i} \in \{1,2\}$ and $i \neq \bar{i})$, the impact between the two gears occurs. From momentum conservation and the simple impact law, two velocities of the two gears after impacting are given by

$$\dot{x}_2^{(i)+} = I_1^{(i)} \dot{x}_2^{(i)-} + I_2^{(i)} \dot{x}_2^{(\bar{i})-} \tag{3}$$

where the superscripts "−" and "+" represent before and after impact, and the corresponding coefficients are

$$\left.\begin{array}{ll} I_1^{(1)} = \dfrac{m_1 - m_2 e}{m_1 + m_2}, & I_2^{(1)} = \dfrac{(1+e)\, m_2}{m_1 + m_2}, \\ I_1^{(2)} = \dfrac{(1+e)\, m_1}{m_1 + m_2}, & I_1^{(2)} = \dfrac{m_2 - m_1 e}{m_1 + m_2}. \end{array}\right\} \tag{4}$$

Once two gear oscillators stick together, equations of motion are for $i = 1, 2$ and $\alpha = 1, 3$

$$\ddot{x}_\alpha^{(i)} + 2\zeta_\alpha^{(i)} \dot{x}_\alpha^{(i)} + (\omega_\alpha^{(i)})^2 x_\alpha^{(i)} = b_\alpha^{(i)} + Q_\alpha^{(i)} \cos \Omega t \tag{5}$$

where

$$\left.\begin{aligned} &\zeta_\alpha^{(i)} = \frac{r_1 + r_2}{2(m_1 + m_2)}, \qquad \omega_\alpha^{(i)} = \sqrt{\frac{k_1 + k_2}{m_1 + m_2}}, \qquad Q_\alpha^{(i)} = \frac{A_0}{m_1 + m_2}; \\ &b_\alpha^{(1)} = \frac{B_0}{m_1 + m_2} \pm \frac{k_2 d}{2(m_1 + m_2)}, \qquad b_\alpha^{(2)} = \frac{B_0}{m_1 + m_2} \mp \frac{k_1 d}{2(m_1 + m_2)}. \end{aligned}\right\} \tag{6}$$

From physics points of view, there is a pair of internal forces during the sticking of two gears, and such internal forces are assumed to be positive in the negative direction, expressed by for $\alpha = 1, 3$

$$\left.\begin{aligned} f_\alpha^{(1)} &= -m_1 \ddot{x}_\alpha^{(1)} - r_1 \dot{x}_\alpha^{(1)} - k_1 x_\alpha^{(1)} + B_0 + A_0 \cos \Omega t, \\ f_\alpha^{(2)} &= -m_2 \ddot{x}_\alpha^{(2)} - r_2 \dot{x}_\alpha^{(2)} - k_0 x_\alpha^{(2)}. \end{aligned}\right\} \tag{7}$$

From the Newton's third law, we have

$$f_\alpha^{(1)} = -f_\alpha^{(2)}. \tag{8}$$

Consider the 2nd gear to be a base reference as in Fig. 1. In region $\alpha = 1$, $f_\alpha^{(1)} > 0$ and $f_\alpha^{(2)} < 0$, but in region $\alpha = 3$, $f_\alpha^{(1)} < 0$ and $f_\alpha^{(2)} > 0$. The stick motion vanishing requires

$$f_\alpha^{(i)} = 0 \quad \text{for } i = 1, 2. \tag{9}$$

The stick condition for two gear oscillators is given for $i = 1, 2$ and $\alpha = 1, 3$

$$f_\alpha^{(i)} \operatorname{sgn}(x_\alpha^{(i)} - x_\alpha^{(\bar{i})}) > 0. \tag{10}$$

Further, the condition for stick vanishing is given by

$$f_\alpha^{(i)} \operatorname{sgn}(x_\alpha^{(i)} - x_\alpha^{(\bar{i})}) = 0. \tag{11}$$

In region $\alpha = 2$, two gear oscillators do not interfere each other. So $f_2^{(i)} = 0$ holds always.

3 Switching Sets and Mappings

As a result of the two gears impacting, the phase plane for each gear is discontinuous. The phase plane domains and boundaries were mathematically defined in Luo and O'Connor [13]. Based on the connectable domain, the mapping structures were introduced to describe possible motions. For the gear transmission system, equations of motion in the absolute frame are from Luo and O'Connor [14]

$$\dot{\mathbf{x}}_\alpha^{(i)} = \mathbf{F}_\alpha^{(i)}(\mathbf{x}_\alpha^{(i)}, t) \tag{12}$$

for $i = 1, 2$ and $\alpha = 1, 2, 3$ with the following vectors

$$\begin{aligned}\mathbf{x}_\alpha^{(i)} &= (x_\alpha^{(i)}, \dot{x}_\alpha^{(i)})^T = (x_\alpha^{(i)}, y_\alpha^{(i)})^T,\\ \mathbf{F}_\alpha^{(i)} &= (\dot{x}_\alpha^{(i)}, F_\alpha^{(i)})^T = (y_\alpha^{(i)}, F_\alpha^{(i)})^T;\end{aligned} \tag{13}$$

where

$$F_\alpha^{(i)} = -2\zeta_\alpha^{(i)}\dot{x}_\alpha^{(i)} - (\omega_\alpha^{(i)})^2 x_\alpha^{(i)} + b_\alpha^{(i)} + Q_\alpha^{(i)}\cos\Omega t, \tag{14}$$

and the superscript "i" represents the ith mass and the subscript "α" represents the α-domain. From discontinuous boundaries in [14], the switching planes based on the two impacting chatter boundaries are defined as

$$\left.\begin{aligned}{}^R\Sigma_{2\infty}^{(i)} &= \left\{(t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \middle| x_k^{(\bar{i})} = x_k^{(i)} - \frac{d}{2}, \dot{x}_k^{(i)} \neq \dot{x}_k^{(\bar{i})}\right\},\\ {}^L\Sigma_{2\infty}^{(i)} &= \left\{(t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \middle| x_k^{(\bar{i})} = x_k^{(i)} + \frac{d}{2}, \dot{x}_k^{(i)} \neq \dot{x}_k^{(\bar{i})}\right\}.\end{aligned}\right\} \tag{15}$$

From now on, $x_k^{(i)} \equiv x^{(i)}(t_k)$ and $\dot{x}_k^{(i)} \equiv \dot{x}^{(i)}(t_k)$ on the separation boundary at time t_k. are switching displacement and velocity. The switching phase is defined by $\varphi_k = \operatorname{mod}(\Omega t_k, 2\pi)$. Based on the above definitions of switching planes, four mappings are defined in the absolute frame as

$$\left.\begin{aligned}&P_2 : {}^R\Sigma_{2\infty}^{(i)} \to {}^R\Sigma_{2\infty}^{(i)}, \qquad P_3 : {}^R\Sigma_{2\infty}^{(i)} \to {}^L\Sigma_{2\infty}^{(i)};\\ &P_5 : {}^L\Sigma_{2\infty}^{(i)} \to {}^L\Sigma_{2\infty}^{(i)}, \qquad P_6 : {}^L\Sigma_{2\infty}^{(i)} \to {}^R\Sigma_{2\infty}^{(i)}.\end{aligned}\right\} \tag{16}$$

To investigate stick motions in the gear transmission system, the switching planes for stick are defined as

$$\left.\begin{aligned}\Sigma_{12}^{(i)} &= \left\{(t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \middle| x_k^{(i)} = {}^R x_{2-}^{(\bar{i})} + \frac{d}{2}, \dot{x}_k^{(i)} = {}^R\dot{x}_{2-}^{(\bar{i})}\right\},\\ \Sigma_{21}^{(i)} &= \left\{(t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \middle| x_k^{(i)} = {}^R x_{2+}^{(\bar{i})} + \frac{d}{2}, \dot{x}_k^{(i)} = {}^R\dot{x}_{2+}^{(\bar{i})}\right\};\\ \Sigma_{23}^{(i)} &= \left\{(t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \middle| x_k^{(i)} = {}^L x_{2-}^{(\bar{i})} - \frac{d}{2}, \dot{x}_k^{(i)} = {}^L\dot{x}_{2-}^{(\bar{i})}\right\},\\ \Sigma_{32}^{(i)} &= \left\{(t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \middle| x_k^{(i)} = {}^L x_{2+}^{(\bar{i})} - \frac{d}{2}, \dot{x}_k^{(i)} = {}^L\dot{x}_{2+}^{(\bar{i})}\right\}.\end{aligned}\right\} \tag{17}$$

The two switching planes can be treated as the same for all mappings. Except for two stick mappings (i.e., P_1 and P_4). the other mappings are the same as in (16). From the stick switching planes, the mappings are defined as

$$\left.\begin{aligned}&P_1 : \Sigma_{21}^{(i)} \to \Sigma_{12}^{(i)}, \qquad P_2 : \Sigma_{12}^{(i)} \to \Sigma_{21}^{(i)}, \qquad P_3 : \Sigma_{12}^{(i)} \to \Sigma_{23}^{(i)};\\ &P_4 : \Sigma_{23}^{(i)} \to \Sigma_{32}^{(i)}, \qquad P_5 : \Sigma_{23}^{(i)} \to \Sigma_{32}^{(i)}, \qquad P_6 : \Sigma_{32}^{(i)} \to \Sigma_{21}^{(i)}.\end{aligned}\right\} \tag{18}$$

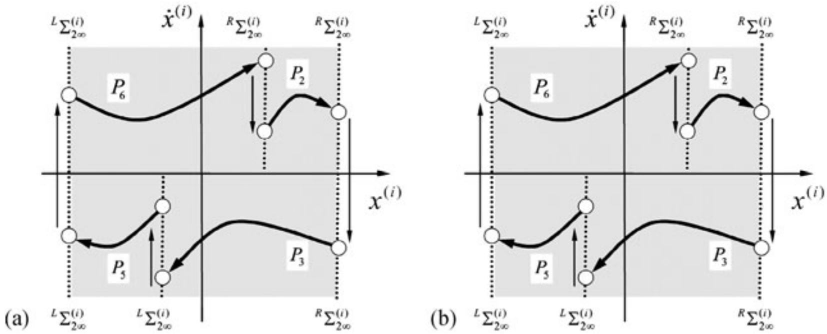

Fig. 2 Basic mappings: (**a**) impacting chatter only and (**b**) with stick switching. The straight line with arrow represents an impact on the boundary

With mixed switching planes, four mappings are defined by

$$\left.\begin{aligned} &P_2 : \Sigma_{12}^{(i)} \to {}^R\Sigma_{2\infty}^{(i)}, \qquad P_2 : {}^R\Sigma_{2\infty}^{(i)} \to \Sigma_{21}^{(i)}; \\ &P_3 : \Sigma_{12}^{(i)} \to {}^L\Sigma_{2\infty}^{(i)}, \qquad P_3 : {}^R\Sigma_{2\infty}^{(i)} \to \Sigma_{23}^{(i)}; \end{aligned}\right\} \tag{19a}$$

$$\left.\begin{aligned} &P_5 : \Sigma_{23}^{(i)} \to {}^L\Sigma_{2\infty}^{(i)}, \qquad P_5 : {}^L\Sigma_{2\infty}^{(i)} \to \Sigma_{32}^{(i)}, \\ &P_6 : \Sigma_{32}^{(i)} \to {}^R\Sigma_{2\infty}^{(i)}, \qquad P_6 : {}^L\Sigma_{2\infty}^{(i)} \to \Sigma_{21}^{(i)}. \end{aligned}\right\} \tag{19b}$$

Among four basic mappings, the two mappings (P_2 and P_5) are local and the other two mappings (P_3 and P_6) are global. The local mapping will map the motion from a switching plane onto itself. However, the global mapping will map the motion from a switching plane to another one. Such mappings are sketched in Fig. 2(a). The corresponding switching planes are labeled. On the impacting chatter boundaries, impacts are expressed by thin straight lines with arrows. The mappings relative to the stick switching planes only are sketched in Fig. 2(b). Only two stick mappings (P_1 and P_2) are new, and the other four mappings are the same as in Fig. 2(a). The mappings based on the sticking and impacting switching planes are presented in Fig. 3(a) and (b).

Set a vector as

$$\mathbf{y}_k \equiv (t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})})^T. \tag{20}$$

For the impacting maps $P_\sigma\,(\sigma = 1, 2, \ldots, 6)$, $\mathbf{y}_{k+1} = P_\sigma \mathbf{y}_k$ can be expressed by

$$P_\sigma : (t_k, x_k^{(i)}, \dot{x}_k^{(i)}, \dot{x}_k^{(\bar{i})}) \to (t_{k+1}, x_{k+1}^{(i)}, \dot{x}_{k+1}^{(i)}, \dot{x}_{k+1}^{(\bar{i})}). \tag{21}$$

From Appendix in Luo and O'Connor [13, 14], the absolute displacement and velocity for two gear oscillators can be obtained with initial conditions $(t_k, x_k^{(i)}, \dot{x}_k^{(i)})$ and $(t_k, x_k^{(\bar{i})}, \dot{x}_k^{(\bar{i})})$. The final state for time t_{k+1} can be given. The switching planes give $x_\gamma^{(\bar{i})} = x_\gamma^{(i)} \pm \frac{d}{2}$ $(\gamma = k, k+1)$.

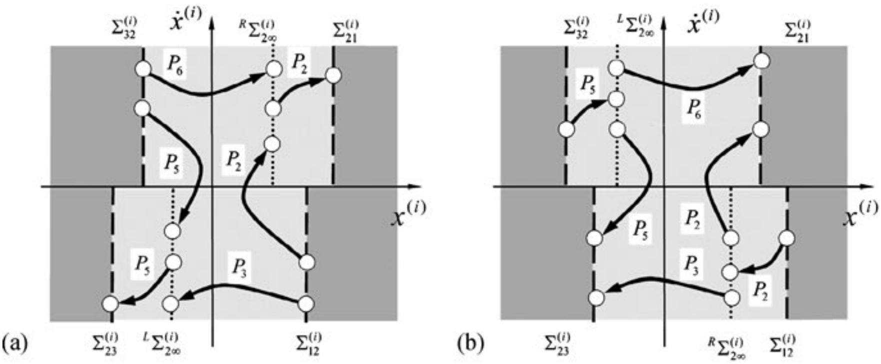

Fig. 3 Mappings between switching planes for stick and impacting

4 Mapping Structures

To describe motions in such a gear transmission system, the notation for mapping actions of basic mappings is introduced as in Luo [10, 15]

$$P_{n_k \cdots n_2 n_1} \equiv P_{n_k} \circ \cdots \circ P_{n_2} \circ P_{n_1} \tag{22}$$

where the mapping P_{n_j} $(n_j \in \{1, 2, \ldots, 6\}, j = 1, 2, \ldots, k)$ is defined in the previous section. Consider a generalized mapping structure as

$$\begin{aligned} &\underbrace{P_{(65^{k_{s4}}4^{k_{s3}}31^{k_{s2}}2^{k_{s1}})\cdots(65^{k_{14}}4^{k_{13}}31^{k_{12}}2^{k_{11}})}}_{s\text{-terms}} \\ &\quad = \underbrace{P_{(65^{k_{s4}}4^{k_{s3}}31^{k_{s2}}2^{k_{s1}})} \circ \cdots \circ P_{(65^{k_{14}}4^{k_{13}}31^{k_{12}}2^{k_{11}})}}_{s\text{-terms}} \end{aligned} \tag{23}$$

where $(k_{\mu\nu} \in \{0, \mathbb{N}\}, \mu = 1, 2, \ldots, s, \nu = 1, 2, 3, 4)$. From the generalized mapping structure, consider a simple mapping structure of periodic motions for impacting chatter. For instance, the mapping structure is

$$P_{65^n 32^m} = P_6 \circ P_{5^n} \circ P_3 \circ P_{2^m} \tag{24}$$

where $m, n \in \{0, \mathbb{N}\}$. Such a mapping structure gives $(m + 1)$-impacts on the right boundary and $(n + 1)$-impacts on the left boundary, which are described by mappings P_2 and P_5, respectively. Through the global mappings P_3 and P_6, the impacting chatters on the two boundaries are connected together. Consider a periodic motion of $P_{65^n 32^m}$ with period $T_1 = k_1 T$ $(k_1 \in \mathbb{N})$. If the mapping structure copies itself, a new mapping structure is:

$$P_{(65^n 32^m)^{2^l}} = P_{(65^n 32^m)^{2^{l-1}}} \circ P_{(65^n 32^m)^{2^{l-1}}}. \tag{25}$$

As $l \to \infty$, a chaotic motion relative to mapping structure $P_{65^n32^m}$ is formed. The prescribed chaos is generated by period-doubling. However, if the grazing bifurcation occurs, such a mapping structure may not be copied by itself. The new mapping structures are combined by the two different mapping structures. For instance,

$$\begin{aligned} P_{65^{n_2}32^{m_2}65^{n_1}32^{m_1}} &= P_{65^{n_2}32^{m_2}} \circ P_{65^{n_1}32^{m_1}}, \\ &\vdots \\ P_{65^{n_l}32^{m_l}\ldots 65^{n_1}32^{m_1}} &= \underbrace{P_{65^{n_l}32^{m_l}} \circ \cdots \circ P_{65^{n_1}32^{m_1}}}_{l\text{-terms}}. \end{aligned} \tag{26}$$

Such a gazing bifurcation will cause the discontinuity of periodic motions, and chaotic motions may exist between periodic motions of $P_{65^{n_l}32^{m_l}\ldots 65^{n_1}32^{m_1}}$ and $P_{65^{n_{l-1}}32^{m_{l-1}}\ldots 65^{n_1}32^{m_1}}$.

For low excitation frequency, the impacting chatter accompanying stick motion exists in the gear transmission system. Consider a simple chatter with stick motion with the following mapping structure

$$P_{645^n312^m} = P_6 \circ P_4 \circ P_{5^n} \circ P_3 \circ P_1 \circ P_{2^m}. \tag{27}$$

From the above mapping structure, m-impacts on the right boundary and n-impacts on the left boundary, which are described by mappings P_2 and P_5, respectively. In addition, both the mth mapping of P_2 and the nth mapping of P_5 map the impacting boundary to the stick boundary, and the stick mappings are P_1 and P_4, respectively. The two global mappings P_3 and P_6 connect the impact and stick boundaries. Similarly, a mapping structure for period-doubling is

$$P_{(645^n312^m)^{2^l}} = P_{(645^n312^m)^{2^{l-1}}} \circ P_{(645^n312^m)^{2^{l-1}}}. \tag{28}$$

Due to grazing bifurcation, the mapping structures are:

$$\begin{aligned} P_{645^{n_2}32^{m_2}65^{n_1}312^{m_1}} &= P_{645^{n_2}312^{m_2}} \circ P_{645^{n_1}312^{m_1}}, \\ &\vdots \\ P_{645^{n_l}312^{m_l}\ldots 645^{n_1}312^{m_1}} &= \underbrace{P_{645^{n_l}312^{m_l}} \circ \cdots \circ P_{645^{n_1}312^{m_1}}}_{l\text{-terms}}. \end{aligned} \tag{29}$$

To help one understand two sorts of mapping structures, the two simple mapping structures are shown in Fig. 4(a) and (b) for the impacting chatter with and without stick of two gear systems. Similarly, the other mapping structures can be discussed through the generalized mapping structure in (25). Periodic and chaotic motions relative to a certain mapping structure can be determined.

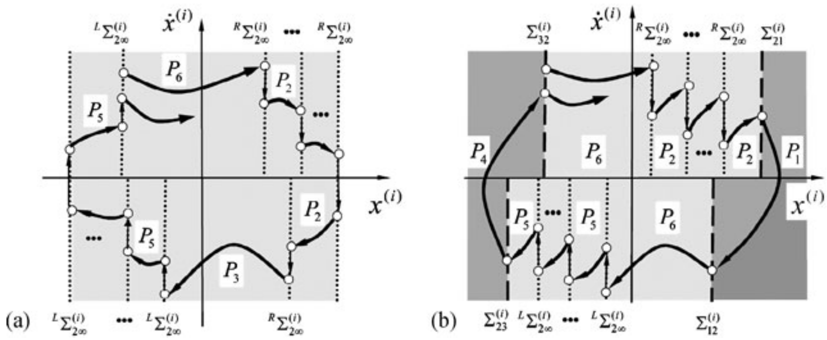

Fig. 4 Mapping structures for (**a**) impacting chatter and (**b**) impacting chatter with stick motion of two gear systems

5 Parameter Maps and Illustrations

The parameter map for excitation frequency Ω versus restitution e are shown in Fig. 5 for parameters ($m_1 = 2, m_2 = 1, r_1 = r_2 = 0.6, k_1 = 30, k_2 = 20,\ Q_0 = 50.0$ and $d = 1.0$. In Fig. 5(a), the entire range of excitation frequency for two masses experiencing interaction is presented. The zoomed view of the parameter map is given in Fig. 5(b) for $\Omega \in [0, 8]$. The chatter with stick possesses a mapping structure of $P_{645^n312^n}$ for $n = 1, 2, \ldots, 70$. The number of impacting chatters increases with increasing e. The region labeled by "Chatter" represents chatter with stick where the chatter impacts number approaches infinity as $e \to 1$. The region just above the region for the chatters with stick has complex mapping structure. Within the "complex motion" region, chaotic and periodic motions of impacting chatter without stick exist, and the corresponding mapping structures are relative to $P_{652^232^2}$ and $P_{653^332^3}$. In additions, the regions relative to periodic motions of $P_{652^232^2}$ and $P_{653^332^3}$ are labeled. With increasing excitation frequency, symmetric and asymmetric periodic motions with the mapping structure of P_{63} are presented. The larger region is symmetric while the smaller region is asymmetric. For higher excitation frequency, the two

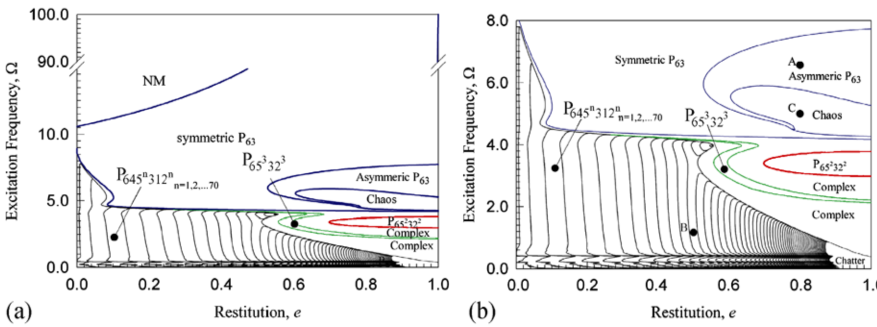

Fig. 5 Parameter map for excitation frequency versus restitution ($m_1 = 2, m_2 = 1, r_1 = r_2 = 0.6$, $k_1 = 30, k_2 = 20,\ Q_0 = 50.0$ and $d = 1.0$)

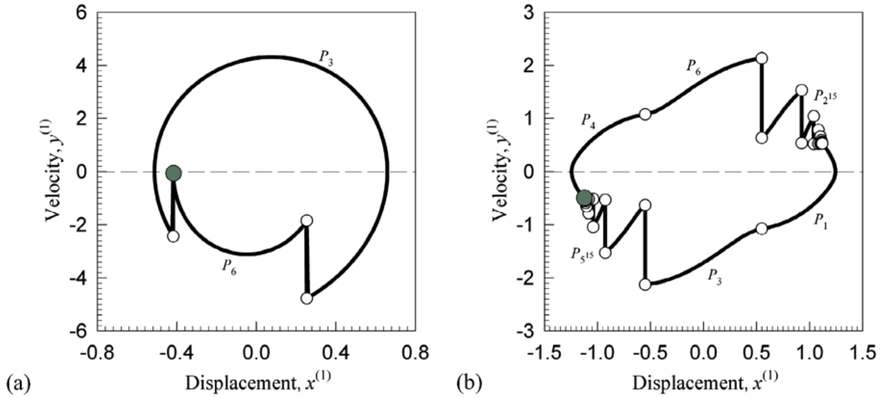

Fig. 6 Phase planes: (**a**) asymmetric impacts $P_{63}(\Omega = 6.6$ and $e = 0.8$; $t_0 \approx 0.8852$, $x_0^{(1)} \approx -0.4179$, $y_0^{(1)} \approx -0.0770$ and $x_0^{(2)} \approx -0.9179$, $y_0^{(2)} \approx -4.0045)$ and (**b**) impact chatter with stick $P_{5^{15}312^{15}64}$ ($\Omega = 1.0$ and $e = 0.5$; $t_0 \approx 2.6000$, $x_0^{(1)} \approx -1.1201$, $y_0^{(1)} \approx -0.5273$ and $x_0^{(2)} \approx -0.6201$, $y_0^{(2)} \approx -0.5271$)

masses will not contact each other, and such a region is labeled by "NM". It means the two gears do not transfer any energy.

To demonstrate motions with specific mapping structures in the parameter map, three sets of excitation frequency and restitution are used, and they are labeled through points A, B and C in Fig. 5(b). At the point "A" , $\Omega = 6.6$ and $e = 0.8$ are selected. For this point, the initial conditions are $t_0 \approx 0.8852$, $x_0^{(1)} \approx -0.4179$, $y_0^{(1)} \approx -0.0770$ and $x_0^{(2)} \approx -0.9179$, $y_0^{(2)} \approx -4.0045$. The corresponding phase plane is plotted in Fig. 6(a). The motion starts with just after the driving gear impacts at the right hand side of the driven gear. The next impact takes place at the left hand side of the driven gear and then returns back to the right side again. The asymmetric motion is relative to mapping P_6 and P_3, its twin asymmetric motion will not presented and the detailed discussion can referred to Luo [15]. For parameters (i.e., $\Omega = 1.0$ and $e = 0.5$) labeled "B" in Fig. 5(b), the periodic motion of impacting chatter with stick $P_{5^{15}312^{15}64}$ is plotted in Fig. 6(b) with initial conditions ($t_0 \approx 2.6000$, $x_0^{(1)} \approx -1.1201$, $y_0^{(1)} \approx -0.5273$ and $x_0^{(2)} \approx -0.6201$, $y_0^{(2)} \approx -0.5271$). The driving gear begins at the onset of stick motion relative to P_4 on the left hand side of the driven gear. Crossing the tooth gap from the left to right side of the driven gear is the mapping of P_6. The two gears impact fifteen times (i.e., $P_{2^{15}}$) before a new stick motion is formed on the right side, and the stick motion is described through the mapping of P_1. The second half of the periodic motion can be described in a similar fashion. Finally, the chaotic motion is demonstrated through Poincaré mapping sections at point C (i.e., $\Omega = 5.0$ and $e = 0.8$). The initial conditions are $t_0 \approx 0.0641$, $x_0^{(1)} \approx -1.5161$, $y_0^{(1)} \approx 6.0031$ and $x_0^{(2)} \approx -2.0161$, $y_0^{(2)} \approx 3.5209$. The switching points are plotted in Fig. 7 for ten thousand periods ($10^4 T$) of the excitation forcing. The Poincaré mapping sections of switching points for the 1st and 2nd masses are given in Fig. 7(a) and (b), respectively. The switching points describe the posi-

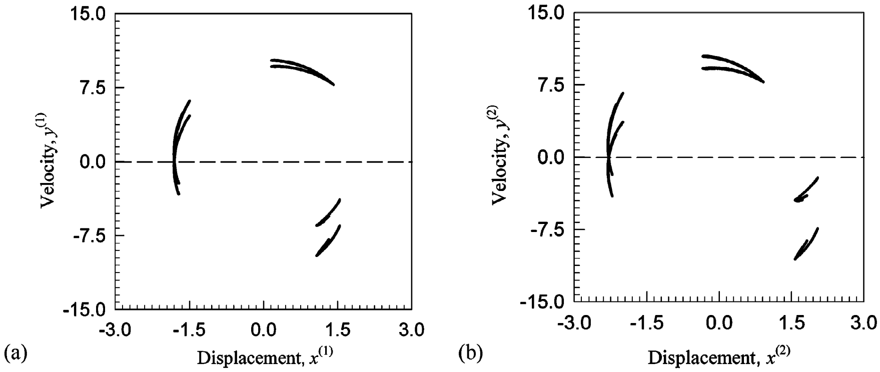

Fig. 7 Poincaré mapping sections for chaos ($\Omega = 5.0$ and $e = 0.8$): (**a**) mass m_1 and (**b**) mass m_2. ($t_0 \approx 0.0641, x_0^{(1)} \approx -1.5161, y_0^{(1)} \approx 6.0031$ and $x_0^{(2)} \approx -2.0161, y_0^{(2)} \approx 3.5209$)

tion and velocity of the driving and driven gears upon impact. The switching points form a strange attractor of chaotic motions for such a gear transmission system. In a similar fashion, the periodic and chaotic motions can be illustrated.

6 Conclusions

Nonlinear dynamical behaviors of a gear transmission system with impacts were investigated through an impact model with possible stick between the two gears. Switching sets and basic mappings were introduced to identify periodic and chaotic motions in such a gear transmission system. To understand the global dynamical behaviors of the gear transmission system, system parameter maps were developed analytically and numerically. Numerical simulations for illustration of periodic and chaotic motions in such a gear transmission system were performed from the parameter maps.

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