Preface

The origin of dual tableaux goes back to the paper by Helena Rasiowa and Roman Sikorski 'On the Gentzen theorem' published in Fundamenta Mathematicae in 1960. The authors presented a cut free deduction system for the classical first-order logic without identity. Since then the deduction systems in the Rasiowa–Sikorski style have been constructed for a great variety of theories, ranging from well established non-classical logics such as intuitionistic, modal, relevant, and multiple-valued logics, to important applied theories such as, among others, temporal, in particular interval temporal logics, various logics of programs, fuzzy logics, logics of rough sets, theories of spatial reasoning including region connection calculus, theories of order of magnitude reasoning, and formal concept analysis.

Specific methodological principles of construction of dual tableaux which make possible such a broad applicability of these systems are:

- First, given a theory, a truth preserving translation is defined of the language of the theory into an appropriate language of relations (most often binary);
- Second, a dual tableau is constructed for this relational language so that it provides a deduction system for the original theory.

This methodology, reflecting the paradigm 'Formulas are Relations', enables us to represent within a uniform formalism the three basic components of formal systems: syntax, semantics, and deduction apparatus. The essential observation, leading to a relational formalization of theories, is that a standard relational structure (i.e., a Boolean algebra together with a monoid) constitutes a common core of a great variety of theories. Exhibiting this common core on all the three levels of syntax, semantics and deduction, enables us to create a general framework for representation, investigation and implementation of theories.

The relational approach enables us to build dual tableaux in a systematic, modular way. First, deduction rules are defined for the common relational core of the theories. These rules constitute a basis of all the relational dual tableau proof systems. Next, for any particular theory specific rules are added to the basic set of rules. They reflect the semantic constraints assumed in the models of the theory. As a consequence, we need not implement each deduction system from scratch, we should only extend the basic system with a module corresponding to the specific part of a theory under consideration.

Relational dual tableaux are powerful tools which perform not only verification of validity (i.e., verification of truth of the statements in all the models of a theory) but often they can also be used for proving entailment (i.e., verification that truth of a finite number of statements implies truth of some other statement), model checking (i.e., verification of truth of a statement in a particular fixed model), and satisfaction (i.e., verification that a statement is satisfied by some fixed objects of a model).

Part I of the book is concerned with the two systems which provide a foundation for all of the dual tableau systems presented in this book. In Chap. 1 we recall the original Rasiowa–Sikorski system and we extend it to the system for first-order logic with identity. We discuss relationships of dual tableaux with other deduction systems, namely, tableau systems, Hilbert-style systems, Gentzen-style systems, and resolution. In Chaps. 2 and 3 classical theories of binary relations and their dual tableaux are presented. It is shown how dual tableaux of these theories perform the above mentioned tasks of verification of validity, entailment, model checking, and verification of satisfaction. Some decidable classes of relational formulas are presented in this part together with dual tableau decision procedures.

Part II is concerned with some non-classical theories of relations. In Chap. 4 we present a theory of Peirce algebras and its dual tableau. Peirce algebras provide a means for representation of interactions between binary relations and sets. In Chap. 5 a theory of fork algebras and its dual tableau are presented. Fork algebras are the algebras of binary relations which, together with all the classical relational operations, have a special operation, referred to as fork of relations. While the relational theories of Chap. 2 serve as means of representation for propositional languages, the fork operation enables us a translation of first-order languages into a language of binary relations. In Chap. 6 we present a theory of typed relations and its dual tableau. The theory enables us to represent relations as they are understood in relational databases. The theory deals with relations of various finite arities and, moreover, each relation has its type which is meant to be a representation of a subset of attributes on which the relation is defined.

In Parts III–V relational formalizations of various theories are presented. In Part III relational dual tableaux are constructed for modal (Chap. 7), intuitionistic (Chap. 8), relevant (Chap. 9), and finitely many-valued (Chap. 10) logics.

Part IV is concerned with the major theories of reasoning with incomplete information. In Chaps. 11 and 12 we deal with logics of rough sets and their relational dual tableaux. Chapter 13 presents a relational treatment of formal concept analysis. In Chap. 14 a monoidal t-norm fuzzy logic is considered and a relational dual tableau for this logic is constructed. In this system ternary relations are needed for representation of the monoid product operation. Next, in Chap. 15 theories of order of magnitude reasoning are considered and their dual tableaux are presented.

Part V is concerned with dual tableaux for temporal reasoning, spatial reasoning, and for logics of programs. The first two chapters of that part refer to temporal logics. In Chap. 16 some classical temporal logics are dealt with and in Chap. 17 relational dual tableaux for a class of interval temporal logics are presented. In Chap. 18 dual tableaux for theories of spatial reasoning are constructed, including a system for the region connection calculus. Chapter 19 includes dual tableaux for various versions of propositional dynamic logic and for an event structure logic.

In Part VI we consider some theories for which dual tableau systems are constructed directly within the theory, without translation into any relational theory. In Chap. 20 we present a class of threshold logics where both weights of formulas and thresholds are elements of a commutative group. In Chap. 21 we present a construction of a signed dual tableau which is a decision procedure for a well known intermediate logic. Chapter 22 includes dual tableaux for a class of first-order Post logics. The reduct of this dual tableau for the propositional part of the logic is a decision procedure. Chapter 23 presents a propositional logic endowed with identity treated as a propositional operation and some theories based on this logic. Dual tableaux for all of these theories are presented. In Chap. 24 logics and algebras of conditional decisions are considered together with their dual tableau decision procedures.

The book concludes with Part VII. In the single Chap. 25 of this part we make a synthesis of what we learned in the process of developing dual tableaux in the preceding chapters. We collect observations on how the dual tableaux rules should be designed once the constraints on the models of the theories or definitions of some specific constants are given. We also discuss some useful strategies for construction of dual tableaux proofs.

All the dual tableau systems considered in the book are proved to be sound and complete. We present a general method of proving completeness of dual tableaux which is shown to be broadly applicable to many theories.

Researchers working in any of the theories mentioned in the titles of the chapters will receive in the book a formal tool of specification and verification of those problems in their theories which involve checking validity, satisfaction, or entailment. Every theory whose dual tableau is presented in a chapter of the book is briefly introduced at the beginning of the chapter and a bibliography is indicated where an interested reader could trace developments, major results, and applications of the theory.

To get an idea of what dual tableaux are and how they are related to the other major types of deduction systems, reading Chap. 1 is recommended. After reading the introductory material from Sects. 1.1, …, 1.4, and Sects. 2.1, …, 2.8, each chapter in Parts III, IV, and V may be read independently. The material of Chap. 7 may be helpful in reading Chapters 11, 12, 16, 17, and 19, since they are concerned with modal-style logics.

Readers interested in the formal methods of deduction and their application to specification and verification will find in the book an exhaustive exposition and discussion of dual tableaux and their methodology illustrated with several case studies.

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