

Optimized Life Using Frequency and Time Domain Approaches

J.S. Rao

Abstract Life estimation of mechanical components has been practiced for quite some time. There are two approaches available: (1) use the response in time domain and do a cycle counting and (2) determine the response in frequency domain and use a cumulative damage calculation to cross the resonance. These two methods apparently look different and generally not connected. Here we examine these methods and their applicability in determining life of mechanical components and find their suitability for different applications. We will also discuss their common aspects and state their relative advantages and disadvantages.

Keywords Lifing · Time domain · Cycle counting · Frequency domain · Optimized life

1 Introduction

All mechanical components are subjected to alternating forces and fatigue is essentially due to the resulting alternating stresses; Wöhler [1] conducted earliest tests to define S - N diagrams. The influence of mean stress component is presented in Goodman diagrams [2]. Bagci [3] proposed a fourth order relation which is shown to give good results for ductile materials. S - N diagrams and mean stress diagrams are combined to define fatigue failure surface under elastic conditions, see Rao [4]. The fatigue failure surface defined thus is applicable when the stress field is entirely in the elastic region.

Ludwik [5] and Hollomon [6] defined the stress–strain relation in plastic region, $\sigma = K\varepsilon^n$ where K is the strength coefficient and n is strength exponent. For globally elastic and locally plastic structures, Neuber [7] hypothesis is used for relating the nominal and local cyclic stresses and strains, e.g., [8, 9]. For fatigue loading,

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using fatigue stress concentration factor K_f , local stress and strain ranges are related to nominal stress range

$$\Delta \varepsilon \Delta \sigma = (K_f \Delta S)^2 / E \quad (1)$$

Socie et al. [10] suggest that K_f can be taken to be the theoretical stress concentration factor. Based on Basquin [11] for endurance and Manson [12] and Coffin [13] works, we have a life relation for crack initiation given by

$$\frac{1}{2} \Delta \varepsilon = \frac{1}{E} \sigma_f' (2N_i)^b + \varepsilon_f' (2N_i)^c \quad (2)$$

Palmgren [14] and Miner [15] gave linear cumulative damage law and Marco and Starkey [16] gave nonlinear damage rule based on the stress amplitude. For other nonlinear relations, see Rao et al. [17]. For details on these aspects, reference may be made to Collins [18] and Rao [19].

Lifing of mechanical components from cumulative damage can be broadly classified into two categories.

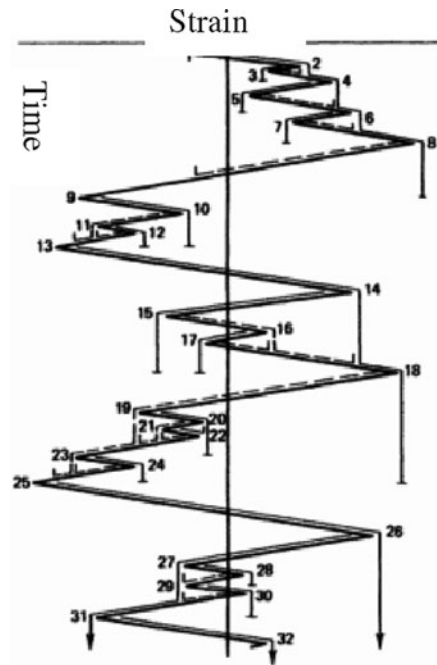
1. Components subjected to relatively low alternating stresses that lead to large life in terms of number of cycles of loading; typically the alternating stresses under these conditions arise out of forced vibration from several sources that produce no resonant conditions; usually cycle counting methods in time domain can be adopted for this purpose. The method of lifing can be termed *Time Domain Method*.
2. Components subjected to resonances lead to large alternating stresses; typically such conditions occur during start up and shut down conditions or a given mission of operation when the component crosses several critical speeds. The alternating stresses can then be high because of resonance and correspondingly the life cycles will also be low. The stress values increase rapidly near resonance in a pure harmonic manner while crossing a critical speed and also fall rapidly to nominal values. Lifing is then done for pure harmonic alternating stresses over a mean stress and this lifing can be termed *Frequency Domain Method*. These two methods are discussed as follows.

2 Time Domain

For complex load, stress or strain time histories a cycle counting method is developed by Dowling [20]. He proposed several cycle counting methods: (1) Peak Count, (2) Peak between Mean Crossing Count, (3) Level Crossing Count, (4) Fatigue Meter Count, (5) Range Count, and (6) Range Mean Count.

Each of the above methods are shown to be deficient in some respect or other by Dowling and two additional methods are proposed, viz., Range Pair Method and Rain Flow Method.

Fig. 1 Range pair cycle counting



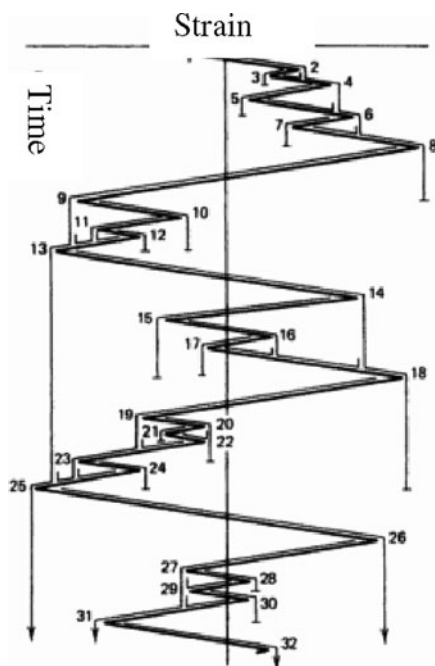
Range Pair Method is illustrated in Fig. 1. A strain range is counted as a cycle if it can be paired with a subsequent straining of equal magnitude in the opposite direction. The counted ranges are marked with solid lines and the paired ranges with dashed lines.

The Rain Flow Cycle Counting Method illustrated in Fig. 2 is more widely used than any other method. The lines connecting strain peaks are imagined to be a series of Pagoda roofs. Rules are imposed on rain dropping down these roofs so that cycles and half cycles are defined.

Rainflow begins successively at the inside of each strain peak and is allowed to drop down and continue except that, if it initiates at a minimum, it must stop when it comes opposite a minimum more negative than the minimum from which it initiated. Let us begin at peak 1 in Fig. 2 and stop opposite peak 9, peak 9 being more negative than peak 1. A half cycle is counted between peaks 1 and 8. Similarly if the rain flow initiates at a maximum, it is stopped when it comes opposite a maximum more positive than the maximum from which it is initiated. If we begin at peak 2 and stop opposite peak 4, we count a half cycle between peaks 2 and 3. Every part of the strain time history is counted once only. We can also build from Rain Flow Cycle counting method the stress strain hysteresis loops and thus account for all cycles of loading accurately.

This type of rain flow cycle counting is adopted in many codes, e.g., Altair HyperWorks Fatigue Process Manager [21].

Fig. 2 Rain flow cycle counting



2.1 Some Observations on Time Domain Method

Complex time domain patterns usually mean that the alternating stresses are not at resonance but caused by several excitation frequencies, as in automotive drive train with excitations arising out of engine harmonics, gear mesh from several gear trains in the transmission unit, road excitations . . . Also in these applications, the natural frequencies are relatively high compared to excitation frequencies from engine speed and harmonics. Traditionally automotive drive train developers resort to laboratory or road tests and the strain signals collected in time domain can be used to make an approximate estimate of life of the structural parts.

In order to obtain correct estimates of life, we need to make first several corrections on the specimen material properties before applying them to the measured time domain signals. They can be size effect, surface finish effect, stress concentration effect, reliability factor . . . The signals can be measured at a suitable location but not on actual stress raiser locations. Complex time domain signals mean long life and predictions with low level stresses become difficult and compare with test values. Nevertheless, the time domain cycle counting methods are employed in determining the life.

Other disadvantages in time domain method for lifing include:

1. Cycle counting method does not recognize the frequency, it is only the number of cycles that matter.

2. Lifting is made by counting cycles irrespective of the order of application of stress whether it is increasing or decreasing with cycles (or time).
3. Since the frequency is not recognized, the life is given in terms of number of cycles of load rather than hours or days.
4. Frequency plays important role in the response of flexible structures yielding dynamically magnified values near resonant conditions; though time domain signals can be processed in frequency domain, this is not the practice as the data collected can be huge and one has no idea whether there is resonance in the response or not.
5. All the cycles whether the stress amplitudes are large or small are to be included in cycle counting; for each cycle, the mean and range are to be calculated and its damage is to be determined.

As a design tool the time domain method has several limitations listed below.

1. Analytical design produces stress response with several harmonics; only some harmonics or modal components with large amplitudes contribute to the damage – time domain analysis does not recognize this.
2. Damping plays significant role and is rarely viscous in nature. Material damping is always nonlinear and depends on strained condition in the structure. In addition bolted or fabricated joints may have friction between interfacial surfaces. Usually transient analysis to determine the response in time domain or conventional steady state analysis can use only linear damping model.
3. Time domain analysis cannot easily identify the deficiencies in the design.

3 Frequency Domain

Frequency domain methods are now gaining wide applications in aerospace and automotive fields amongst others. They are useful both at the design and test stages. They can be broadly classified into two approaches, viz., (1) when a structural component goes through a critical speed suffering damage and (2) when the structure experiences continuous complex response as considered in time domain analysis.

3.1 *Structural Components Crossing Critical Speeds*

The first step in this analysis is to determine damping. The damping model in a rotating structure is first obtained experimentally, Rao et al. [22]; their test rig is shown in Fig. 3. They used electromagnets in place of nozzles and thus provide excitation.

The test results are quantified as shown in Fig. 4, which allowed the development of a nonlinear damping model with the equivalent viscous damping expressed as a function of strain amplitude at a reference point in a given mode of vibration at a given speed of rotation.

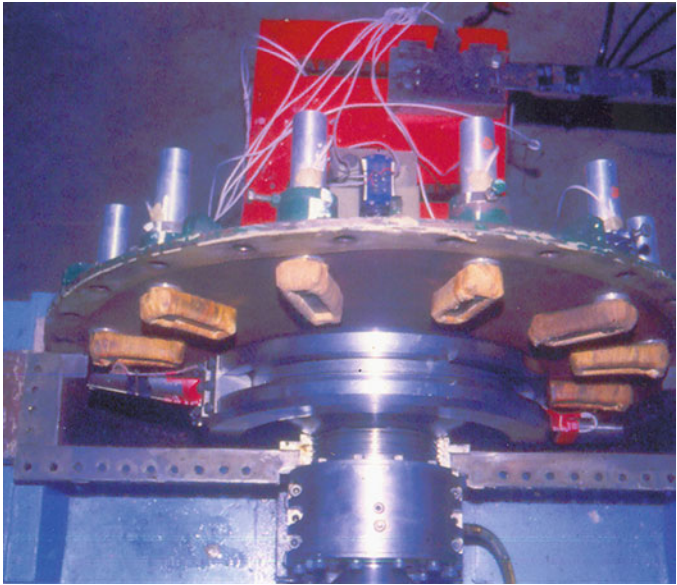


Fig. 3 Blade test rig for damping [22]

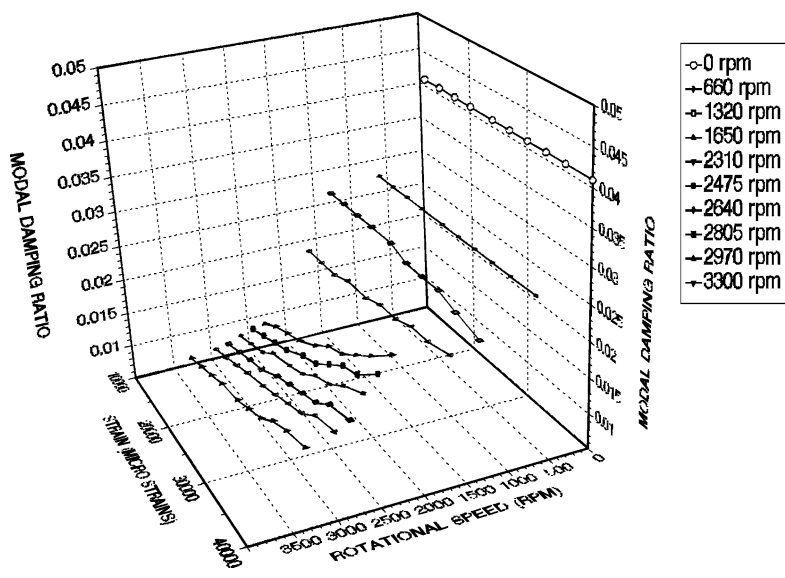


Fig. 4 Nonlinear damping model [22]

Rao and Saldanha [23] developed an analytical procedure using Lazan's hysteresis law [24] to determine such a nonlinear model with material damping. From hysteresis tests with specimens loaded under tension with stress σ , Lazan correlated the damping energy D through hysteresis area to give

$$D = J \left(\frac{\sigma}{\sigma_e} \right)^n \quad (3)$$

where J and n are material constants and σ_e is fatigue strength. If we consider the FE model of a structure, in this case, a bladed disk with each of the element as a test specimen, the total damping energy D_0 (Nm), loss factor η and equivalent viscous damping C (N-s/m) and damping ratio ξ in a given mode of free vibration be obtained from

$$D_0 = \int_0^v D dv; \eta = \frac{D_0}{2\pi W_0}; C = \frac{\eta K}{\omega}; \xi = \frac{C_e}{2\sqrt{Km}} \quad (4)$$

where v is the volume, W_0 is the potential energy in the mode, ω is the natural frequency (rad/s) and K is the modal stiffness (N/m). We can start with an orthonormal mode and choose a convenient reference point to determine the material damping as a function of reference strain amplitude. For increased strain amplitudes the orthonormal reference strain amplitudes, stress and strain energy are multiplied by a factor F to obtain the equivalent viscous damping C_e at various strain amplitudes as given in Eq. (5).

$$\varepsilon' = \varepsilon F; W_0' = W_0 \times F^2; \eta' = \frac{D_0'}{2\pi W_0'}; C_e' = \frac{\eta' K}{\omega_n} F^2; \xi' = \frac{C_e}{2\sqrt{Km}} F^2 \quad (5)$$

A plot of equivalent viscous damping ratio as a function of reference strain amplitude in the chosen mode of vibration defines the nonlinear material damping model. Figure 5 shows a typical material damping model in the first two modes of vibration of a blade rotating at 200 rpm.

3.2 Friction Damping

The friction damping characteristic is obtained by determining the transient response due to an impulse excitation at a suitable point on the blade to simulate the desired mode of vibration and assessing the decay curve. With the help of transient response we can obtain a nonlinear friction damping model of the blade as shown in Fig. 6. Likewise, a combined material and friction damping model can also be developed.

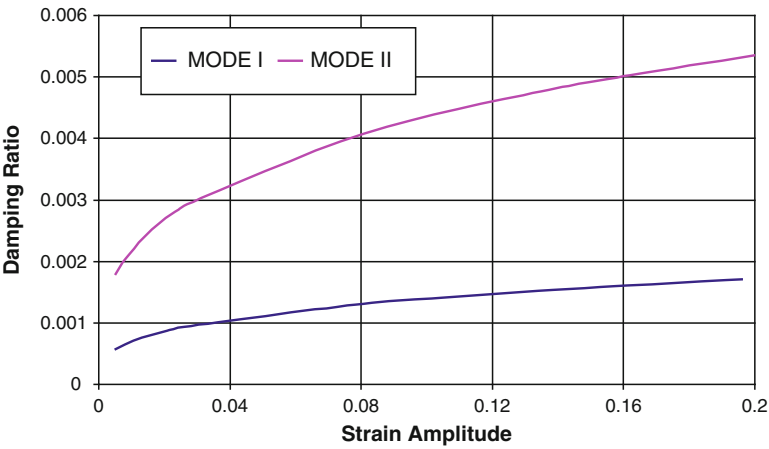


Fig. 5 Material damping model

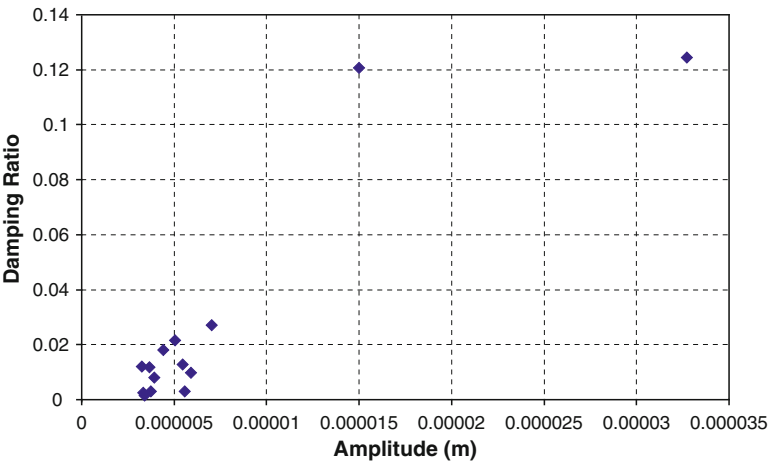


Fig. 6 Friction damping model

3.3 Identification of Critical Speed

Fatigue damage occurs whenever a structure goes through a resonance with the running speed or its harmonics coinciding with any of the natural frequencies. First the natural frequencies are determined as a function of speed and a Campbell diagram is plotted. From the Campbell diagram the critical speeds are identified. When a structure goes through this resonance, large stresses result in and fatigue damage will be predominant here. A typical Campbell diagram and critical speeds are shown in Fig. 7 along with stress response at one critical speed.

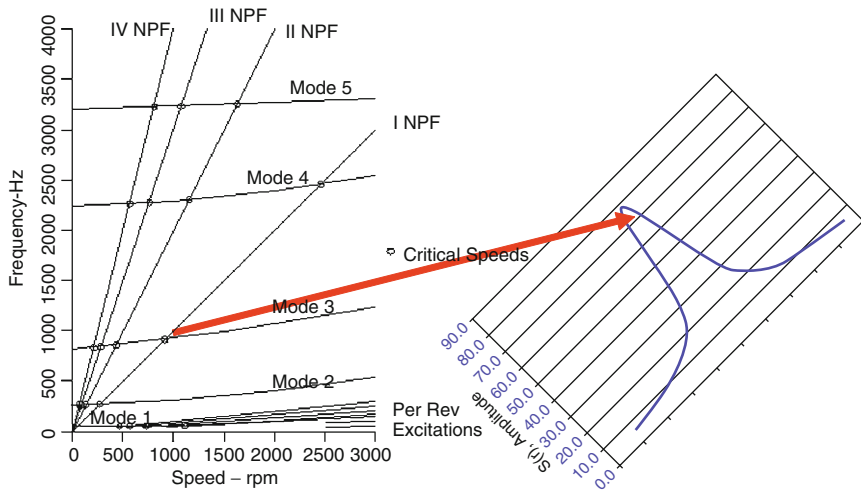


Fig. 7 Resonant response at one of the critical speeds on Campbell diagram

3.4 Resonant Response

This is best determined from fundamental modal properties rather than performing a forced vibration analysis at resonance. The damping is so small in structures, resonance is very sharp and it is difficult to make the excitation frequency and natural frequency coincide. Resonant response is best determined by using quality factor $Q = 1/2\xi$, see Rao and Gupta [25]. Treating the unsteady pressure field or excitation force as steady, steady state response is first determined and multiplied by the quality factor.

The damping ratio is however not known. The damping model developed is highly nonlinear and therefore we need to develop a procedure to handle this damping model. Rao and Vyas [26] developed an iterative procedure for this purpose. Through this procedure one determines the exact resonant stress amplitude through a critical speed.

Once the resonant stress is determined, it is a matter of using modal dynamic magnifier relation [25] given by

$$H(\omega) = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \quad (6)$$

where r is frequency ratio ω/p . This procedure is developed in Rao et al. [27,28] *BLADE* that operates on *HyperWorks* platform now called *Altair TurboManager*.

3.5 Cumulative Damage Through Resonance

The stress response before and after resonance and above endurance limit at each critical speed is divided into several steps and a linear [14, 15] or nonlinear cumulative damage [16, 17] is adopted to determine the damage fraction while crossing each critical. Knowing the acceleration with which the blade passes the resonance, each step through resonance is considered with the stress amplitude and the elapsed number of cycles in that step time period. Cumulative damage is then estimated to reach resonance and return to stress levels below fatigue limit. Since the application of stress levels may play a significant role a nonlinear rule is recommended. It is found that decreasing stress from resonant value to fatigue limit consumes more cycles of life; further while the machine is shut down, these damages could be different because acceleration rates will be different. It may be noted that the acceleration through resonance may affect the resonant amplitude as well as speed at which resonance takes place and this may be incorporated in determining the resonant stress, see Rao et al. [27]. The total damage fractions for each start up and shut down operation can be next obtained to give the life in terms of start ups. *TurboManager* is developed to determine this life in frequency domain.

3.6 Example of Propeller Shaft Life in Frequency Domain

Consider the case of simulation of a drive train of an automobile given in Rao et al. [29] and shown in Fig. 8. The drive train has several rotating components operating at different speeds at any given instant depending on which sets of gears are participating in the transmission. Each of the transmission units may contain several

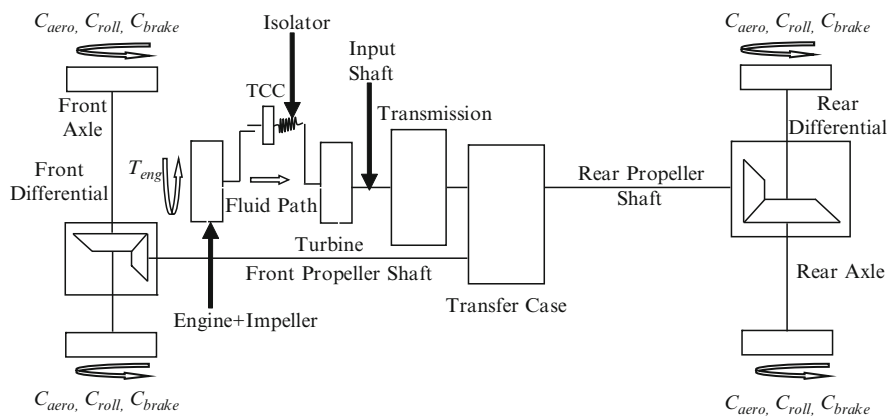


Fig. 8 Drive train model [29]

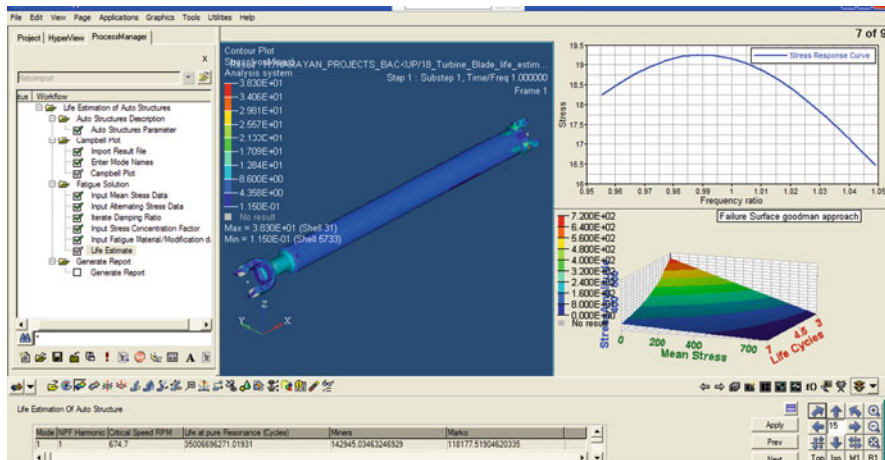


Fig. 9 Stress response and fatigue failure surface

flexible elements transmitting the torque from the engine. The mean torque provides the mean stresses and can be determined from simulation. Maximum value of stress in the propeller shaft of such a train is 28 MPa.

One of the modes of vibration of the drive train in III Mode torsion is 44.98 Hz at engine speed 674.7 rpm [29]. The drive train in Fig. 8 incorporates damping in torque converter and other elements which is fairly larger than the material damping and therefore no material damping is estimated. The alternating stress range at the stress raiser at this speed is found to be 38.3 MPa [30]. The resonant response is then obtained as given in Fig. 9. Using fatigue strength modification factors with the material properties as described in Rao et al. [27], the fatigue failure surface is also shown in Fig. 9.

For stress level above the fatigue limit, assuming a constant stress operation at resonance the life cycles are 3.5×10^{11} , whereas with ten steps in each rise and fall of stress, linear cumulative damage gives 142,945 and non linear cumulative damage gives 118,177 crossings at 674.7 rpm for life.

4 Lifing and Optimization

While lifing technologies have improved providing accurate simulation as discussed above, the next question a designer has – How can I get more life out of this structure within the available constraints? Typically stress raiser locations in any structure give stress concentrations that may be unacceptable and as local plastic conditions are reached under extreme loading the local strains control the life. To get optimum life, a shape optimization in the vicinity of stress raiser can be made to minimize the

stress concentration or minimize the local strain under local plastic conditions. The best shape to give more life is achieved in earlier days through tests and now we can perform a shape optimization and increase life through simulation.

A typical rotating blade [31] under a given unsteady pressure field has its fundamental mode at 344.5 Hz at the operating speed 8,500 rpm. Under self excited vibration from low back pressure under power generation well below rated value, life of the blade was predicted from *Blade* [28] as 101.592 minutes.

The blade root experiences a local plastic region. It should be noted that stress in the plastic region does not decrease significantly as the material is yielding, however we can reduce the strain and increase life. Figure 10 shows the space in which we can allow the shape to be altered consistent with manufacturing and design constraints.

Rao and Suresh [32] adopted an optimization using *HyperStudy* [33] and the range of variables given in Table 1. The optimization can be achieved either using elastic or elasto-plastic analysis.

The optimized shapes at 8,500 rpm from *HyperStudy* are given in Table 2.

The maximum stress at stress raiser decreased marginally from 768 to 746 MPa by 22 MPa (2.86%) from baseline elasto-plastic analysis for 8,500 rpm; however the peak plastic strain reduced from 0.0153 to 0.01126 by 26.4%. This is the major advantage in optimization for a blade root shape.

With the optimized shape, the life of the blade estimated by *Altair TurboManager* at 344.51 Hz, increases to 420.9 minutes which is 4.14 times the base line value of 101.592 minutes.

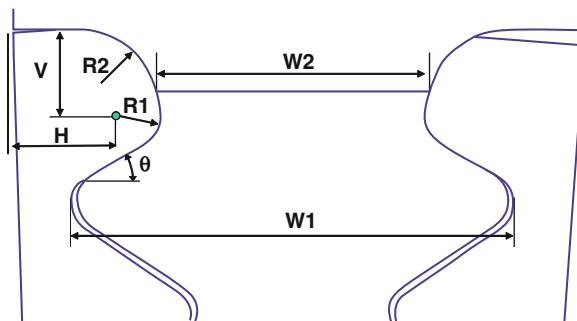


Fig. 10 Shape variables for optimization

Table 1 Shape variable definitions

Min value (mm)	Max value (mm)
W1 = 22.17	W1 = 25.76
W2 = 13.65	W2 = 13.86
R1 = 1.70, H = 5.67,	R1 = 2.14, H = 4.85,
V = 4.13, R2 = 4.0	V = 4.06, R2 = 3.37
$\theta = 29.86^\circ$	$\theta = 16.25^\circ$

Table 2 Optimized configurations under elastic and elasto plastic analyses

Optimized shape (mm) 8,500 rpm – Elastic	Optimized Shape (mm) 8,500 rpm – EP
W1 = 25.43	W1 = 25.48
W2 = 13.86	W2 = 13.82
R1 = 1.89, H = 5.17, V = 4.01, R2 = 4.0	R1 = 2.08, H = 4.96, V = 4.07, R2 = 4.0
$\theta = 16.50^\circ$	$\theta = 17.15^\circ$

5 Concluding Remarks

Lifing techniques in time domain and frequency domain are discussed and the advantages of frequency domain method for advanced structures are highlighted.

A special feature in today's technologies is the ability to analytically determine a nonlinear damping model of a rotating structure that enables an accurate estimation of resonant stress at a critical speed. This analytical development allows a true simulation and faster turn around design cycle time.

Optimization techniques are now increasingly employed to minimize stress concentration factors under elastic conditions or minimize local strains for globally elastic but locally plastic structures considering design constraints. A shape optimization thus performed gives the best possible life.

The lifing and optimization procedures together reduce testing costs and bring an accurate solution to advanced structures in getting maximum life.

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