

Preface

There are good theoretical reasons to consider seriously the possibility that gravity is not described precisely by Einstein's General Relativity but rather by some alternative theory. First, attempts to renormalize General Relativity in the 1960s and 1970s showed clearly that counterterms must be introduced which alter the theory significantly and make its field equations of fourth order instead of second. From the physical point of view, this fact implies that extra degrees of freedom, in addition to the usual spin two graviton, need to be introduced. Unfortunately, the corrected theory is not free of ghosts, which makes it non-unitary. The corrections introduced by renormalization are quadratic in the algebraic invariants of the curvature tensor and were successfully employed in R^2 -inflation in the early universe. By retaining only corrections quadratic in the Ricci scalar R or, by extension, corrections which are general non-linear functions of R (and no longer motivated by renormalization), one obtains the so-called class of $f(R)$ theories of gravity.

Second, when one tries to approach gravity (and the other interactions) from the high energy side and then obtain low-energy physics, one does not recover Einstein's theory. Adopting, at least as a temporary model, string theory as a full theory of quantum gravity which also unifies the known interactions, one can take a low-energy limit which, again, does not reproduce General Relativity but gives instead a Brans-Dicke theory. Scalar-tensor theories of gravity have been known for a long time and were developed following initial suggestions by Dirac, Jordan, Fierz, and Thiry, culminating in the 1961 paper by Brans and Dicke introducing what is now known as Brans-Dicke theory. The original motivations for Brans-Dicke theory were rooted in the need to implement Mach's principle, which is not fully incorporated in General Relativity, in a relativistic theory of gravity. After Brans-Dicke theory (the prototype of scalar-tensor theories of gravity) was born, research on Mach's principle went its own way and, without doubt, the interest of modern physicists in scalar-tensor gravity arises more from string theories than from Mach's principle. Dilaton fields and their non-minimal couplings to the spacetime curvature are unavoidable features of string theories, shared with scalar-tensor gravity.

It seems, therefore, that first loop corrections or attempts to fully quantize gravity necessarily introduce significant deviations from General Relativity and extra degrees of freedom. The recent thermodynamics of spacetime approach to emergent gravity pictures General Relativity as a thermodynamical state of equilibrium

among a wider spectrum of gravity theories and it only makes sense that, if extra degrees of freedom are allowed in addition to the standard spin two graviton of General Relativity, deviations from this equilibrium state will correspond to the excitation of these extra degrees of freedom and to deviations from Einstein's theory. From the theoretical point of view, going beyond General Relativity is a necessity and exploring the wider landscape of theories becomes a cultural need.

From the experimental point of view, General Relativity has been tested directly in the Solar System in its weak-field, slow motion approximation. Binary pulsars, most notably the Hulse-Taylor system PSR 1913 + 16, allow for indirect tests outside the Solar System, in the same regime. However, strong gravity tests are still missing and gravity is tested very poorly at the scale of galaxies and clusters, witnessing the fact that even Newtonian gravity is doubted at galactic scales, which has led to the introduction of *MOND* and *TeVes* theories to replace galactic dark matter. Cosmology cannot be advocated as a precise test of General Relativity at large scales: in fact, almost all theories of gravity admit the Friedmann-Lemaître-Robertson-Walker line element as a solution of their field equations, with perfect fluids or other reasonable matter sources. Indeed, it is from cosmology that comes the indication that gravity may not be described exactly by General Relativity. The 1998 discovery that the present expansion of the universe appears to be accelerated, made using the luminosity distance versus redshift relation of type Ia supernovae, has left cosmologists scrambling for an explanation. In order to explain the cosmic acceleration within the context of General Relativity, one needs to introduce the mysterious dark energy, which is very exotic (its pressure P and energy density ρ must satisfy $P \simeq -\rho$), diluted, comprises approximately 75% of the energy content of the universe, and is not detected in the laboratory. Dark energy seems very much an *ad hoc* solution of the problem of the present acceleration of the universe and, understandably, alternatives have been looked for. Attempts to explain away dark energy using the backreaction of inhomogeneities on the dynamics of the background universe have been, so far, unconvincing. In 2002, the idea was advanced by one of us, soon followed by other authors, that perhaps we are observing the first deviations from General Relativity on the largest scales. $f(R)$ theories of gravity (although not of the quadratic form obtained by renormalization) were resurrected in an attempt to explain this phenomenon. Curiously, $f(R)$ gravity admits a scalar-tensor representation. Since these first attempts, the literature on $f(R)$ and scalar-tensor gravity and their applications to cosmology has flourished, and scalar fields or $f(R)$ modifications of gravity are now even proposed as alternatives to dark matter. This book attempts to organize the available knowledge about these classes of theories and the vast literature into a coherent view. The book is not meant to be a comprehensive review of half a century of literature, including its recent explosion: it is conceived more as an advanced introduction to this expanding area of research.

It would be premature and unjustified to claim that gravity is described by any of the theories described in this book. However, even if none of these extended theories of gravity ultimately proves to be correct, they are simple enough to solve many problems while still allowing us to peek into the vast landscape of theories beyond Einstein gravity and to understand many ways in which gravity can be enlarged with respect to Einstein's conception.

Notations and conventions: the following notations and conventions are used in this book. The metric signature is $-+++$ in four spacetime dimensions. Units are used in which the speed of light c and the reduced Planck constant \hbar assume the value unity. G is Newton's constant, the Planck mass is $m_{pl} = G^{-1/2}$, while $\kappa \equiv 8\pi G$. Greek indices assume the values 0, 1, 2, 3 and Latin indices assume the spatial values 1, 2, 3. A comma denotes ordinary differentiation, ∇_a is the covariant derivative operator, and g denotes the determinant of the metric tensor $g_{\mu\nu}$, while $\eta_{\mu\nu}$ is the Minkowski metric, which takes the form $\text{diag}(-1, 1, 1, 1)$ in Cartesian coordinates in four dimensions. $\varepsilon^{\alpha\beta\mu\nu}$ is the totally antisymmetric Levi-Civita tensor. Round and square brackets around indices denote symmetrization and antisymmetrization, respectively, which include division by the number of permutations of the indices, for example:

$$A_{(\alpha\beta)} \equiv \frac{A_{\alpha\beta} + A_{\beta\alpha}}{2}, \quad A_{[\alpha\beta]} \equiv \frac{A_{\alpha\beta} - A_{\beta\alpha}}{2}.$$

The Riemann and Ricci tensors are given in terms of the Christoffel symbols $\Gamma_{\alpha\beta}^\delta$ by

$$R_{\alpha\beta\gamma}^\delta = \Gamma_{\alpha\gamma,\beta}^\delta - \Gamma_{\beta\gamma,\alpha}^\delta + \Gamma_{\alpha\gamma}^\mu \Gamma_{\mu\beta}^\delta - \Gamma_{\beta\gamma}^\mu \Gamma_{\mu\alpha}^\delta,$$

$$R_{\alpha\gamma} \equiv R_{\alpha\beta\gamma}^\beta = \Gamma_{\alpha\gamma,\beta}^\beta - \Gamma_{\beta\gamma,\alpha}^\beta + \Gamma_{\alpha\gamma}^\delta \Gamma_{\delta\mu}^\mu - \Gamma_{\beta\gamma}^\delta \Gamma_{\delta\alpha}^\beta,$$

and $R \equiv g^{\alpha\beta} R_{\alpha\beta}$ is the Ricci curvature. $\square \equiv g^{\alpha\beta} \nabla_\alpha \nabla_\beta$ is d'Alembert's operator. A tilde usually denotes quantities defined in the Einstein frame and the subscript 0 identifies quantities evaluated at the present instant of time in the history of the universe.

Naples and Sherbrooke
November 2010

Salvatore Capozziello
Valerio Faraoni

Beyond Einstein Gravity

A Survey of Gravitational Theories for Cosmology and
Astrophysics

Capozziello, S.; Faraoni, V.

2011, XIX, 428 p., Hardcover

ISBN: 978-94-007-0164-9