Preface

Matter consists from atoms and elementary particles. It is desirable to get information about atoms and use it in various fields of technology. It was discovered in the twentieth century that elementary particles, atoms and molecules are governed by quantum laws which are different from the classical Newton mechanics. The information treated at the level of atom is called quantum information which is based on the description by quantum states of various quantum systems.

Achievements of the fundamental research are going to be useful in the technological industrial applications. Nanotechnology or quantum technology is the ability to engineer at atomic or molecular levels, it is the creation of materials, devices and systems through the control of matter on the nanometer (one-billionth (10^{-9}) meter) scale. Nanotechnology requires quantum mechanics and quantum information theory to reach its goals.

Quantum information theory is a rather old subject. Quantum mechanics was created by Heisenberg, Schrödinger, Dirac and others in the 1920s. Quantum information theory is based on quantum mechanics, and it has its origins in the 1930s when von Neumann introduced quantum entropy and Einstein, Podolsky and Rosen considered a sort of information gained as a result of measurement on the entangled particle.

In 1950s, in particular after Shannon, the sciences named "information" and "communication" were extensively developed. In the same vein, the processing ability of computers has been increasing each year, and now information technology (IT) expands to almost all our daily life. However, one looks for more accurate, speedier computers and safer communication, and the present stage of computers and IT is not in fullness. Further development will be possible in some conceptually different category, whose candidate is due to quantum mechanics. Mathematical foundations of quantum information were considered by Umegaki, Stratonovich, Ingarden and others in the early 1960s and developed by many other researchers. Important contributions to the modern development of quantum computing and nanoscience was made by Feynman. Information theory and technology based on quantum mechanics are called quantum information theory and nanotechnology, respectively.

In this book, we will discuss some mathematical features of quantum information and quantum computer (mainly computation) as mentioned above. Moreover, we will discuss the following topics: (1) basics of classical and quantum probability, functional analysis, stochastic analysis; (2) mathematical foundations of classical and quantum communication theory; (3) quantum entropy, relative entropy, mutual entropy, information and adaptive dynamics and their use in information communication and description of chaos; (4) Bell's type inequalities, quantum entanglement and their dependence on spatial variables; (5) various quantum algorithms and a new algorithm for quantum computations which goes beyond the quantum Turing machine paradigm, with an application to NP complete problems; (6) classical and quantum cryptography; (7) quantum teleportation; (8) some topics in quantum measurements, quantum electrodynamics and quantum fields; (9) applications to life science.

Further, we will consider applications of these mathematical methods to quantum dots and some topics in nanoscience which have the potential to be useful in future technology. We will also discuss some bio-systems in terms of mathematics treated in this book.

These areas are being quickly developed, and the book does not pretend to give a complete description of these topics. We try to describe the mathematical foundation of these fields.

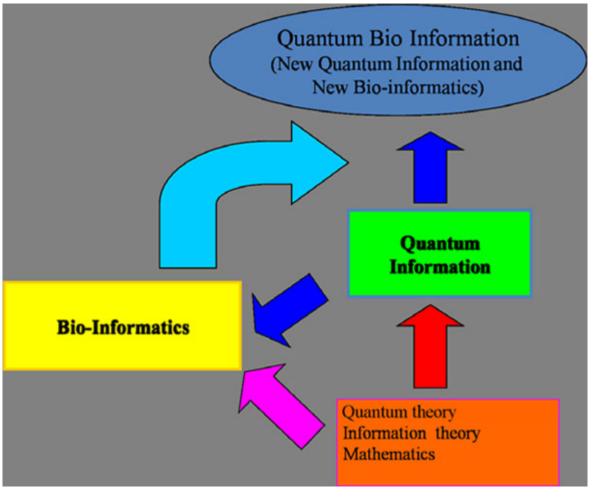
Some of these topics have been already discussed in several textbooks; see the bibliography. This book contributes to the consideration of these topics by treating them from the mathematical points of view. For instance, (1) our description is basically done in an infinite dimensional Hilbert space which is essential for studying quantum systems; (2) the dynamics of systems and its observation are considered from their modes of existence; (3) we investigate some basic notions of quantum information from the point of view of the quantum field theory, e.g., we investigate the spatial dependence of the entangled states which leads to a new perspective to the so-called quantum non-locality and Bell's theorem.

Various applications of mathematics developed in quantum and quantum-like methods to bio-systems will be considered, including the recent experimental discovery of quantum effects in photosynthesis.

The immensely long DNA (sequence of the four bases in the genome) contains information on life, and decoding or changing this sequence is involved in the expression and control of life. In quantum information, meanwhile, we produce various "information" by sequences of two quantum states, and think of ways of processing, communicating and controlling them. It is thought that the problems we can process in time T using a conventional computer can be processed in time nearly $\log T$ using a quantum computer. However, the transmission and processing of information in the living body might be much faster than of quantum information. Seen from this very basic viewpoint, developing the mathematical principles that have been found in quantum information should be useful in constructing mathematical principles for life sciences which have not been established yet. The mechanism of processing information in life is also expected to be useful for the further growth of quantum information.

So it will be very natural to try to find a new bridge between quantum information and life science [41–43]. We believe that the mathematics or mathematical notions discussed in this book are or will become important for bioscience and further developments of quantum information and nanotechnology.

Interrelation between mathematics, information and life science as it is presented in this book is shown in the following figure.



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