

Quantification of uncertain and variable model parameters in non-deterministic analysis

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Abstract A multitude of models for non-deterministic structural analysis have been developed. They are all designed to predict how non-nominal input parameter values propagate through the different phases of the calculation procedure. A literature review on a number of publications that present practical examples shows that the relation between the numerical formalism that describes the uncertain or variable quantity and the physical reality is not so clear. In almost all cases the authors (have to) make assumptions on the non-deterministic nature of the physical quantity, especially for material properties. However, the sensitivity of the structural response to material parameter changes can be very significant. The authors recommend that the numerical formalism for model parameters should be well adapted to physically observed variations.

1 Introduction

Numerical analysis is used throughout in technical analysis and scientific research. The paper takes structural finite element analysis as a reference. In many cases precise numerical data on one or more model parameters are not available, either because the parameter does not have a single value or because its value is not precisely known. Unless the analyst is satisfied with assumptions to assign certain values for each of these parameters, a non-deterministic analysis may be viable. However, some conditions must be met in order for a non-deterministic analysis run to yield

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practically relevant results. The paper discusses these conditions, the types of analysis that are feasible, the requirements on the input data, and the availability of useful data.

In dynamic analysis the effect of uncertainty and variability depends to a very significant extent on frequency, with both effects strongly increasing with increasing frequency. Whereas natural frequencies of the first few modes usually do not change much for moderate changes of input parameter values, this effect is much more pronounced at medium and high frequencies. This paper considers only the low frequency regime, where individual local model parameters determine structural behaviour. They include three types: stiffness, mass and damping. Stiffness and mass parameters depend mainly on the geometrical lay-out and material selection of the structure and its components, and boundary conditions also play a role. Although local damping characteristics of specific materials and treatments may be important, damping is usually a rather global property of a built-up structure, and it is often modelled with one or just a few global model parameters. This paper focusses on local stiffness and mass parameters.

There are two basic categories for non-deterministic analysis: probabilistic analysis is feasible in case of aleatory uncertainty, and non-probabilistic analysis can be used in case of epistemic uncertainty. Input data require a specific numerical formalism, with probability density functions for aleatory uncertainty and interval or fuzzy numbers for epistemic uncertainty. In addition to these distinct categories, there are intermediate categories. In a first section, this paper briefly presents a consistent structure for the representation of uncertain data in each of the cases of non-determinism.

The second section of the paper gives a wide selection of non-deterministic model data as they are reported in numerical analyses in journal articles and conference papers. Despite the apparent simplicity of data formats, the authors observe that in almost all cases the analyst makes assumptions on the nature of the non-determinism of the problem and on the quantification of the uncertain or variable parameters. The unavailability of validated input data is a circumstance that is often encountered, but that does not justify inadvertent assumptions. It is observed that most authors assign the non-deterministic nature of the problem mostly to uncertainty and/or variability of the material characteristics. For this reason, this paper gives an overview of some sources of material data. It is shown that engineering material properties may be very sensitive to production-related aspects of structural components. This phenomenon is very pronounced for composite materials.

2 Numerical representation of parameter uncertainty and variability

Engineering design is the activity of design and development of technical products. A technical product is built to fulfil a well specified function under more or less well prescribed conditions of utilisation. This process consists of a number of anal-

ysis verifications on a virtual product. A common procedure for design verification is finite element analysis, a numerical method for the simulation of the effect of mechanical or thermal loads on a product. As most product parameters are undetermined in the initial phases of design, a range of non-deterministic properties have to be taken into account. This paper discusses the effects of non-determinism on engineering analysis using the finite element method (FE).

2.1 Definitions

In literature, the use of the terminology *error*, *uncertainty* and *variability* is not unambiguous. Different researchers apply the same terminology but the meaning attached to these is rather inconsistent. This necessitates a profound clarification of the terminology for each publication which treats uncertainties. This work does not propose a new terminology, but applies the terminology proposed by OBERKAMPF [1]. Some additional nuances are, however, necessary in order to enable clear distinction between probabilistic and non-probabilistic quantities.

The term *variability* covers *the variation which is inherent to the modelled physical system or the environment under consideration*. Generally, this is described by a distributed quantity defined over a range of possible values. The exact value is known to be within this range, but it will vary from unit to unit or from time to time. Ideally, objective information on both the range and the likelihood of the quantity within this range is available. Some literature refers to this variability as *aleatory uncertainty* or *irreducible uncertainty*, referring to the fact that even when all information on the particular property is available, the quantity cannot be deterministically determined.

An *uncertainty* is *a potential deficiency in any phase or activity of the modelling process that is due to lack of knowledge*. The word *potential* stresses that the deficiency may or may not occur. This definition basically states that uncertainty is caused by incomplete information resulting from either vagueness, nonspecificity or dissonance [2]. Vagueness characterises information which is imprecisely defined, unclear or indistinct. It is typically the result of human opinion on unknown quantities (*“the density of this material is around x ”*). Nonspecificity refers to the availability of a number of different models that describe the same phenomenon. The larger the number of alternatives, the larger the nonspecificity. Dissonance refers to the existence of conflicting evidence of the described phenomenon, for instance when there is evidence that a quantity belongs to disjoint sets. Possibly, limited objective information is available, for instance when a range of possible values is known. In most cases, however, information on uncertainties is subjective and based on some expert opinion. Others in literature refer to this uncertainty as *reducible*, *epistemic* or *subjective uncertainty*.

An *error* is defined as *a recognisable deficiency in any phase of modelling or simulation that is not due to lack of knowledge*. The fact that the error is recognisable states that it should be identifiable through examination, and as such is not

caused by lack of knowledge. A further distinction between *acknowledged* and *un-acknowledged* errors is possible. Errors will not be considered further in this paper.

2.2 Discussion and extension of the definitions

The above definitions of uncertainty and variability are fairly straightforward and comprehensible. However, they are not mutually exclusive, since a variability could be subject to lack of knowledge when information on its range or likelihood within the range is missing. This is for instance the case for every design dimension subject to tolerances, but without further specification of manufacturing process or supplier. The tolerances represent the bounds on the feasible domain, but there is no information on the likelihood of the possible values within these bounds. Consequently, because there is a lack of knowledge, such a variability is also an uncertainty. It is referred to here as an *uncertain variability*. Some vague knowledge may be available (“the mean value is approximately x ”) but also nonspecificity may play an important role in the uncertainty, for instance in choosing an appropriate model to describe a random quantity. Opposed to the uncertain variability, a *certain variability* refers to a variability the range and likelihood of which are exactly known.

On the other hand, it appears logical to state that every property in a numerical model corresponding to a physical quantity is a variability, since it will eventually have a range of possible values and a likelihood inside this range in the physical model. This argumentation implies that all uncertainties are also variabilities. In practice however, the majority of model properties are implemented as constant deterministic values in the numerical model. Though they are subject to variation, the influence of their variability on the analysis result is considered to be negligible. Often, uncertainties refer to a possible lack of knowledge in these deterministic properties. This type of uncertainty is referred to as *invariable uncertainty*. The word *invariable* in this case does not mean that the property cannot change over different analyses. According to the definition of uncertainty, it will change when additional information is gathered that decreases the amount of uncertainty. The invariable uncertainties typically occur in model properties for model parts that are difficult to describe numerically, but considered constant in the final physical product (connections, damping, ...). Other examples are design properties which have negligible variability but which are not defined exactly in an early design stage. Figure 1 gives a graphical illustration of the proposed subdivision of the definitions for uncertainty and variability.

The group of variabilities may be further subdivided into two categories. *Inter-sample variability* is the property of a population of nominally identical realisations of a particular product, with each individual element of the population possibly exhibiting scatter. *Intra-sample variability* is a property of one particular realisation — of which other realisations possibly exist — that exhibits one or more properties that may change over time, due to temperature differences, ageing, ...

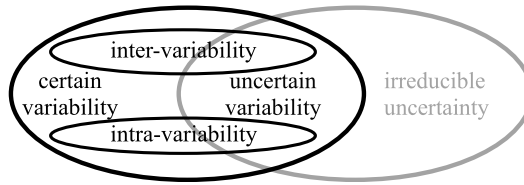


Fig. 1: Classification of variabilities and uncertainties in numerical modelling

3 Literature review on uncertain model and material data

The authors have conducted a review of journal publications in the field of non-deterministic analysis in structural dynamics. Only the numerical examples are considered, with a focus on how the input data are described. Most of the papers that listed are published in the journal for Computer Methods in Applied Mechanics and Engineering. Uncertainty mostly applies on material constitutive data, and for this reason several materials related journals are listed as well.

3.1 Non-probabilistic models

Non-probabilistic models are used in different conditions, usually when limited data are available, and when a probabilistic interpretation is not required. The following examples illustrate when these conditions are met:

- **concept models** are used in early stages of engineering design, when only general and approximate information on a design case is available. MOENS et al. [3] have built a concept model of a truck-trailer combination. The model consists of discrete mass elements for the major components and systems in the truck, and individual springs. The truck manufacturer uses simple MatLab models to investigate vehicle dynamics in an early stage, and the objective of this study was to predict maximum response levels for different excitations. In a later stage of product design, MASSA et al. [4] have used a fuzzy description to investigate different variants of a suspension triangle. The same authors have also used this approach on an impactor [5].
- **investigation of the effect of production tolerance** is done using interval analysis, possibly extended with subjective interpretation using fuzzy numbers. MOENS et al. [6] have investigated the effect of typical plate thickness on the natural frequencies of the stiffened cylinder of the CoRoT satellite.
- **uncertainty of specific model parameters** occurs for specific quantities that are hard to quantify, such as the stiffness of a polymer layer in the windshield of a vehicle. The effect of uncertain thickness of the polymer layer on natural frequencies was also taken into account [7]. Another uncertain parameter that was investigated on the same structure was the curvature of the windshield [8].

- **model parameters with imprecise values** may occur when incomplete data are available. NOOR et al. [9] have calculated the nonlinear response of a stiffened composite aircraft fuselage panel with a circular cut-out. Young's modulus and Poisson ratio are sensitive to the alignment of fibre orientations, and they are modelled with triangular fuzzy numbers with a $\pm 15\%$ support. The flange width and the web height of the T-stiffener are also considered to be uncertain.
- **the imprecise effect of process steps** occurs in several production processes. KHALED et al. [10] have used a fuzzy set approach in conjunction with FE analysis to predict the residual stress field in the heat affected zone of a welded structure that undergoes martensitic transformations during the cool-down part of the weld thermal cycle.

In conclusion, non-probabilistic models are fed with input data that are only subjectively linked to realistic problem data. If the bounds of the interval are well defined, and if the non-probabilistic analysis procedure does not introduce artificial conservatism, the output is a realistic set of bounds on output quantities.

3.2 Probabilistic models

Probabilistic models are used for several decades already, in a very wide range of applications. The list below is a selection of applications in structural dynamics and also in static structural analysis with random stiffness characteristics.

1. LIONNET and LARDEUR [11] have developed a hierarchical model for the effect of variability on the booming noise in a passenger car. They identified three different sources of variability: engine vibrations, the dynamic stiffness of the engine mounts and the transfer function from the vibrating body to the noise level. Each of these sources is measured independently, and modelled with a normal distribution. Interior noise level at the driver's ear shows measured intra-variability level between 2 and 20 dB depending on the engine speed. The inter-variability of booming noise was also measured, and it is found to be lower than the intra-variability one.
2. PELLISSETTI et al. [12] conducted a reliability analysis on vibration levels of the INTEGRAL satellite. Extensive material data were available. No less than 1319 independent random variables were defined with coefficients of variation ranging from 4% (for mass density) to 12% (mainly for composite material properties). Particular emphasis is given to the effect of the uncertainty in the damping on the reliability of the considered structure. Hence, various levels of the uncertainty in the damping have been investigated. In all the cases the damping ratios have been assumed to follow a log-normal distribution and to be mutually independent.
3. SCHUËLLER [13] applied Monte Carlo simulation for the reliability analysis of a 12-storey building subjected to earthquake excitation. 244 random variables model the stiffness of confined reinforced concrete, and the covariance matrix is modelled with 80 Karhunen-Loève terms.

4. CHUNG et al. [14] developed a stochastic finite element model of Glare, a metal laminate with a glass fibre reinforced layer in between two aluminum layers. The properties of the metal layers and the Poisson ratio of the glass prepreg layers are considered to be deterministic. The elastic modulus of the fibre reinforced layer is assumed to be a random process, and it is modelled with a Karhunen-Loève expansion. The covariance function for the prepreg layers is modelled with an exponential function with a correlation length which is longer in the fibre direction than in the transverse directions. The values for the correlation length are based on assumptions.
5. SARKAR and GHANEM [15] have integrated a number of frequency-domain dynamic analysis procedures of randomly disordered structural systems in the medium frequency range into the stochastic finite element method with an application to the analysis of the dynamics of a coupled uncertain rod assembly subjected to an external excitation. Young's modulus of each rod is assumed to be an independent and homogeneous Gaussian random field with a coefficient of variation equal to 5%. The autocovariance function of the process is chosen to be of the form $R(x, y) = e^{-|x-y|/b}$, where b is the correlation length, assumed to be equal to half of the length of each rod.
6. AGARWAL and ALURU [16] propose a stochastic framework to handle uncertain coupled electromechanical interaction, arising from variations in material properties and geometrical parameters such as gap between the microstructures, applicable to the static analysis of electrostatic MicroElectroMechanical Systems. The stochastic mechanical analysis quantifies the uncertainty associated with the deformation of MEM structures due to the variations in material properties and/or applied traction, and the stochastic electrostatic analysis quantifies the uncertainty in the electrostatic pressure due to variations in geometrical parameters or uncertain deformation of the conductors. The Young's modulus is assumed to be a uniformly distributed random variable with a mean value of 169MPa and a coefficient of variation of no less than 20%.
7. FALSONE and FERRO [17] present a procedure that gives the exact relationship between the response and the random variables representing the structural uncertainties in structures that are built up of beam-like components, under the assumption that a point-discretisation method is used for the representation of the uncertain random field. An uncertain Young's modulus is considered for each finite element, with "high" correlation (COV equal to 70%) for adjacent elements and "low" correlation (COV equal to 40%) for other cases
8. STEFANO and PAPADRAKAKIS [18] present a stochastic formulation of the triangular composite facet shell element for the case of combined uncertain material (Young's modulus, Poisson's ratio) and geometric (thickness) properties. These properties are assumed to be described by uncorrelated two-dimensional homogeneous stochastic fields. The spatial variability in Young's modulus and thickness of the shells is described by two uncorrelated homogeneous stochastic fields with coefficient of variation equal to 10%. The same assumption is made for the stochastic fields describing the random variation of Young's modulus and

Poisson's ratio. A prescribed form for the power spectral density function that characterises the two stochastic fields in both cases is assumed.

Only the first two publications [11] and [12] give a reference to data that are based on measurements or on thorough analysis. The authors of the other papers content themselves with assumptions on the nature and the quantification of stochasticity. Quite different levels are assumed for the coefficients of variation (ranging from 4% to 30%). Sometimes the coefficients of variation are different for different properties, and sometimes they are not. Sometimes a correlation between different properties is assumed, and sometimes properties are independent. The models for spatial variation are very diverse, with different assumptions for correlation length.

For uncertain variabilities, a representation by a single random quantity is generally not sufficient. Engineering scientist FREUDENTHAL [19] stated in 1961 that "*... ignorance of the cause of variation does not make such variation random.*". By this, he means that when crucial information on a variability is missing, it is not good practice to model it as a probabilistic quantity represented by a single random PDF. On the contrary, in this case it is mandatory to apply a number of different probabilistic models to examine the effect of the chosen PDF on the result. For instance, when the range of the variability is known but the information on the likelihood is missing, all possible PDFs over the range should be taken into consideration in the analysis. The analyst will generally select only a few probabilistic models which he considers consistent with the limited available information or most appropriate to obtain as much knowledge as possible on the result. Another important criterion in the selection of the type of distribution is the nature of the distribution function itself and its relation to the phenomena that it represents. The risk function is a useful indicator in this respect.

One conclusion is firm however, all authors apply variability on material characteristics, mostly on stiffness parameters. Material models are the most uncertain parameters in variability analysis (not taking into account damping).

3.3 *Material data*

With the observation that material parameters are the main source for non-determinism in probabilistic models, the literature study is extended to materials data.

The mechanical properties of most common structural materials, especially metals and unreinforced polymers, are relatively well known. However, the range of materials is very wide, and properties may differ with precise chemical composition, with thermal treatment and they may even be different with different manufacturers. In addition, material properties have some scatter. However, over all physical and mechanical properties that a material exhibits, mass and stiffness usually are fairly close to their nominal values, unlike strength, which depends strongly on chemical composition and heat treatment. Thickness of the unworked piece also plays a role in material strength, with strength decreasing with increasing thickness.

Non-determinism in the properties of a specific material is a case of variability. Real materials are characterised in experimental measurements. The size of the set of measurements that are taken in identical conditions determines if a probability density function can be established with sufficient accuracy. If a sufficient number of individual measurements are available, the variability can be considered certain.

3.3.1 Metals and polymer materials databases

The first source for metals and polymer materials data is a materials database. The MIL-handbook [20], which is now published on the web, and other web based databases such as *matweb* [21] and *efunda* [22] contain a large number of records for many material variants, even from different manufacturers. They usually specify nominal values, sometimes complemented with an indication of the probability distribution.

These databases do not give any indication on the spatial scatter within a test coupon. The test procedure implies that stiffness values are averaged numbers of the length of the sensor that is used, whereas strength is based on a local value in the section of fracture. Experimental data on spatial scatter are not available.

3.3.2 Composites properties

An important class of materials that will continue to gain importance is composites. The advantage of these materials is their excellent ratio of mechanical properties over mass density, which is a crucial asset especially in the transportation industry. These materials also offer wide opportunities for tailored solutions. The designer has many degrees of freedom, including the selection of raw materials for both the matrix and the fibre reinforcement, the architecture of the fibre reinforcement (unidirectional fibres, woven fabrics, knitted fabrics, braided fabrics, non-crimp fabrics, . . . , each with its own variants), the fibre volume fraction, the number of layers and the orientation of layers. For the analyst, this large set of design degrees of freedom translates into a wide range of model parameters, and inevitably also a wide range of uncertain or imprecise material data. Figure 2 illustrates some of these effects. The left hand side of the figure shows the variation of the elastic orthotropic stiffness constants for different orientations of a uniaxially reinforced glass fibre composite lamina with respect to the applied uniaxial tensile load. E_{11} is the modulus in the longitudinal direction along the fibre orientation, and E_{xx} is the modulus in the loading direction that has an angle θ with respect to the fibre direction. The graph shows a significant decrease of stiffness with increasing misalignment of the fibre. The right hand side of the figure is valid for a cross-ply (0° - 90°) carbon-epoxy system. The graph shows the variation of the Young's modulus for different alignments of the the fibre orientations with respect to the loading direction. The graphs show that the equivalent material stiffness depends strongly on the fibre placement. An imprecise placement of the fibre inevitably leads to a change of stiffness with respect to

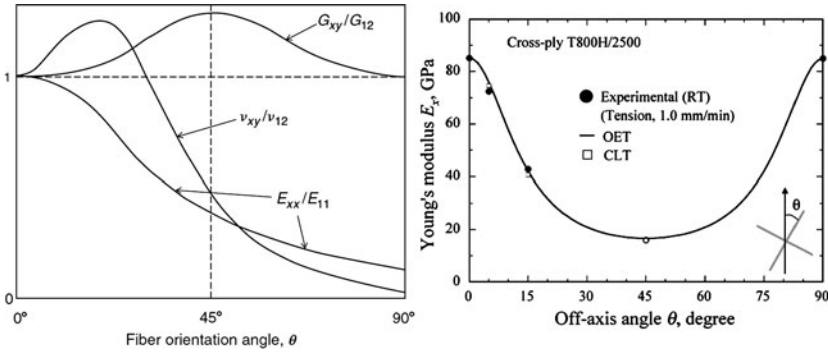


Fig. 2: Dependency of in-plane material parameters on the orientation θ of the major fibre axis to the loading direction; left: variation of the elastic constants of a continuous E-glass fibre lamina [23]; right: variation of the tensile Young's modulus for a cross-ply carbon-epoxy composite [24]

the nominal values. The left hand side of the graph also shows that the orthotropic elastic constants are inter-related.

Another geometrical parameter that determines the homogenised stiffness characteristics of a textile composite material is the so-called crimp factor. It is a measure of the waviness of the yarn through the thickness of the panel. A general tendency is that the equivalent modulus of a textile composite increases with decreasing crimp.

3.3.3 Multi-scale models for spatial variation of material properties

Recent research has brought forward significant advances in models that describe different aspects of material non-homogeneity. Extensive research efforts are currently ongoing to develop a multi-scale modelling procedure at successive scales. Depending on the type of material, the micro-scale describes properties with a reference length in the order of $10^{-6} - 10^{-4}$ m, the meso-scale describes properties with a reference length in the order of $10^{-4} - 10^{-2}$ m, and entire component structural behaviour is described on the macro-scale, with reference lengths in the order of $10^{-2} - 10^0$ m and above. The step from a lower level to a higher level is made using homogenisation procedures, that assign overall properties at a higher scale based on lower scale data. So far, these models are mainly deterministic. When these models will be well established, they present an excellent opportunity to introduce variability at the appropriate level, and to predict the propagation of their effect to a higher level, and ultimately to the entire component. CHARMPIS and SCHUËLLER [25] have already made proposals to materials researchers to develop these models. Experiments will however always be required to validate these models.

Multi-scale models also have the advantage that spatial variation of homogenised properties can be described based on lower scale characteristics. This presents op-

opportunities for realistic quantification of random fields, for which experimental data are currently missing.

Some initial efforts to establish stochastic models for specific purposes have already been taken in other applications. As an example, the spatial distribution of crystal orientations affects plastic behaviour [26]. Other types of non-homogeneous materials, such as metal foam are also described using stochastic models [27].

3.4 Other model properties

In addition to material properties and geometrical dimensions and shapes, other FE model characteristics exhibit some kind of uncertainty or variability as well.

A delicate property is the boundary condition with which a structure is attached to the environment. Only one reference has been identified that addresses uncertainty on boundary conditions for buckling analysis of cylindrical shells with random boundary geometric imperfections [28]. FE models typically use either pinned or fixed conditions. In a pinned connection displacements are prescribed and rotations are free, and in a fixed connection both displacements and rotations are fixed. These conditions correspond to an infinitely stiff connection, which can never be realised in practice. The stiffness of the connection may be very small or very large, but it is always finite. The non-determinism has definitely a character of uncertainty, and an interval number or a fuzzy number seems to be the best representation.

Damping is another unknown quantity. Physically realistic models for damping are not available, and it may even be hard to characterise damping from experiments. An interval number is again the most appropriate model.

3.5 Alternative approaches: non-parametric model concept and info-gap theory

An alternative strategy is the non-parametric modelling concept, that was originally introduced by SOIZE in 2000 [29]. Rather than modelling the variability on each individual parameter, the generalised matrices of a mean reduced matrix model of the structure are replaced by random matrices whose probability model is constructed with the maximum entropy principle [30]. This is a promising unified approach that brings together uncertainty and probability. PELLISSETTI et al. [12] have applied this concept in the reliability analysis of a satellite structure subjected to harmonic base excitation in the low frequency range with respect to the exceedance of critical frequency response thresholds. The results indicate that for low levels of uncertainty in the damping, the non-parametric model provides conservative predictions about the exceedance probabilities. For high levels of damping uncertainty the opposite is the case.

The same research group has set up a procedure for the experimental identification and the validation of a non-parametric probabilistic approach allowing model uncertainties and data uncertainties to be taken into account in the numerical model developed to predict low- and medium-frequency dynamics of structures [31]. The analysis is performed for a composite sandwich panel representing a complex dynamical system which is sufficiently simple to be completely described and which exhibits not only data uncertainties, but above all model uncertainties. In a more recent paper [32] the same author has extended this approach to structural vibrations and vibro-acoustics.

Another alternative approach is info-gap theory, introduced by BEN-HAIM. PIERCE et al. present a case study [33].

3.6 Summary of observations

- Probabilistic methods provide more information than non-probabilistic methods; however, both families are highly complementary.
- The number of publications on probabilistic methods exceeds the non-probabilistic ones.
- Almost all publications refer to aleatory uncertainty in material parameters, but there are very few references to uncertainty on other important FE model parameters that are not precisely known, such as boundary conditions, although FE results are highly sensitive to them.
- Very few publications refer to validated data, and most authors who publish in the leading scientific journal content themselves with assumptions on the non-deterministic nature of the model parameters.
- Very different values are assumed for the coefficients of variation on material parameters such as Young's modulus: from 4% to even 30% for isotropic materials.
- Literature does not provide any evidence on values for spatial scatter; correlation length is based on assumptions, apparently related to the length of the component correlation between model parameters is not taken into account.

4 Conclusions

Researchers follow diverse strategies when they introduce non-determinism in their engineering analysis, and the type of data that are available does not necessarily match with the objectives of the analysis. The availability of data determines the type of non-deterministic analysis that can be executed without unintentional misrepresentation of data and inadvertent introduction of unvalidated assumptions. Inversely, a specific type of analysis can only be executed when the model data are available in a suitable format. The appropriate data format depends on the phase of

development of the structure that is considered and on the type of parameter that is modelled: material data, geometrical data, loads data, boundary conditions and spatial distribution of model parameters.

The authors perceive a need for a coordinated effort by the scientific research community to collect reliable data on different types of model parameters in an appropriate format for non-deterministic analysis and to make available these data to their fellow researchers and to the engineering community.

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