

Chapter 2

Scaling Laws and Complexity in Fire Regimes

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2.1 Introduction

Use of scaling terminology and concepts in ecology evolved rapidly from rare occurrences in the early 1980s to a central idea by the early 1990s (Allen and Hoekstra 1992; Levin 1992; Peterson and Parker 1998). In landscape ecology, use of “scale” frequently connotes explicitly spatial considerations (Dungan et al. 2002), notably *grain* and *extent*. More generally though, scaling refers to the systematic change of some biological variable with time, space, mass, or energy. Schneider (2001) further specifies *ecological scaling* sensu Calder (1983) and Peters (1983) as “the use of power laws that scale a variable (e.g., respiration) to body size, usually according to a nonintegral exponent” while noting that this is one of many equally common technical definitions. He further notes that “the concept of scale is evolving from verbal expression to quantitative expression” (p. 545), and will continue to do so as mathematical theory matures along with quantitative methods for extrapolating across scales. In what follows, we operate mainly with this quoted definition, noting that other variables can replace “body size”, but we also use such expressions as “small scales” and “large scales” somewhat loosely where we expect confusion to be minimal. We examine the idea of contagious disturbance, how it influences our cross-scale understanding of landscape processes, leading to explicit quantitative relationships we call *scaling laws*. We look at four types of scaling laws in fire regimes and present a detailed example of one type, associated with correlated spatial patterns of fire occurrence. We conclude briefly

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with thoughts on the implications of scaling laws in fire regimes for ecological processes and landscape memory.

Landscape ecology differs from ecosystem, community, and population ecology in that it must always be spatially explicit (Allen and Hoekstra 1992), thereby coupling scaling analysis with spatial metrics. For example, characteristic scales of analysis such as the stand, watershed, landscape, and region are associated with both dimensional spatial quantities (e.g., perimeter, area, elevational range) and dimensionless ones (e.g., perimeter/area ratio, fractal dimension). Similarly, properties of landscapes such as patch size distributions are also associated with spatial metrics. The tangible physical dimensions of landscapes obviate the often circuitous methods required to define and quantify scales in communities or ecosystems.

2.2 Scale and Contagious Disturbance

A contagious disturbance is one that spreads across a landscape over time, and whose intensity depends explicitly on interactions with the landscape (Peterson 2002). Some natural hazards (Cello and Malamud 2006), such as wildfires, are therefore contagious, whereas others, such as hurricanes, are not, even though their propagation may still produce distinctive spatial patterns. By the same criterion, biotic processes can be contagious (e.g., disease epidemics, insect outbreaks, grazing) or not (e.g., clearcutting). Contagion has two components: momentum (also see energy, Chap. 1) and connectivity. Together they create the aforementioned interaction between process and landscape. For an infectious disease—the best-known contagious process—a sneeze can provide momentum, while the density of nearby people provides connectivity. For fire, momentum is provided by fire weather via its effects on fireline intensity and heat transfer, whereas connectivity is provided by the spatial pattern and abundance of fuels.

Momentum and connectivity covary in a contagious disturbance process such as fire. Increases in momentum generally increase connectivity, and changes in connectivity can be abrupt when the number of patches susceptible to fire reaches a percolation threshold (Stauffer and Aharony 1994; Loehle 2004). For example, Gwozdz and McKenzie (unpublished data) found that decreasing humidity across a mountain watershed (momentum provided by fire weather) can abruptly change the connectivity of fuels when the percentage of the landscape susceptible to fire spread crosses a percolation threshold.

Interactions between momentum and connectivity may appear to be scale-dependent in that they yield qualitative changes in the behavior of landscape disturbances when viewed at different scales, even though the mechanisms of contagion per se do not change across scales. For example, the physical mechanisms of heat transfer remain the same across scales, and fire spread does depend on local connectivity of fuels, but estimates of connectivity across landscapes are sensitive to spatial resolution (Parody and Milne 2004).

2.3 Extrapolating Across Scales

Much study has gone into understanding how spatial processes change across scales (Levin 1992; Wu 1999; Miller et al. 2004; Habeeb et al. 2005). Scale extrapolation is universally seen to be obligatory, because detailed measurements are often only available at fine spatial scales (McKenzie et al. 1996), but also difficult. Given a set of observations at coarse scales, however, it is important to understand the distinction between *average behavior* of fine-scale processes and the *emergent behavior* (Milne 1998; Levin 2005) of a system. Emergent behavior “appears when a number of simple entities (agents) operate in an environment, forming more complex behaviors as a collective”.¹ In the first case, the principal difficulty in extrapolation is error propagation, producing biased estimates of the average or expected behavior at broad scales because of the cumulative error from summing or averaging many calculations (Rastetter et al. 1992; McKenzie et al. 1996). In the second case, the difficulty is more profound, in that one must identify scales in space and time at which qualitative changes in behavior occur.

Some qualitative models can partition scale axes in tractable ways. For example, Simard (1991) developed a classification of processes associated with wildland fire and its management that spanned many orders of magnitude on space and time axes. This “taxonomy” of wildland fire, though not derived quantitatively from data, was enough to build a logical connection to the National Fire Danger Rating System (NFDRS—Cohen and Deeming 1985) that was of practical use (Simard 1991). Nevertheless, the limitations of such models are clear, in that qualitative changes in system behavior and key variables are established *a priori*. In order to relate processes quantitatively across scales, whether one is interested in average behavior or emergent behavior, a tractable theoretical framework is needed.

Scaling laws are quantitative relationships between or among variables, with one axis (usually X) often being either space or time. Many scaling laws are bivariate and linear or log-linear, and are developed from statistical models, theoretical models, or both. Most commonly they are based on frequency distributions or cumulative distributions wherein variables, objects, or events with smaller values occur more frequently than those with larger values. The simplest scaling law is a *power law*, for which a histogram in log-log space of the frequency distribution follows a straight line (Zipf 1949, as cited in Newman 2005). Following Newman (2005), let $p(x) dx$ be the proportion of a variable with values between x and dx . For histograms that are straight lines in log-log space, $\ln p(x) = -\alpha \ln x + c$, where α and c are constants (Newman 2005). Exponentiating both sides and defining $C = \exp(c)$, we have the standard power law formulation

$$p(x) = Cx^{-\alpha} \quad (2.1)$$

¹ Wikipedia contributors, “Emergence,” Wikipedia, the Free Encyclopedia, <http://en.wikipedia.org/wiki/Emergence>. Accessed 25 Jan 2010.

The parameter of interest is the slope α (always negative for frequency distributions), whereas C serves as a normalization constant such that $p(x)$ sums to 1 (Newman 2005). In the case of a frequency distribution, where Y values in a histogram are counts, C can be rescaled in order to compare slopes among distributions. Power-law relationships are often fit statistically by various binning methods, with subsequent regression of bin averages on event size, but more complicated maximum-likelihood methods may be more robust (White et al. 2008; Chap. 3).

Newman (2005) gives 12 examples of quantities in natural, technical, and social systems that are thought to follow power laws over at least some part of their range. His diverse examples include intensities of wars (Roberts and Turcotte 1998), magnitude of earthquakes (National Geophysical Data Center 2010), citations of scientific papers (Redner 1998), and web hits (Adamic and Huberman 2000). Newman (2005) specifically excludes fire size distributions, while admitting that they might follow power laws over portions of their ranges. Current opinion is divided among those who would globally assign power laws to fire-size distributions (Minnich 1983; Bak et al. 1990; Malamud et al. 1998, 2005; Turcotte et al. 2002; Ricotta 2003) and those who would attribute them only to portions of distributions or rule them out altogether in favor of alternatives (Cumming 2001; Reed and McKelvey 2002; Clauset et al. 2007; Chap. 3).

2.4 Scaling Laws and Fire Regimes

Wildfires affect ecosystems across a range of scales in space and time, and controls on fire regimes change across scales. The attributes of individual fires are spatially and temporally variable, and the concept of *fire regimes* has evolved to characterize aggregate properties such as frequency, severity, seasonality, or area affected per unit time. These aggregate properties are often reduced to metrics such as means and variances, thereby simplifying much of the complexity of fire by focusing on a single scale and obscuring ecologically important cross-scale interactions (Falk et al. 2007).

Scaling laws can deconstruct aggregate statistics of fire regimes in two ways: via frequency distributions that exhibit scaling laws, or by examining the scale dependence of individual metrics. Fire-size distributions are an example of the first, in that frequency distributions of fire sizes often follow power laws over at least portions of their ranges (Malamud et al. 1998, 2005; Turcotte et al. 2002; Moritz et al. 2005; Millington et al. 2006). Fire frequency, fire hazard, and spatial patterns of fire occurrence in fire history data are examples of the second, in that these statistics often change systematically and predictably across the spatial scale of measurement (Moritz 2003; McKenzie et al. 2006a; Falk et al. 2007; Kellogg et al. 2008). Here we briefly discuss both the scaling *patterns* that have been found within each of these

four metrics of fire regimes (size, frequency, hazard, spatial pattern) and the more problematic attribution of *mechanisms* responsible for the scaling patterns.

2.4.1 Fire Size Distributions

Power laws have been statistically fit to fire size distributions from simulation models and empirical data at many scales, from virtual raster landscapes generated by the “Forest Fire Model” (Bak et al. 1990) to historical wildfire sizes throughout the continental United States (Malamud et al. 2005). Not all scaling relationships found in fire-size distributions are power laws. For example, Cumming (2001) found that a truncated exponential distribution, which defines an upper bound to fire size, had the best fit to data from boreal mixedwood forests in Canada. Reed and McKelvey (2002) suggest that the power law serves as an appropriate *null model*, but that additional parameters in a “competing hazards” model improved the fit to empirical data at regional scales. Ricotta (2003) suggests that power law exponents can change with spatial scale, based on hierarchical fractal properties of landscapes, providing a rejoinder to detractors of the power-law paradigm. An excellent review of this topic, with discussion, is found in Millington et al. (2006). These authors state, and we concur, that the value of discerning power-law behavior, or alternative, more complex nonlinear functions, would increase greatly if the ecological mechanisms driving such behavior could be identified (West et al. 1997; Brown et al. 2002).

Two mechanisms in particular have been proposed to explain power-law behavior in fire-size distributions. Self-organized criticality (SOC—Bak et al. 1988) refers to an emergent state of natural phenomena whereby a system (be it physical, biological, or socioeconomic) evolves to a state of equilibrium characterized by variable event sizes, each of which resets the system in proportion to event magnitude. In theory, the frequency distribution of events will approach a power law because the recovery time from “resetting” varies with event magnitude. SOC has been associated mainly with physical systems, particularly natural hazards such as earthquakes and landslides (Cello and Malamud 2006), but its attribution to power laws in fire regimes has typically been only at small scales (Malamud et al. 1998) or inferred from small-scale behavior (Song et al. 2001).

In contrast to SOC, highly optimized tolerance (HOT) emphasizes structured internal configurations of systems that involve tradeoffs in robustness (Carlson and Doyle 2002; Moritz et al. 2005), rather than the emergent outcomes of stochastic though correlated events as in SOC. For example, a HOT model that can be applied to wildfires is the probability-loss ratio (PLR) model (Doyle and Carlson 2000; Moritz et al. 2005), a probabilistic model of tradeoffs between resources (e.g., some ecosystem function in natural systems or efforts to protect timber in managed systems) and losses (e.g., from fire). Solving the PLR model analytically produces a

frequency distribution of expected fire sizes that follows a power law (Moritz et al. 2005). HOT provides a theoretical framework for examining ecosystem resilience in response to fire events (Chap. 3).

2.4.2 Fire Frequency

The terms *fire frequency* and *fire-return interval* (FRI) are part of the currency of ecosystem management. Fire frequency is often compared among different geographic regions and between the current and historical periods. For example, considerable FRI data exist across the western United States (NOAA 2010), which can be compared and used to build regional models of fire frequency (McKenzie et al. 2000). Both comparisons and model-building assume that all FRI data points represent a *composite fire return interval* (CFRI)—the average time between fires that are observed within a sample area, but the likelihood of detecting a fire event clearly increases as the search area is expanded. FRIs are inherently scale-dependent, despite sophisticated methods for unbiased estimation of fire-free intervals (Reed and Johnson 2004).

Scaling laws in fire frequency thus quantify the relationship between the area examined for evidence of fire and the estimated fire return interval. This *interval-area* relationship (IA—Falk et al. 2007) appears in low-severity fire regimes producing fire-scars on surviving trees, mixed-severity fire regimes where fire perimeters are estimated, and raster simulation models that produce a range of fire severities and fire sizes (Falk 2004; McKenzie et al. 2006a; Falk et al. 2007). In each case, the IA can be fit to a power law, whose slope (exponent) captures other aggregate properties of the fire regime (Fig. 2.1). For example, larger mean fire sizes produce less negative slopes, because small-area samples are more likely to detect large fires than small fires. Simulations suggest that greater variance in fire size, given equal means, also produces less negative slopes, for reasons that are presently unclear (see Falk et al. 2007 for details).

In theory, then, the intercept in log-log space of the IA relationship reflects the mean point fire-return interval (sample area=0 in the case of a point, or the area of the minimum mapping unit otherwise), providing a “location” parameter to the scaling law (Falk et al. 2007). Also in theory, the exponents in the IA relationship could be derived from the properties of fire-size distributions, possibly means and variances alone, although extreme values (rare large fires) make this difficult. This connection to fire size is useful because predictive modeling of fire sizes, though subject to substantial uncertainty, is less problematic than predicting fire frequency (McKenzie et al. 2000; Littell et al. 2009). Further work is necessary, though, to connect the IA relationship to estimates of fire sizes, or fire-size distributions.

Another metric of fire frequency, the *fire cycle*, or *natural fire rotation*, refers, on a particular landscape, to the time it takes to burn an area equal to that landscape. The fire cycle is presumably independent of spatial scale if the sample landscape is much larger than the largest fire recorded within it (Agee 1993), but calculating it

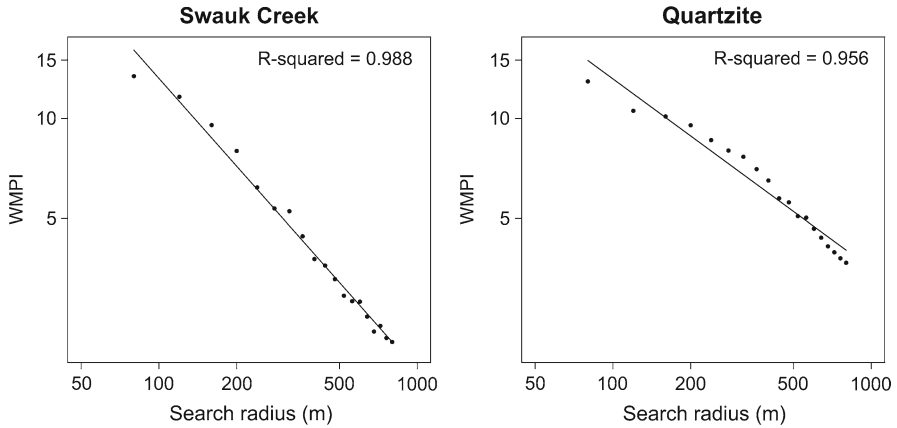


Fig. 2.1 Interval-area (IA) relationships (power laws) in log-log space for two watersheds in eastern Washington. WMPI=Weibull median probability interval. The more negative slope in Swauk Creek is a result of smaller fire sizes and more frequent fire occurrence than in Quartzite. Quartzite displays a minor but noticeable (concave down) departure from linearity (Redrawn and rescaled from McKenzie et al. (2006a))

depends on accurate estimates of the sizes of every fire in the sample. This is a difficult task in historical low-severity fire regimes, in which most fire-frequency work has been done (Hessl et al. 2007; Chap. 7). Furthermore, Reed (2006) showed that the mathematical equivalence between the fire cycle and the mean point FRI only holds if all fires are the same size, limiting the usefulness of the fire cycle as a metric of fire frequency.

2.4.3 Fire Hazard

Fire hazard in fire-history research quantifies the instantaneous probability of fire, and is derivable from distribution functions of the exponential family (e.g., negative exponential and Weibull) associated with the fire cycle (stand-replacing fire—Johnson and Gutsell 1994) and the distribution of fire-free intervals (fire-scar records—McKenzie et al. 2006a). The *hazard function* may be constant over time, reflecting a memory-free system in which current events do not depend on past events, and producing exponential age class distributions of patches in stand-replacing fire regimes (Johnson and Gutsell 1994). In contrast, an increasing hazard of fire over time (or decreasing, but this is rarely seen in fire regimes) reflects a causative factor, i.e. the growth of vegetation and buildup of fuel that facilitates fire spread. This increasing hazard is represented mathematically by a shape parameter in the Weibull distribution that is significantly greater than 1 (if this parameter is 1 the distribution reduces to the negative exponential—Evans et al. 2000). Moritz (2003) observes, however, that the ecological significance of the shape parameter

covaries with the scale parameter, representing, with fire, the mean fire-free interval. For long fire-free intervals, shape parameters ≤ 2 represent fire hazard that increases negligibly over time (Moritz 2003).

When the hazard function changes with spatial scale, it reflects changing controls on fire occurrence. McKenzie et al. (2006a) and Moritz (2003) identified patterns in hazard functions that were associated with the relative strength of transient controls on fire occurrence and fire spread. In low-severity fire regimes in dry forests of eastern Washington state, USA, McKenzie et al. (2006a) sampled composite fire records at different spatial scales to examine the scale dependence of fire frequency and fire hazard. At small sampling scales, hazard functions were significantly greater than 1 (increasing hazard over time), particularly in watersheds with complex topography, but declined monotonically with increasing sampling scale (Fig. 2.2). McKenzie et al. (2006a) suggest that fire hazard on eastern Washington landscapes increases over time at spatial scales associated with a characteristic size of historical fires, reflecting the effects of fuel buildup within burned areas.

In high-severity fire regimes of shrublands in southern California, USA, Moritz (2003) found no scale dependence in the hazard function except for one landscape whose location and topography protected it from extreme fire weather (Fig. 2.3). Fire hazard increased in response to the increasing flammability of fuels over time. Over most of the region, however, fuel age-classes burned with equal likelihood, because almost all large fires occurred during extreme fire weather, providing sufficient inertia to overcome the patchiness of fuels and rendering the hazard function essentially constant. In both these examples, then, scaling laws in fire hazard were

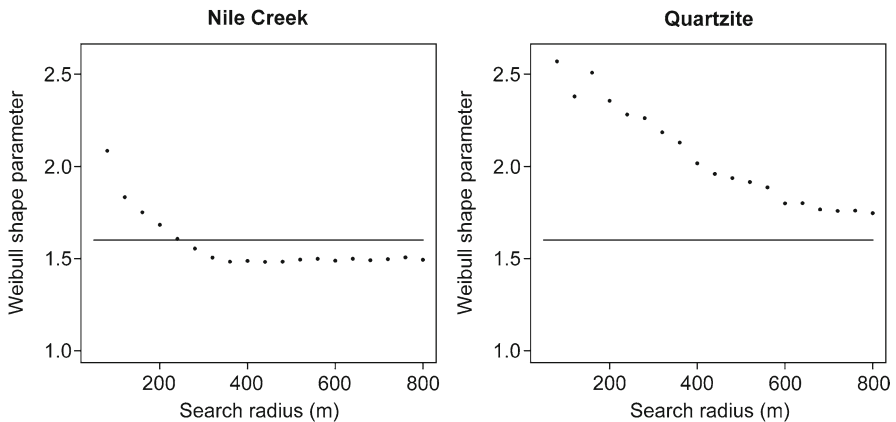


Fig 2.2 The Weibull shape parameter decreases with scale of sampling in two watersheds in eastern Washington. WMPI = Weibull median probability interval. Horizontal line marks the value (1.6) at the 95% upper confidence bound for testing whether the parameter is different from 1.0—meaning no increasing hazard over time. Fires were larger and less frequent in Quartzite than in Swauk Creek, so a shape parameter significantly greater than 1.0 may still be negligible ecologically, because shape and scale parameters co-vary (Moritz 2003 and Fig. 2.3) (Redrawn from McKenzie et al. (2006a))

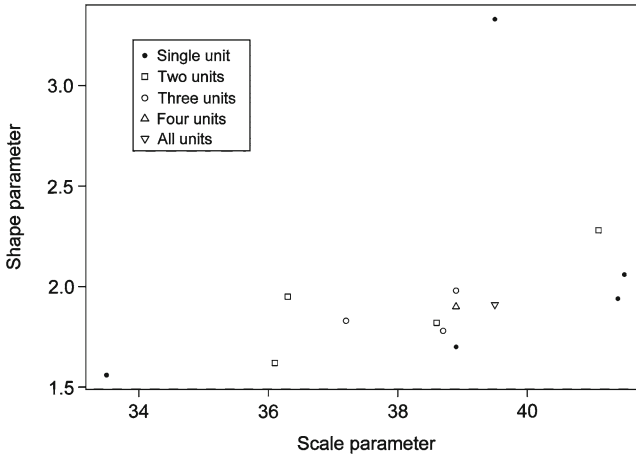


Fig. 2.3 Hazard function scale and shape parameters sampled at different scales in high-severity fire regimes in shrublands of southern California. The single point in the upper right represents one sample at the finest spatial scale that was protected from extreme fire weather and shows significantly increasing hazard over time. The positive covariance of the two parameters widens confidence intervals on significance tests of the shape parameter’s difference from 1.0, *sensu* McKenzie et al. (2006a) and Moritz (2003), such that even values ≈ 2.0 may not indicate increasing fire hazard with time (Redrawn from Moritz (2003))

apparent only when controls were “bottom-up” (Kellogg et al. 2008, Chaps. 1 and 3), i.e., produced by interactions between fine-scale process (the buildup of fuels over time) and landscape pattern (topography and the spatial variability in fuel loadings), and where extreme fire weather was uncommon.

2.4.4 Correlated Spatial Patterns

We emphasized earlier that a key property of landscape fire is contagion. The relative connectivity of landscapes with respect to fire spread and the momentum provided by fire intensity and fire weather jointly affect the probability that two locations will experience the same fire event. If this probability attenuates systematically with distance, it can in theory be represented by a scaling law related to contagion.

The cumulative effect of these probabilities over time can be seen clearly as the similarity between two locations of the time series of years recording fire. In low-severity fire regimes, this similarity is measured between two recorder trees (point fire records) or area samples (composite fire records). Kellogg et al. (2008) compiled these time series for every recorder tree in each of seven watersheds in Washington state, USA. They used a classical ecological distance measure, the Jaccard distance (closely related to the Sørensen’s distance [see below]—Legendre

and Legendre 1998), to compare pairs of recorder trees at different geographic distances, generating scatterplots analogous to empirical variograms (hereafter *SD variograms*). Spherical variogram models, and power-law functions, were fit to these aggregate data for each watershed (McKenzie et al. 2006b; Kellogg et al. 2008; and the example below). Both types of models had better explanatory power in more topographically complex watersheds.

2.4.5 Mechanisms

Power laws abound in nature and society, but to date explicit mechanisms that produce them, and the parameters associated with their variability, have been difficult to identify. Purely stochastic processes can produce power laws (Reed 2001; Brown et al. 2002; Solow 2005), as can general dimensional relationships among variables, the most familiar being Euclidean geometric scaling (Brown et al. 2002). Brown et al. (2002) suggest that when scaling exponents in power laws (α in Eq. 2.1) take on a limited or unexpected range of values they are more likely to have arisen from underlying mechanisms. Examples of this are in organismic biology, where the fractal structure of networks and exchange surfaces clearly leads to allometric relationships (West et al. 1997, 1999, 2002) and in ecosystems in which there are strong feedbacks between biotic and hydrologic processes (Scanlon et al. 2007; Sole 2007).

How might we identify the mechanisms behind scaling laws in fire regimes? We propose two general criteria, based on our overview above, as hypotheses to be tested. Criterion #1 suggests how mechanisms produce scaling laws, whereas criterion #2 provides necessary conditions for scaling laws in fire regimes to be linked to driving mechanisms.

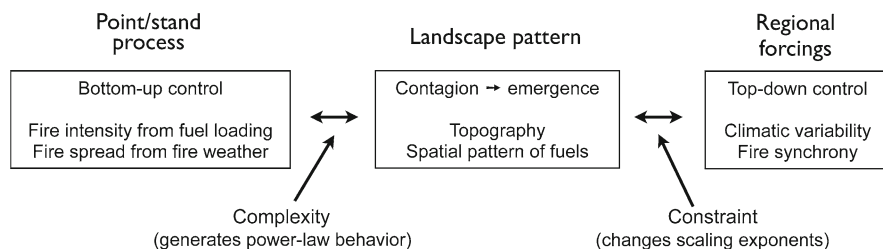


Fig. 2.4 Scaling laws in fire regimes are expected when bottom-up controls predominate and they interact strongly with landscape elements. For the contagious process of fire, fine-scale mechanisms provide momentum and topography and spatial pattern of fuels control connectivity (see text for discussion of contagion). In contrast, top-down controls (climate) increase fire size and therefore fire synchrony on landscapes where they are dominant, e.g., with gentle topography or continuous fuels. This favors irregular frequency distributions and lessens the scale dependence of fire frequency, hazard functions, and spatial patterns

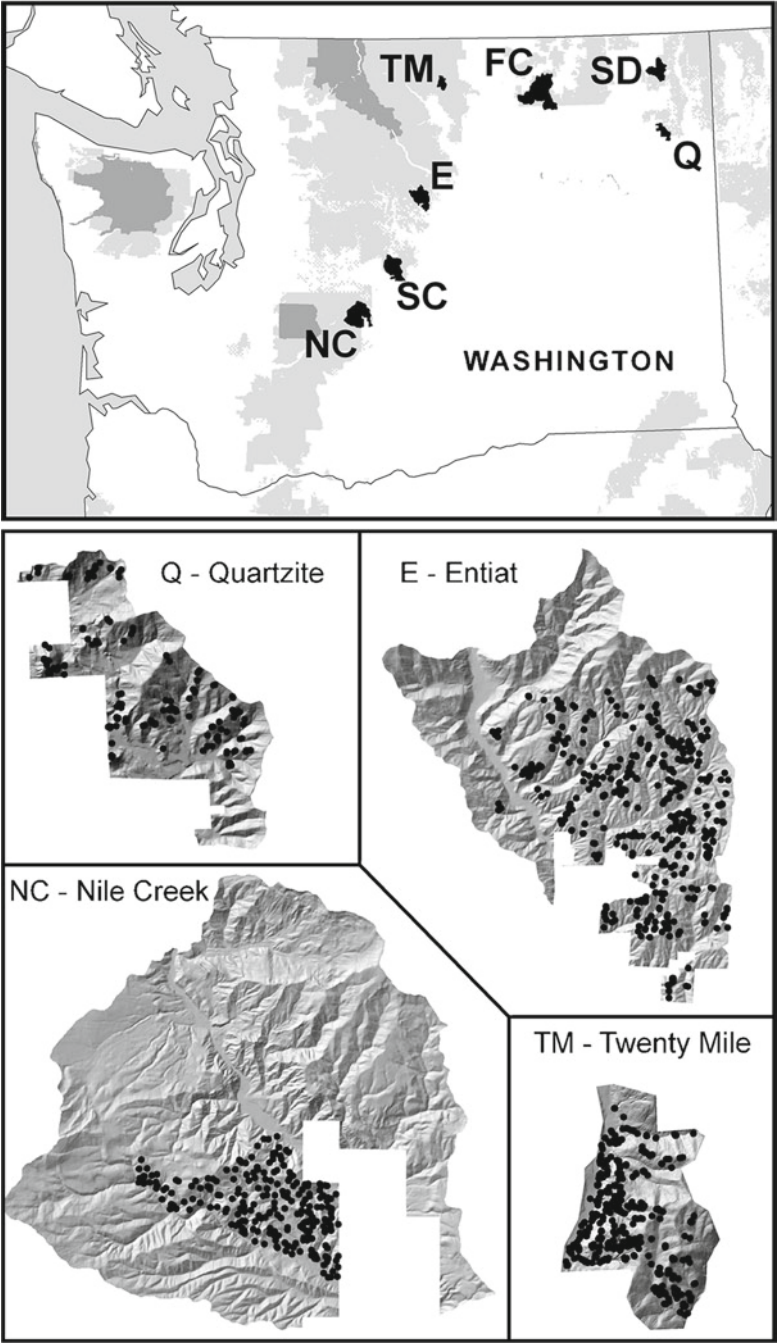
1. Bottom-up controls are in effect: Drawing on O'Neill et al. (1986), we propose a hierarchical view of fire regimes that focuses interest on landscape scales (Fig. 2.4). Mechanisms at a finer scale below drive fire propagation, and interactions between process (fire spread) and pattern (topography and fuels) generate complex spatial patterns. When landscape spatial complexity is sufficient, fire spread and fuel consumption produce the spatial patterns that are reflected in the IA relationship, the hazard function, and the SD variogram. Conversely to one paradigm of complexity theory that posits that simple generating rules can produce complex observable behavior, we therefore see that relatively simple aggregate properties of natural phenomena—scaling laws—are the result of complex interactions among driving mechanisms.
2. Contagion provides a linkage among observations: We submit that if events (fires) are separated by more distance in space or time than some limit of contagion, observed scaling laws cannot be reasonably linked to a driving mechanism. Mechanism requires “entanglement” (as in the quantum-mechanical sense). For example, both SOC and HOT, mentioned above, require that events within a domain influence each other, whether one event resets system properties in proportion to its magnitude (SOC) or multiple events interact as they propagate through a system (HOT). The range limit of contagion clearly changes as a function of variation in fine-scale drivers. As we said earlier (see also Chap. 1), increasing energy (momentum) effectively increases connectivity, e.g., when extreme fire weather overcomes barriers to fire spread that are associated with landscape heterogeneity (Turner and Romme 1994).

Criterion #2 does not preclude some mechanism for power-law behavior across continental-to-global scales; it just limits the hierarchical interpretation in criterion #1 to spatial scales at which contagion occurs. Other explanations for power laws in nature and society do exist, however, including the purely mathematical (Reed 2001; Solow 2005).

2.5 Example: Power Laws and Spatial Patterns in Low-Severity Fire Regimes

We now turn to an example, briefly alluded to above, from low-severity fire regimes of eastern Washington state, USA (Everett et al. 2000; Hessl et al. 2004, 2007; McKenzie et al. 2006a; Kellogg et al. 2008; Kennedy and McKenzie 2010). Detailed fire-history data were collected in seven watersheds east of the Cascade crest, along a southwest–northeast gradient (Fig. 2.5). In contrast to most fire history studies, exact locations of all recorder trees were identified, creating an unprecedented opportunity for fine-scale spatial analysis (McKenzie et al. 2006a; Hessl et al. 2007; Kellogg et al. 2008). For a detailed description of the data and methods, see Everett et al. (2000) or Hessl et al. (2004).

a



b

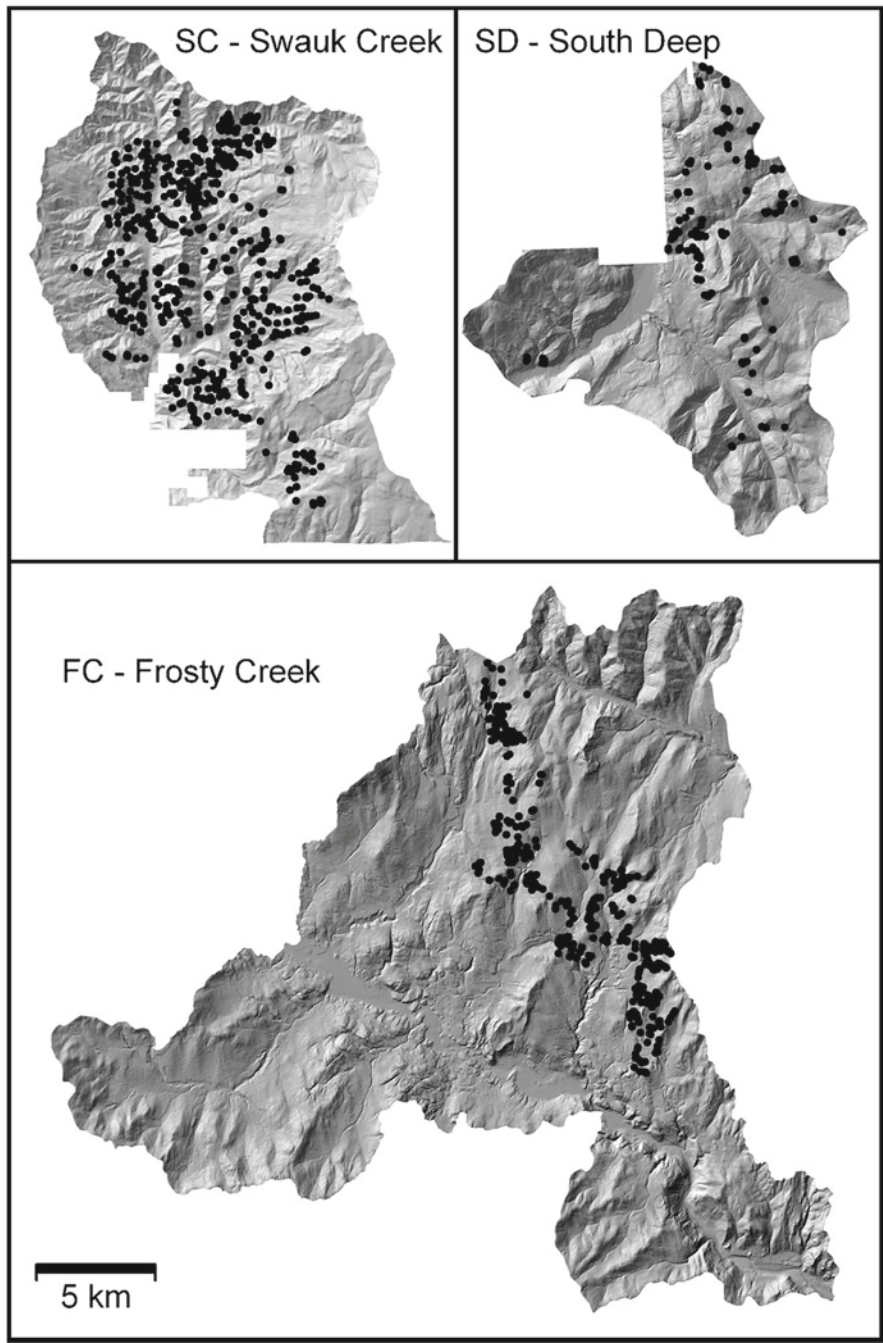


Fig. 2.5 Fire history study sites, east of the crest of the Cascade Mountains, Washington, USA. (a) Watershed locations. (b) Inserts that display hill shaded topography with dots representing the locations of recorder trees

Kellogg et al. (2008) fit the aforementioned empirical SD variograms to spherical models, in keeping with standard practice in geostatistics (Rossi et al. 1992), which uses variograms chiefly for spatial interpolation. Interpolation is generally only feasible with spherical, exponential, or Gaussian variogram models, due to certain mathematical conveniences (Isaaks and Srivastava 1989), but the spherical model in particular is a rather cumbersome artifact, with two separate equations applying to observations within or beyond the range (Kellogg et al. 2008). McKenzie et al. (2006b) examined the same empirical variograms in double logarithmic space and found that for some watersheds, the variograms seemed linear or nearly so, both graphically and when fit with linear regression. This suggested that power laws govern the correlated spatial pattern of fire histories. The observed pattern in these variograms was consistent across varying distance lags used to construct the variogram. We seek to test the hypothesis in criterion #1 (above) by trying to replicate the power-law behavior by controlling fine-scale processes (bottom-up control), using a neutral landscape model (Gardner and Urban 2007).

2.5.1 *Neutral Model for Fire History*

McKenzie et al. (2006a) developed a simple neutral fire history model to simulate recorder trees on landscapes that are scarred by fires of different sizes and frequencies. The purpose of the neutral model is to separate intrinsic stochastic processes from the effects of climate, fuel loadings, topography and management. We have enhanced the model to spread fires probabilistically on raster landscapes (Kennedy and McKenzie 2010; Fig. 2.6). The raster model produces 200-year fire histories on a neutral landscape, with homogenous topography and fuels. The raster landscape is initialized with a spatial point pattern of recorder trees; this pattern is simulated as a Poisson pattern of complete spatial randomness (CSR—Diggle 2003). A mean fire return interval (μ_{fri}) is specified for the whole “landscape”, yielding a random number of fires (n_{fire}), drawn from a negative exponential distribution, within the 200-year fire history. For each fire, a random fire size is drawn from a gamma probability distribution (Evans et al. 2000) with the scale and shape parameters adjusted to produce a specified mean fire size (μ_{size}). For each fire in the fire history, an ignition point (pixel) is randomly assigned and the fire is spread until it reaches the randomly drawn fire size (i.e., area), or until all tests for fire spread fail in a given iteration. When a pixel is burned, each of the four immediate neighbors that are not yet burned is tested for fire spread against the spread probability (p_{burn}). After the neighbors are tested for fire spread, the burned pixel can no longer spread fire.

In a given fire, if a pixel is burned, then all trees located in that pixel are tested independently for scarring in the same time step. This is a simple probability test, with a specified scar probability (p_{scar}) that is uniform across all trees. This neutral model was produced in particular to evaluate whether the pattern in the observed SD variogram could be replicated by a simple stochastic model of fire spread, and

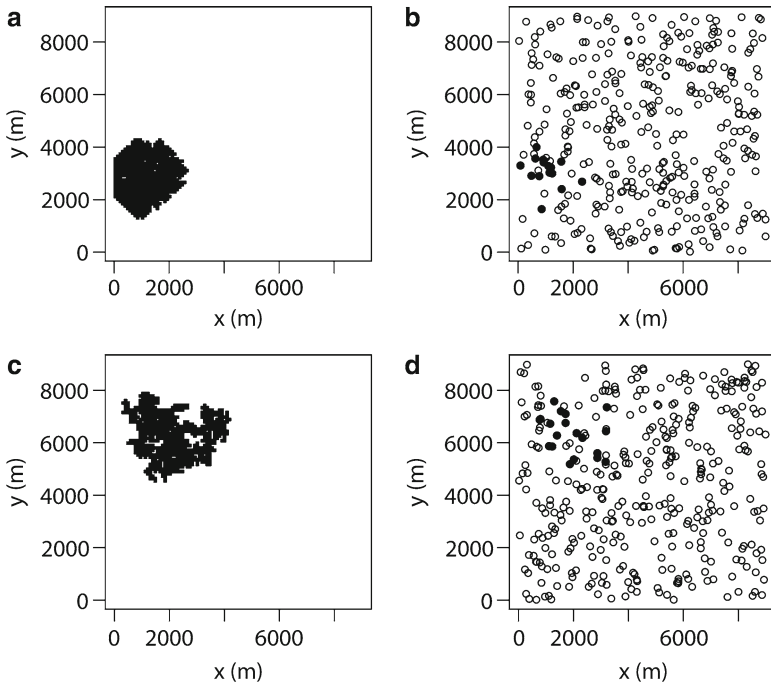


Fig. 2.6 Fire spread for (a) $p_{\text{burn}} = 0.75$ and (c) $p_{\text{burn}} = 0.50$. A complete spatial randomness (CSR) process generates recorder trees (points), with trees scarred by associated fire (black-filled points in b and d). A higher p_{burn} yields a more regular fire shape, although the difference in fire shape is difficult to discern visually in the scar pattern

to explain what differentiates variograms that appear linear in log-log space from those that do not. In order to satisfy the second goal, we considered whether the value of Sørensen's distance between two trees could be predicted by features of the neutral model.

2.5.2 Prediction of Sørensen's Distance

The Sørensen's distance can be analytically derived from conditional probabilities associated with fire spread and the scarring of recorder trees. Within the context of this neutral model, and under several assumptions verified by simulation, Kennedy and McKenzie (2010) found that the Sørensen's distance (SD) for a pair of trees a given distance apart is predicted by two features of the neutral model. The first is the probability a tree in a burned pixel is scarred (p_{scar} , which is spatially independent), which in the neutral model is constant across all recorder trees in the simulated landscape. The second model feature is the probability that two trees

are both in a burned pixel in a given fire year (but not necessarily the same burned pixel). Specifically, for the pair of trees A and B, we calculate the probability that tree B is in a burned pixel (B_{fire}) given that tree A is in a burned pixel ($P(B_{\text{fire}}|A_{\text{fire}})$). For the stochastic model we consider the expected value of SD, and we found that it is predicted by (Kennedy and McKenzie 2010)

$$E(SD) = 1 - P(B_{\text{fire}} | A_{\text{fire}})^{p_{\text{scar}}} \quad (2.2)$$

The probability the second tree is in a burned pixel given the first is in a burned pixel is not constant across pairs of trees, as it depends on the distance between the two trees, the fire size, and fire shape (Fig. 2.7).

As the distance between two trees approaches 0, then the conditional probability the second is in the fire given that the first is ($P(B_{\text{fire}}|A_{\text{fire}})$) approaches 1, and Eq. 2.2 reduces to

$$E(SD) = 1 - p_{\text{scar}} \quad (2.3)$$

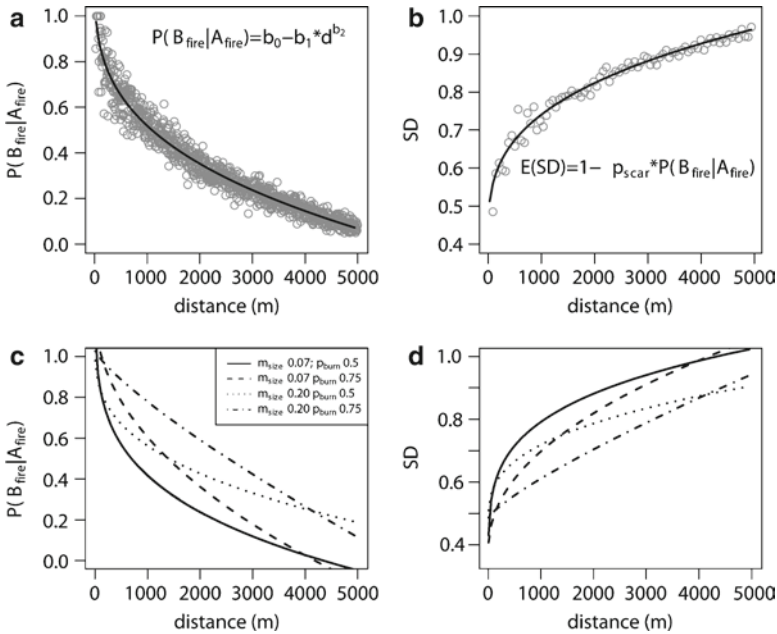


Fig. 2.7 Verification of the derivation of $E(SD)$ via simulation and nonlinear regression. (a) $P(B_{\text{fire}}|A_{\text{fire}})$ with distance (d) predicted by 3-parameter model (neutral model $\mu_{\text{size}}=0.15=1500$ pixels). (b) The fit to $P(B_{\text{fire}}|A_{\text{fire}})$ $\{\hat{b}_0, \hat{b}_1, \hat{b}_2\}$, with the p_{scar} set in the simulation ($=0.5$), used to predict $E(SD)$ and compared to calculated SD variogram from the same simulation (i.e., Eq. 2.5). It fits well. (c) The relationship of $P(B_{\text{fire}}|A_{\text{fire}})$ with distance changes with mean fire size (μ_{size}) and fire shape as modified by the burn probability (p_{burn}); (d) these differences are also shown in changes to the shape of the SD variogram

Therefore, one can estimate the p_{scar} from an empirical SD variogram by the mean SD at the smallest distance bin. Simulations confirmed that the value of p_{scar} would be \geq the mean value at the smallest distance bin.

We used a least-squares nonlinear regression algorithm in the R statistical program (nls; R Foundation 2003) to fit simulated $P(B_{\text{fire}} | A_{\text{fire}})$ against distance (up to half the maximum distance between simulated recorder trees—the same criterion used to evaluate SD), for three candidate functions (Kennedy and McKenzie 2010). The best fit with respect to an information-theoretic criterion (AIC) was found with a three-parameter function:

$$P(B_{\text{fire}} | A_{\text{fire}}) = b_0 - b_1 d^{b_2} \quad (2.4)$$

and, therefore,

$$E(SD) = 1 - p_{\text{scar}} (b_0 - b_1 d^{b_2}) \quad (2.5)$$

The coefficients $\{b_0, b_1, b_2\}$ thereby characterize the change in $P(B_{\text{fire}} | A_{\text{fire}})$ with distance, and consequently the change in SD with distance. The estimates of b_0 , b_1 and b_2 in the neutral model change with increasing fire size, in a manner that depends on the shape of the fire (Fig. 2.7). Fire shape is closely associated with p_{burn} , with lower values of p_{burn} producing more irregular and complex shapes (Fig. 2.6). As the fire becomes larger and more regular, then the relationship between $P(B_{\text{fire}} | A_{\text{fire}})$ approaches a straight line with intercept b_0 and slope $-b_1$, i.e., b_2 gets closer to 1 (Fig. 2.7c; Table 2.1), and the slope (b_1) becomes less negative. In contrast, for irregularly shaped fires characteristic of $p_{\text{burn}} = 0.5$, the decline of $P(B_{\text{fire}} | A_{\text{fire}})$ remains non-linear with estimates of b_2 well below 1 across a range of values for μ_{size} (Fig. 2.7c).

Note also that when $b_0 = 1/p_{\text{scar}}$, a power law describes the SD variogram, because we have:

$$E(SD) = p_{\text{scar}} b_1 d^{b_2}, \quad (2.6)$$

which is the power-law relationship presented in Eq. 2.1.

Recall that the relationship $P(B_{\text{fire}} | A_{\text{fire}})$ is independent of p_{scar} , and values of $\{b_0, b_1, b_2\}$ change with p_{burn} and μ_{size} . It is therefore possible to calibrate the values

Table 2.1 Parameter estimates for neutral model results with varying μ_{size} (0.07, 0.20) and p_{burn} (0.5, 0.75), and for the observed variograms (Twentymile, Swauk). Note that the coefficients b_1 are all negative, also indicated, for clarity, by the minus sign in Eq. 3.4

		b_0	b_1	b_2
μ_{size} 0.07	p_{burn} 0.50	1.430	-0.1990	0.235
	p_{burn} 0.75	1.240	-0.0247	0.469
μ_{size} 0.20	p_{burn} 0.50	1.060	-0.0437	0.351
	p_{burn} 0.75	1.030	-0.0010	0.805
Twentymile	p_{scar} 0.704	0.979	-0.0008	0.788
Swauk	p_{scar} 0.689	1.492	-0.2270	0.195

of μ_{size} , p_{burn} and p_{scar} to make $b_0 * p_{\text{scar}}$ arbitrarily close to 1, and thus manipulate simulated results to produce a power-law relationship in the SD variogram. In the neutral model this is a consequence of the mathematical relationships that we have found, yet the exercise of calibrating the parameters reveals under what conditions, as represented by μ_{size} , p_{burn} and p_{scar} , power laws should be expected. These can then be compared to the patterns observed in real landscapes, and indicate the ecological conditions under which power laws are produced.

The challenge, then, is to evaluate the relevance of the neutral model results for real landscapes insofar as the derived mathematical relationships are able to predict the patterns observed. We fit Eqs. 2.3, 2.5, and 2.6 to the SD variograms of real landscapes on a gradient of topographic complexity; first we estimate p_{scar} as the mean SD at the smallest distance bin in the observed SD variogram, then we fit Eq. 2.5 to the variogram in order to estimate the coefficients $\{b_0, b_1, b_2\}$. Here we compare the two watersheds from Kellogg et al. (2008) that are at opposite ends of this topographic gradient: Twentymile (least complex) and Swauk Creek (most complex). Coefficient estimates are in Table 2.1, and Fig. 2.8 shows the contrasting fits of the SD variograms from Twentymile and Swauk Creek in log-log space.

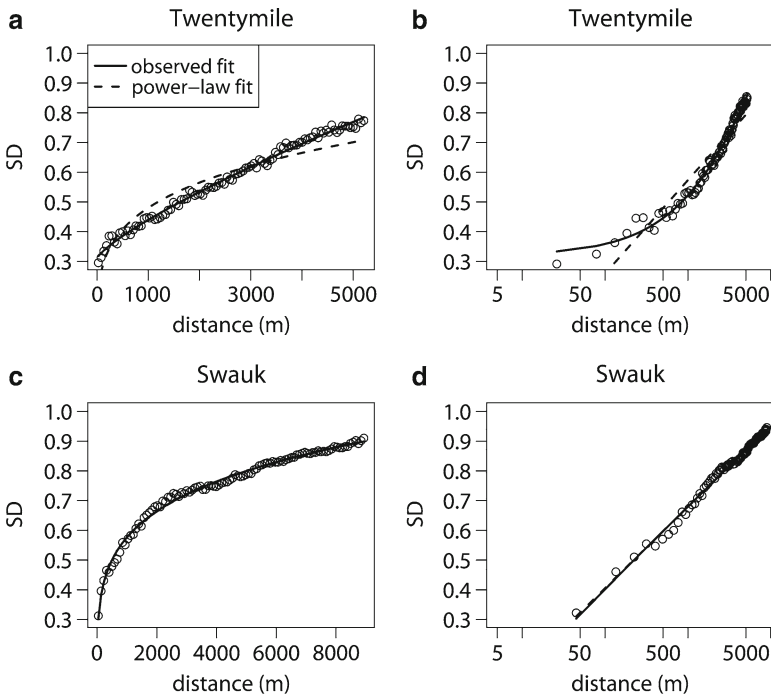


Fig. 2.8 Observed SD variograms for the least (Twentymile; **a,b**) and most (Swauk; **c,d**) topographically complex sites. Swauk increases more rapidly at smaller distances, and reaches a higher value. The Swauk fit is almost indistinguishable from the power-law prediction, with a small separation at the lowest distance bins

Clearly the relationship for Swauk Creek follows a power law ($b_0 * p_{\text{scar}} = 1.492 * 0.689 = 1.028 \approx 1$; Eq. 2.6), whereas Twentymile does not ($0.7 * 0.979 = 0.685$).

These results suggest preliminary support for the hypothesis associated with Criterion #1 (above): Topographic complexity provides a bottom-up control on the spatial patterns of low-severity fire, producing relatively small fires and irregular fire shapes (SD increases more rapidly with distance, and reaches a higher peak, in Swauk Creek than Twentymile; Fig. 2.7). Neutral model runs with $p_{\text{burn}} = 0.5$ (irregular fire shapes; Fig. 2.6a) and relatively small mean fire sizes produced coefficient estimates similar to Swauk ($\{b_0, b_1, b_2\}$; Table 2.1) and SD variograms that followed power laws with p_{scar} near that estimated for Swauk. In contrast, neutral model runs with $p_{\text{burn}} = 0.75$ (regular fire shapes; Fig. 2.6c) and larger mean fire sizes produced coefficient estimates and SD variograms similar to those from Twentymile (Table 2.1).

What do we gain, then, by deconstructing these scaling laws via simulation; e.g., can we back-engineer a meaningful, preferably quantitative, description of fire regime properties that is relevant for landscape ecology and fire management? Certain combinations of the probability of scarring, the probability that a cell burns given that a neighboring cell has burned, and the mean fire size produce power-law behavior in an aggregate measure—the SD variogram—that represents the spatial autocorrelation structure of fire occurrence. For example, a low probability of scarring suggests variable fire severity at fine scales. A moderate likelihood of a cell's burning given that its neighbor has burned (i.e., $p_{\text{burn}} = 0.5$) suggest fine-scale controls on fire spread (topography and spatial heterogeneity of fuels). Mixed-severity fires subject to fine-scale landscape controls over time (decades to centuries) engender complex patterns that nonetheless produce simple mathematical structures (power laws). Further simulation modeling such as we describe here should illuminate what additional structures and scaling relationships can arise from the universe of complex interactions between the contagious process of fire and landscape controls.

2.6 Conclusions and Implications

Scaling laws in fire regimes are one aggregate representation of landscape controls on fire. Cross-scale patterns can reflect landscape memory (Peterson 2002). For example, fire-size distributions on landscapes small enough for fires to interact hold a memory of previous fires (Malamud et al. 1998; Collins et al. 2009), as do shape parameters of the hazard function on landscapes in which fuel buildup is necessary to sustain fire spread (Moritz 2003; McKenzie et al. 2006a). Scaling laws in our SD variograms hold a memory of all historical fires registered by recorder trees. We have conjectured above that scaling laws arise when bottom-up controls are in effect, but an additional possibility is that scaling relationships may be non-stationary over time, reflecting changes or anomalies in top-down controls, specifically climate (Falk et al. 2007). Mean fire size, fire frequency, and fire severity change

with changes in climate and land use (Hessl et al. 2004; Hessburg and Agee 2005; Littell et al. 2009). A rapidly changing climate may at least change the parameters of scaling relationships, such as exponents in power laws derived from frequency distributions, and at most make them disappear altogether. Such behavior could indicate that a fire-prone landscape had crossed an important threshold (Pascual and Guichard 2005), with implications for ecosystem dynamics and management.

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